An Overview on Network Diagrams: Graph-Based Representation

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***Abstract*— Graph theory is an important area in mathematics. A network is a graph-based representation which represents a problem as a graph to provide a different point of view to the problem. A problem is much simpler when it is represented as a graph since it can provide the appropriate tools for solving the problem. Hence, graph or network acts as an excellence modeling tool in representing several fundamental issues in network such as connectivity, routing, data gathering, mobility, topology control, traffic analysis, finding shortest path and load balancing. In this regard, this paper first presents concepts of graph theory and their associated applications in various networking field. Subsequently, relevant applications of graph-based representation in technological fields are focused and discussed.**

***Index Terms*— Graph theory; Network; Graph-based representation; Technological networks.**

I. Introduction

Graph theory has wide real life applications in the field of networking. Two main areas are involved in analyzing graph theory application in networking, which are graph based representation and network theory. A graph has two components which are nodes and edges. In a graph based representation, these components have natural correspondences to the problem elements. In general, nodes in a graph represent entities and edges represent interactions between entities. Most of the complex systems are graph-like problems, such as transportation network, business ties of an industry in a country, genetic interaction network and internet. On the other hand, network theory works as a tool that provides a set of techniques for analyzing a graph and applying network theory using a graph representation. In this paper, applications in networking field will be focused and discussed.

There are two different ways for graph theory to be used in generalization of networks. First, graph theory is used to derive quantitative measures of topological characteristics of nodes and arcs [1-2]. Second, it is used to identify the important topological information and represent it as a network. This requires a rich understanding of relationships between objects and sets of objects according to their relevance, context and proximity to salient features [3].

II. Methodology

Our world is a connected world where all sorts of complex systems in our lives, ranging from interaction between people, economic of a country, information networking and molecules within living creatures can be displayed as networks. Networks are systems which connecting elements or components. In mathematical language, graph theory is the name used to describe networks. The first use of network can be credited to Leonhard Euler where he has successfully translated the transportation problem of the seven bridges of Konigsberg into a network in 1736. This is followed by electrical networks which was developed by Kirchhoff in 1847, organic chemistry by Cayley in 1857 and puzzles by Hamilton in 1857 [4]. Since then the use of graph theory has been extended to many fields of study such as computer science, physics, ecology and telecommunication where random, small-world and scale-free network models were introduced and implemented.

III. Some Types Of Networks

As discussed earlier, a network is graph with a set of entities which commonly known as nodes or vertices connected by links or edges. Networks of relationships appear in various shapes and sizes, there is no single way of representing networks that encircles all applications. There are various systems taking the form of networks that can be found in the real world applications. In general, there are four categories of networks diagram for graph-like representations which will be discussed in more detail in the following sections.

A. Social and Economic Networks

Social networks represent a set of people or groups of people with certain interaction characteristics between them. For example, Facebook, LinkedIn, scientific collaboration network, business ties in a particular industry and labor markets. It permeates our social and economic lives. In this respect, social networks play a crucial role in the transmission of information about job opportunities, and are critical to the trade of many goods and services. They are the basis of the provision of mutual insurance in developing countries. Social networks are also important in determining which products have high demands, how people vote, how diseases spread among human or animals, how much education people obtain, and how one's likelihood succeeding professionally [5].

B. Information Networks

An information network is generally used to represent the connections between "information" objects. In 2010, Easley and Kleinberg [6] suggested that the information we deal with on-line has a fundamental network structure. For example, links among Web pages in World Wide Web show how these pages are related, how they are grouped in different communities and which pages are the most prominent. Next example of information network is citation. In research world today, citation is a reference that allows authors to quote the source used in a formal academic article. It gives readers the key information to locate those sources. As papers and citations represented by nodes and edges, it forms a network of citations between academic articles. Another good example of information network is semantic. Semantics appear as an information network as well since it shows the relationships between words.

C. Biological Networks

Networks are found in biological systems from food webs in ecology to various biochemical nets in molecular biology. Complex networks are best illustrated with interactions between genes, proteins and metabolites in a cell. In biological networks, the most common network is the metabolic network. The complementary representations of a metabolic network is called a substrate graph where its vertex set consists of all chemical compounds (known as substrates) that occur in the network. Two substrates are adjacent if they occur in the same chemical reaction [7]. Protein interactions are another popular research area in cellular network. It can be considered as nodes and the links represent the physical interactions or binding [8]. This is similar to protein domain networks where protein domains and their co-occurrence in proteins are considered as nodes and links.

