A Review Study on Flexible Link Manipulators

Esmail Ali Alandoli[1], Marizan Sulaiman[1], M.Z.A. Rashid[1], H.N.M Shah[1], Z. Ismail[2]
[1]*Faculty of Electrical Engineering, Universiti Teknikal Malaysia Melaka,*
Hang Tuah Jaya, 76100 Durian Tunggal, Melaka, Malaysia.
[2]*Faculty of Electrical Engineering, Universiti Teknologi Malaysia,*
Jalan Sultan Yahya Petra (Jalan Semarak),
54100 Kuala Lumpur, Malaysia.
zamzuri@utem.edu.my

***Abstract*— This paper presents a review on various studies of flexible link manipulators which cover mathematical modeling and control of single link, two link and multi-link manipulators. This review has shown that the effectiveness of some controls such as Model Predictive Control (MPC) scheme alone and combined with Piezoelectric actuators (PZT), combination of robust nonlinear and fuzzy compensator, employing piezoelectric actuators by taking into account their position along the link, linear quadratic regulator (LQR), and fuzzy logic controllers. Based on the reviews, these control approaches are better than other control schemes to control the flexible link manipulators and vibrations suppression.**

***Index Terms*— Control of flexible system; Flexible link manipulator; Mathematical modeling.**

I. Introduction

Robotic manipulators are highly demanded to work in dangerous, routine, and difficult jobs instead of human in order to achieve accurate, faster, and economical operations. Traditional solution to avoid the end effector vibration and achieving good position accuracy is using heavy material and bulky designs to achieve a high stiffness for robotic manipulators. The high power consumption, low speed, danger, and less productivity are the disadvantages of the heavy rigid manipulators used of the most existing robotic manipulators [1].

In order to build modern robotic manipulators to satisfy the needs of industrial applications, it is important to ensure by decreasing the weight of the link manipulators and utilize flexible materials to satisfy the special needs of industries. Significant control problems appear in flexible link manipulators such as vibration, and static deflection from external effects, and designing errors. These factors can decrease the end effector accuracy, increase settling time, and complicate the controller design scheme. Currently, flexible link manipulators have been designed and have the following advantages: (i) greater the ratio of payload weight to robot weight, (ii) use of less powerful actuators which reduces the energy consumption, (iii) cheaper construction, (iv) faster motion, (v) safe operation ([2]and [3]).

Modeling, analysis, and control of the flexible mechanism started in 1970s with a great effort. The modeling and controlling the vibrational phenol- mena characterizing flexible mechanisms allow engineers to design and build lighter robot manipulators which would offer the demanded advantages [4].

Various fields request for flexible manipulators, but controlling and suppression the vibration of the flexible link manipulators are still considered huge challenges [5].The problem of controlling robotic mechanisms which are attached with flexible link have been investigated by control engineers and roboticists for nearly a decade [6]. As reported in [7], the control difficulty is caused by the fact that since the manipulator is a distributed system, a large number of flexible modes are required to accurately model its behavior. Further, complications arise because of the highly nonlinear nature of the system.

Due to the advantages of flexible link manipulator, it is suitable to be applied in engineering applications such as space arm manipulators, telesurgery operations, aerospace industry, and safely operates near to human due to their flexibility and lower inertia ([8] and [9]).

II. Modeling Of Flexible Link Manipulator

This section describes various modeling techniques and grouped depending on the number and type of the links.

A. Single link manipulator

In this part is the single link manipulator modeling is reviewed.