D. Technological Networks

Technological networks are physical networks that form the backbone of modern technological societies. They are designed typically for distribution of a commodity or service. These includes infrastructure networks such as internet, telephone networks, power grids, transportation networks, distribution networks, and also temporary networks such as ad hoc communication networks, sensor networks and autonomous vehicles.

Internet is the massive network of physical data connections between computers and related devices. It is a packet switched data network, where the message is decomposed into small chunks of data, called packets, sent separately over the network and reassembled into a complete message again at the other end.

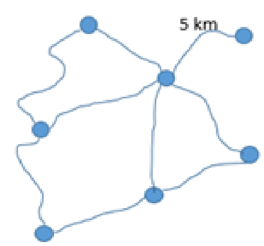
Transportation networks include road and rail networks and airline routes while distribution networks include oil and gas pipelines, water supply network, sewerage lines, mailing and courier delivery routes. In transportation network, route and junction between routes are commonly treated as edge and node, respectively. It is similar to distribution network where supply line and interconnection of supply line could be represented by edge and node, respectively.

A telephone network is a communication network used for phone calls between two or more parties. It is made up of landlines and wireless links that transmit telephone calls. Power grids are referring to the transmission system for electricity. Power providers and consumers that are connected by transmission and distribution lines are synchronized into a network operated by control centers.

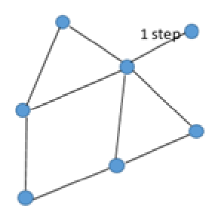
IV. Application Of Technological Networks

A. Transportation Networks

In the modern world, network not only refer to internet connection, it also refers to routes to move people, transport goods, communicate information and control the flow of matter and energy. For instances, roads, railways, cables and pipelines are frequently represented and analyzed as a network. Therefore, cost and time required to solve a network problem are depending on the complexity and type of the network. In 1995, Thomson [9] stated that graph representations offer a convenient way in handling the topological and associated information describing a road network. Figure 1 shows how the real world road system can be transformed into topologic geometric system. The common graph techniques such as the shortest path between network nodes and spanning trees, are sufficient to provide an optimal solution to the problem. Spanning trees can be used in addition to maintain connectivity between destination points during attenuation.



Real world distance



Topologic (geometric) distance

Figure 1: Real world system and topologic geometric system

There are many well-known transportation and network problems had been proposed and solved by mathematicians using graph theory approach in history. For example, the Konigsberg bridge problem, the Chinese postman problem, the travelling salesman problem, the maximum flow problem and the minimum cost flow problem [4].

Planning efficient routes is essential in business industry. In courier service industry, finding the shortest path to deliver product to end users is a big challenge to them. These involve shortest path problem to find a path between two vertices (or nodes) in a directed weighted graph in such a way the sum of the weights of its constituent edges is minimized. There are plenty of algorithms to be applied in solving shortest path problem in graph based network system. These algorithms aim to reduce the complexity of network path, cost and time to build and maintain the network system. Sadavare and Kulkarni [10] had reviewed most of the recent works carried out by researchers in finding shortest path in solving transportation and network problem, for instance Dijkstra's Algorithm, Bellman ford Algorithm and Warshall's Algorithm by considering network base systems in Cable network (Telephone cabling, Electricity power supply network) and water supply system network.

B. Communications Networks

In this technology world, people are connected by all kinds of communication devices even they are thousand miles apart. All these communication interactions and processes can be modeled and represented by directed graphs. A communication network is made up of terminals, links and nodes which enable communication between users of the terminals [11]. In general, terminals which are represented by nodes are the input and output points of a network while an edge or link represents transmission channel which help in data transmission. Each terminal has a unique address so that messages can be routed to the correct recipients. The communicated data is in the form of a packet that consists of a fixed amount of data and a collection of data form a message, such as a webpage.

A communication network can be modeled and presented in different mathematical structure where congestion, switch size and switch count are used to compare the pros and cons of the network representation. Congestion of a set of paths is the largest number of paths that pass through a single switch. A lower congestion is preferable since packets can be delayed at an overloaded switch. Switch size refers to the number of incoming edges and outgoing edges while switch count is the number of switch in the network. One of the graph-based representations for communication network is complete binary tree. Figure 2 shows the complete binary tree representation of a communication network where the squares are terminals, and the circles are the switches. The unique path from input 1 to output 3 is shown in bold.