The Euler–Bernoulli beam theory and the assumed modes method are used by Geniele et al. [7] to express the deflection of a point located at a distance x along the arm, and to derive the equation of motion. The position of any point along the link is defined by Figure 1 and by Equation (1). By assuming that the cross-sectional area of the link is small compared to its length h for a cantilever beam, the effects of rotary inertia and shear deformation are ignored.

$$y(x,t) = x\theta(t) + w(x,t) \tag{1}$$

Andres San-Millan et al. [9] defined the tip angle of a single link manipulator with a payload by Equation (2).

$$\theta_t(t) = \theta_m(t) + \frac{w(L,t)}{L} \tag{2}$$

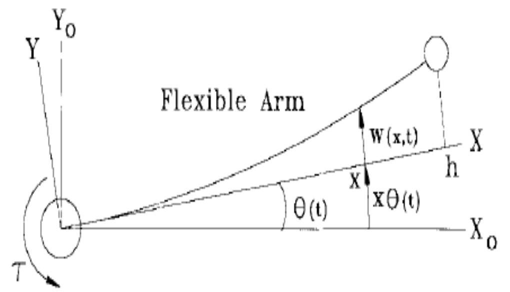

Figure 1: Single link manipulator [7]

where, $w(x,t)$ is the elastic displacement measured from the rigid beam at the point x in the time instant$t$, and $\theta_m(t)$ is the motor angle. The time derivative of the tip angle defines the tip speed, Equation (3).

$$v_t(t) = \dot{w}(L,t) \tag{3}$$

Raouf Fareh et al. [10] derived the equation of motion of an n-degree of freedom link manipulator using Lagrangian formulation. Computed torque method, and adaptive control are used in this investigation. In [11], the dynamic equations for a flexible link manipulator are derived by Alaa Shawky et al. on basis of Lagrange's equations of motion. Two sets of equations are obtained, the rigid body degree of freedom is the first set and the elastic degrees of freedom are the second set.

Farruggio and Menini [12] obtained the finite dimensional approximate model using the Eigen functions of the beam instead of an arbitrary complete set of functions. In general, it is quite easy to reproduce satisfactorily the resonances of the exact transfer function, but it not easy to obtain its zero.

The total displacement $y(x,t)$of any point along a flexible link manipulator at a distance x from the hub is a function of both the rigid body motion $\theta(t)$ and elastic deflection $w(x,t)$ as expressed in Equation (1) [13]. In [13] Mohammad Khairudin et al. derived desired dynamic equations of motion using the finite element method and Bernoulli-Euler beam theory.

In [14], Kerem Gurses et al. used the finite element method to model a flexible link manipulator as a slender flexible beam rigidity attached to a rotating hub, the flexible link considered as a piezo-beam element because it is bonded by PZT patches as vibration control actuators.

Dong Sun et al. [15] derived the complete system equation of the motion for a single link flexible manipulator as a uniform cantilever beam with a tip mass and represented the beam deflection using the assumed mode model.

B. Two link manipulators

Vakil et al. [16] derived the dynamic model for planar Flexible-link Flexible-joint Manipulators of two link manipulators by combining the assumed mode and the Lagrange's equation with taking in account the tip mass and mass moment of inertia. In [17], authors used partial differential equation (PDE) to describe the dynamics of structural flexibility, and ordinary differential equation (ODE) to represent the rigid dynamics. This type of modeling is convenient for observer and control design. Zhang Chunyi and Bai Guangchen [18] used Monte Carlo method to obtain the reliability of the dynamical strength and the dynamic stiffness for two flexible link manipulators. The link manipulators are assumed as a homogeneous Euler beam and the dynamic equations are derived based on the Lagrange's equations. In [19-21], Linjun Zhang and Jinkun Liu used partial differential equation (PDE) dynamic model, they derived the dynamic model that consists of PDE governing equations and ODE boundary conditions, and this model derived reduces the modeling complexity as it is in an identical structure and simple boundary conditions.

Jimoh O. Pedro et al. [22] derived the dynamics equations of motion for planar two-flexible link manipulators using the Lagrangian approach and the actuators dynamics are included in the modeling, the links are assumed as Euler-Bernoulli beam. The PDE (equation of motion) is reduced into ODE using assumed mode method.

Cao Qingsong et al. [23] established a coupled dynamic model by calculating generalized force, kinetic energy, potential energy (the deformation, electric potential energy of piezoelectric element) of each smart link to describe two-link piezoelectric flexible manipulators based on assumed mode method and Lagrange equation. The authors used modal function to obtain the stress of surface layer of the second flexible link in order to determine the optimal sensor/actuator placement on the second link.

Authors in [24] modeled two flexible link robot manipulators as Euler-Bernoulli beams using Lagrange and assumed mode method. The authors emphasized that the most method used to reduce the PDE to ODE is the assumed mode method, and Lagrange approach is adequate to describe the system dynamics.

Tahmina Zebin and M. S. Alam [25] derived a mathematical modeling for two flexible link manipulators by utilizing the finite element method and Lagrangian approach. Initially, the authors derived a mathematical model for a single link and the extended the modeling for multi-links. They combined the kinetic and potential energies to derive the system dynamic model, and the overall model is represented in a state-space form. In [26], authors studied two joint robot arms using Lagrange equation, and they derived the kinetic equations of the model.

C. Multi-link flexible manipulators

This part of the review focuses on the flexible manipulators with more than two links.

Luo Jian et al. [27] expressed the deflection of a n-link flexible manipulator using series finite terms and Euler-Bernoulli beam model. The Lagrangian method is applied for the dynamic system to obtain the dynamic equation for general n-link flexible manipulator. Singular Perturbation theory is used as a model reduction to divide the couple system to two time-scaled subsystems; the slow varying system and the fast varying system. The authors reported that the finite element method and the modal expansion method are the most popular methods used for the purpose of modeling flexible link manipulators.

Meldrum et al. [28] modeled a multi-link flexible manipulator as a series of finite elements and presented an inverse Jacobian solution. This inverse Jacobian solution obtains a quick endpoint control. Madani and Moallem [29] employed the Lagrange equation to derive the equation of motion for a three linkages rigid links and the flexible link is connected to the three linkages rigid links. The manipulator only moves in the XY plane and consists of eight passive and four active revolute joints connected by 10 rigid links to form the three linkages. The forward and inverse kinematics equations of the manipulator are obtained. David Wang et al. [30] used the 4×4 homogeneous transformation matrices of Denavit and Hartenberg method to derive the dynamic equations for a class of multilink manipulators which consists of elbow manipulator and five bar linkages manipulator with the last flexible link. The authors in [30] simplified the dynamic equations by neglecting small elements of the inertia matrix and recalculated the dynamic equations; this is because the simpler equations are desired for the purpose of control.

Pritam Kumar Sarkar et al. [31] derived the dynamics and the resulting equations of motion based on finite element method using Lagrange's equations for multi-links flexible manipulators, the links are considered as Euler-Bernoulli beams.

III. Control Of Flexible Link Manipulator

This section of review covers some control methods for the purpose of regulating the motion of manipulators and suppression the vibration of flexible link manipulators.

A. Single flexible-link manipulator control review

Single flexible link manipulator is easier in controlling the motion and vibration suppression due to less degree of freedom.

For the purpose of keeping the rotation angle of the single-link at desired position and to eliminate the oscillation angle of end effector, Ismail H et al. [2] designed PID and State Feedback control scheme. They compared the experimental results for the two control methods and the PID is more satisfactory due to better step responses and simpler structure. The authors continued their study and designed fuzzy logic control [3]. The experiments of FLCs that are implemented show robust to internal and external effects, and show efficient performance. Overall, the performance of the FLCs is quite good.

Boscariol et al. [4] introduced the Model Predictive Control (MPC) scheme to control the position and vibration of a single flexible-link, the prediction ability of the MPC control is very effective for vibration reduction in fast dynamic system so this advantage motivated the MPC control. The authors proved that the MPC control is very effective compared to a standard PID control. Before this, Hassan et al. [32] presented MPC strategy using piezoceramic actuators in order to suppress the vibration of a single flexible-link having multiple inputs and multiple outputs. The authors studied four control schemes; the first scheme only joint angle is controlled, the second scheme suppresses the beam vibration by motor only, the third scheme suppresses the beam vibration by piezoelectric actuators only, the fourth scheme suppresses the beam vibration by both the motor and piezoelectric actuators. The combination of piezoelectric actuators and strain gauge sensing demonstrates good control performance in vibration suppression.