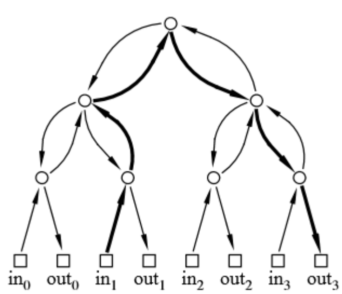


Figure 2: A communication network represented by a complete binary tree [12]

In a wireless cellular network, channel assignment is a discrete-time event process that requires the precise mapping of the channels to the caller-receiver pairs. A dynamic technique is proposed [13] where the graph is transformed into a single-row network [14-15] and the pins in the network represents the channels. This transformation works as a part of the circuitry in the multi-switching center.

Besides the representing of message transmission in communication by using a network, the cost of a telecommunications networks can also be estimated by using the fractal structure of the human population distribution. Sierpinski triangle may represents an idealized fractal population distribution. Figure 3 shows a simple network with a single node that serves the whole population. The optimal location for the node is at the middle of population with minimum total link cost.

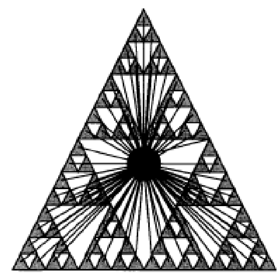


Figure 3: Sierpinski population served by single node [16]

Appleby [16] focused on the telecommunication networks of a star topology. The author exploited the strong-scale invariant structure present in the spatial distributions of the populations of Great Britain and the United States and they found that it can be related to the cost of a star network to serve those populations.

C. Supplying Networks

Given the complexity of the power distribution network and the sheer amount of networks' topological data, the distribution network operators are facing great challenge in analyzing and predicting the performance of the future network development. In addition, the uncertainties involved in the power distribution networks such as the area types, load segments, network service connection and etc have led to the idea of utilizing fractal technique to model power distribution networks particularly at low voltage level.

In this respect, S. A Smith [17] has pioneered the work on generic network generation using fractal theory [18]. The aim is to provide a tool that can be used to investigate network design policy by computer modelling on a reasonably large number of statistically networks. More importantly, it has the capability to resemble different area types such as rural, urban or mixed areas. S. Curcic [19] C. K. Gan [20] has extended his work to incorporate load flow analysis and performed optimal network design. Similar idea has been extended to upper voltage levels by utilizing grid-matrix approach as proposed in [21]. Figure 4 and Figure 5 show the urban and rural networks generated by fractal model using two seed numbers, respectively.

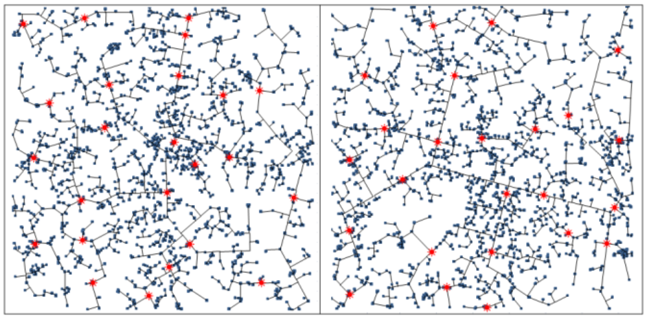


Figure 4: Statistically similar urban networks with two different seeds [20]

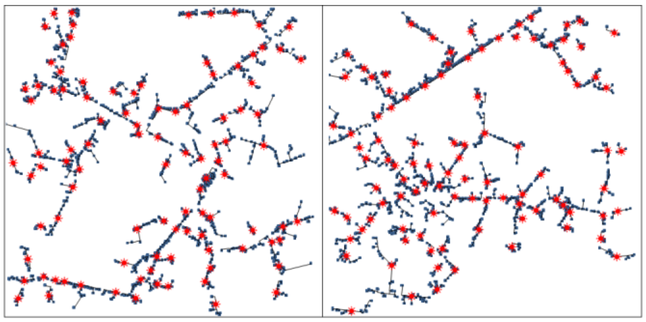


Figure 5: Statistically similar rural network with two different seeds [20]

V. CONCLUSION AND SUGGESTIONS

As a conclusion, graph theory allows the identification of the topological information from a real life network into a graph-based representation to enable the process of problem solving. Besides, graph theory contributes to the generalization which aims to find the representative characteristics of nodes and arcs or a data set. These lead to a better understanding on network-based problems and followed by the efficient problem solving.

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