Sang-Myeong Kim et al. [5] illustrated non-collocated vibration control using servo motor and an accelerometer attached to the tip of the single flexible-link manipulator. In spite the actuator and the sensors are not collocated the experimental results of the controller show that this method is effective in non-collocated control of a flexible link manipulator. The authors in [7] focused on the tip position control of a single flexible link manipu- lator. They designed a nonminimum phase system consists of a part which has a feed through term assigns the transmission zeros of the system at desired locations in the complex plane, and a feedback term which moves the poles of the system to appropriate locations in the left-half plane. Also the system has second part which is a loop of the feedback servo to allow tracking of the desired trajectory. This system revealed a significant impr- ovement for the tip position control.

Hamed Ghazavi Khorasgani et al. [8] applied a robust nonlinear approach to ensure the system stability and reduce the degrading effects of uncertainties for trajectory control of single link flexible manipulator. They employed a fuzzy compensator to solve the problem of a significant difference between a tip position and a new defined output .The combination of the proposed control strategy has demonstrated the effectiveness of trajectory tracking, but there was a big error at the beginning time showed in the experimental results due to mechanical delay and motor saturation.

A sliding mode technique is used in [10] to establish two nonlinear controllers computed torque and adaptive control for a single flexible link manipulator. This control is more capable to suppress the vibration than the PD control, and it achieves a good precision. The authors in [11] developed a nonlinear State Dependen Riccati Equation (SDRE) to control the tip position of one link flexible manipulator with a payload at the end-effector. The results of this model showed it is better than PD is near to optimal and robustly.

Mohammad Khairudin et al. [13] based on the linear quadratic regulator (LQR), they developed a robust control of a single flexible link manipulator to solve flexible link robustness and input tracking capability of hub angular position. A satisfied performance is achieved compared with PID controller.

Subhash Chandra et al. [33] compared the pole placement method and linear quadratic regulator (LQR) technique in decreasing the vibration and increasing the system stability of a flexible single link manipulator. The results and based on the plotted graphs the linear quadratic regulator showed a better stability than state feedback control scheme.

Kerem Gurses et al. [14] introduced a combined scheme of PD-based hub motion control and a series of PZTs bounded to the surface of a single link flexible manipulator to actively suppress the vibration of the flexible link manipulator. The authors used a unique fiber optic curvature sensor array, named "ShapeTape" for the purpose of providing linear and angular velocity feedback which helps in vibration control that the experimental results have showed its effectiveness. This approach motivated by [15], the authors used the same combination of control scheme to analyze the stability and the vibration damping. The vibration of the link could be suppressed effectively with the proposed control scheme.

B. Two flexible-link manipulators control review

In this section there are briefly reviewed some control techniques that have been studied to control two flexible link manipulators.

Vivek G. Moudgal et al. [6] employed fuzzy logic controller to suppress links vibrations of two flexible link manipulators. The fuzzy logic contr-oller actively suppresses the vibrations as have been proven by the results, and it performs much better than PID controller. Linjun Zhang and Jinkun Liu [19] designed an observer and controller based on the partial differential equation (PDE) in order to reach a desired tip position and eliminate the vibrations. This work needs further study to enhance the effectiveness of the proposed approach. Later on, [20] the authors designed optimal boundary control based on the partial differential equation (PDE) in two steps due to the difficulty of the PDE systems. First, apply the differential evolution algorithm to produce the optimal trajectory that reduces the total energy consumption. Then, proposed a boundary control scheme to regulate the joints along the optimal trajectory and suppress the vibrations. The validity of the effectiveness of the proposed control scheme is appeared by the simulated results. Continuing for well achieving the same objectives with same authors [21], they designed an adaptive boundary control scheme based on PDE model for the two flexible link manipulators with changeable payload at the end-position. The numerical simulations verified the effectiveness of the control technique.

A hybrid control scheme consists of neural network based model predictive controller (NNMPC) and PID controller is illustrated by Jimoh O. Pedro and Thando Tshabalala [22] which employed for motion tracking and vibration suppression respectively with taking into account payload variations. The neural network started with the size of 10 hidden layers and increased by 10 for five times. The proposed hybrid control scheme has performed much better than hybrid PD/PID controller as reported in [24]. However, the network takes too long to be trained with the increment of the hidden layers and there was no signification improvement in its performance.

Cao Qingsong and Yu Ailan [23] aimed to obtain the optimal location of the piezoelectric actuator/sensor for two-link piezoelectric flexible manipulator to suppress the links' vibrations using the fuzzy self-tuning PID controller and the stress of structure surface. The fuzzy self-tuning PID controller suppresses effectively the vibration and the accurate location is determined in the second link with a maximum strain ($X_2 = 0$), second maximum strain($X_2 = 0.5l_2$), and third maximum strain($X_2 = 0.7l_2$) . A fuzzy logic control is optimized by adding Genetic Algorithm (GA) as reported in [25] by Tahmina Zebin et al. The proposed control method reduces a noticeable amount of vibration with less overshoot, rise time, settling time, and steady state error.

Xiaoguang et al. [34] illustrated a PID controller with PB-ANN feed forward control to easily suppress the elastic vibrations for two-links manipulator moving in vertical planar and to control joint trajectory tracking. Based on the simulation results, the proposed technique can restrain the vibrations and track the tip trajectory of manipulator with a slight error.

C. Multi-link flexible manipulators control review

This section discusses three and above Multi-link flexible manipulators. Controlling the multi-link flexible manipulator is much difficult that the single or two link flexible manipulators due to the increment of the degree of freedom.

Luo Jian et al. [27] proposed an adaptive RISE control algorithm for multi-link flexible manipulator. The authors proposed the gain-varying RISE control algorithm to achieve trajectory tracking as a slow-varying subsystem and a sliding mode control method is applied to restrain the vibration as a fast-varying subsystem. The proposed algorithm is not enough confident as the results are only simulated for a single-link flexible manipulators to show its performance.

Madani et al. [29] developed and implemented a hybrid position/force control algorithm for a closed chain planar manipulator with a last link flexible to investigate simultaneous position and force control. The position and force control is decoupled by a selection matric, position control is controlled by PID controller and the control performance has been enhanced by computed torque control. Errors in position and force are not fully eliminated and it increase for faster position trajectories and larger forces.

Ge et al. [35] used an integral sliding mode controller with piezoelectric patches to control the angular displacement and overcome the effect of the parametric variations and external disturbance forces. The effectiveness of the proposed control scheme is shown by the simulation results and for more accurate results more piezoelectric actuators should be employed on the links, but increasing the number of piezoelectric actuators effects in weight and flexibility of the beam.

Malzahn et al. [36] designed a robust control scheme combined with linear control and feed forward input shaper for a 3 DOF flexible link robot subject to deflections caused by gravity. For the purpose of positioning control and vibration suppression, this combined technique proved its ability to almost damp the oscillation and accomplish fee end-effect positioning.

IV. CONCLUSION

This review has briefly discussed the modeling for single, two, and multi flexible link manipulators respectively. Furthermore, controlling the flexible link manipulators is the main contribution of the current review. To briefly conclude the better control methods in his review, comparing the MPC scheme combined with PZT actuators and without using the piezoelectric actuators, with PZT actuators is better in performance but more complicated control scheme. Fuzzy logic controllers with more hidden layers take too long to train the network which limits increasing the hidden layers and no more improvement in its performance. Increasing the number of PZT actuators bounded along the flexible link manipulators influence the flexibility of the robot arm and increases its mass. Finally, we recommend for a future study and investigation the linear quadratic regulator (LQR) combined with PZT actuators to solve the problem of increasing the piezoelectric actuators which will effectively restrain the vibrations and control the tip position of flexible link manipulators.

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