Gram-Schmidt Orthogonalisation-based Antenna Selection for Pre-Coding Aided Spatial Modulation

Saetbyeol Lee, Manar Mohaisen
Department of EEC, Korea University of Technology and Education, Cheonan 330-708, Korea.
byeol4632@koreatech.ac.kr, manar.subhi@koreatech.ac.kr

***Abstract*—In this paper, we introduce a computationally-efficient antenna selection algorithm for the pre-coding aided spatial modulation (PSM) that is applicable in both the under-determined and over-determined multiple-input multiple-output (MIMO) systems. The proposed algorithm is based on a modified Gram-Schmidt orthogonalisation, where the optimisation function is the sum of scalars that is computed successively. The proposed algorithm does not only select antennas one-by-one with low computations, but it can also remove one or two antennas per iteration, leading to further reduction in the computational complexity. Simulation results show that the proposed algorithm achieves the optimal diversity with tolerable degradation in the bit-error-rate.**

***Index Terms*—Pre-Coding Aided Spatial Modulation; MIMO; Antenna Selection; Gram-Schmidt Orthogonalisation.**

I. INTRODUCTION

Spatial modulation (SM) is a multiple-antenna technique that uses both a data symbol selected from a constellation set and the index of a transmit antenna chosen from a set of transmit antennas to convey information from a transmitter to a receiver [1]. In pre-coding aided spatial modulation (PSM), the data symbol is pre-coded and the index of a single receive antenna is selected from the available set of received antennas [2]. SM and PSM were extended to the multiple-receive and -transmit antennas cases in [3] and [4], respectively.

With the emergence of new MIMO systems, where terminals and base stations are equipped with a relatively high number of antennas (c.f. [5]), antenna selection becomes an attractive choice in over-determined and under-determined systems. In particular, antenna selection provides gains in the signal-to-noise ratio (SNR) and receiver complexity. The criterion based on which a subset of the available antennas at either the transmitter or the receiver (or both) is chosen depends on the transmitter's and receiver's structure. Maximising the capacity, minimising the symbol error rate or maximising the post-processing SNR at the receiver side are frequently researched criteria for antenna selection [6].

In this paper, we propose an antenna subset selection algorithm for PSM based on the Gram-Schmidt orthogonalisation technique. The proposed algorithm sub-optimally maximises the post-processing SNR, while achieving huge gains in terms of computational complexity. Such gain is contrasted to the optimum selection algorithm and a recently proposed fast antenna selection algorithm due to Zheng [7], where we achieve more than 90% gain of their performance.

In Section II, the system model is introduced and in Section III, the optimal antenna selection method and the fast receive antenna selection is reviewed, and then the QR-based antenna selection (QRAS) algorithms is introduced. In Section IV, the computation complexity of the proposed method is analysed and reported. Then in Section V, simulation results are provided followed by concluding remarks in Section VI.

The following notations are used in this paper. The notations $\mathrm{Tr}(\mathbf{A})$, $\mathbf{A}^H$, and $\mathbf{A}^{-1}$ are the trace, the Hermitian transpose and the inverse of the matrix **A**, respectively. $\sigma_i(A)$ is the i-th singular value of **A**, a_i is the i-th column of **A**, and $\mathbf{A}^{-i}$ is the matrix **A** after removing the i-th column. $CN(\mu,\sigma^2)$ is a circularly-symmetric Gaussian random variable with mean and variance of μ and σ^2, respectively.

II. SYSTEM MODEL AND RELATED WORK

In this paper, we consider a PSM system with $n_T = 2^N$ transmit antennas, for any positive integer N, and n_R receive antennas. Without the loss of generality, we consider an over-determined system, i.e., s.t. $n_R > n_T$. Optimally, the goal is to select the subset of n_T receive antennas that maximises the post-processing SNR. $\mathbf{H} = [\mathbf{h}_1, \cdots, \mathbf{h}_{n_T}] \in C^{n_R \times n_T}$ is the channel matrix whose element $h_{i,j}$, which couples the i-th receive and j-th transmit antenna, is modelled as a circularly-symmetric Gaussian random variable with mean and variance of zero and one, respectively. The pre-coding matrix $\mathbf{P} = [\mathbf{p}_1, \cdots, \mathbf{p}_{n_T}] \in C^{n_R \times n_T}$ is based on the zero-forcing criterion, which is given as:

$$\mathbf{P} = \sqrt{n_T/\gamma}\,\hat{\mathbf{H}}^{-1}, \tag{1}$$

where $\hat{\mathbf{H}}$ is the channel matrix after antenna selection and $\gamma = \mathrm{Tr}[(\hat{\mathbf{H}}\hat{\mathbf{H}}^H)^{-1}]$ is a scaling factor that limits the transmit power to n_T. In this paper, we are concerned with the zero-forcing (ZF) criterion (c.f. [8] for further details on the post-processing SNR of the ZF and the minimum-mean square error (MMSE) criteria). As such, the pre-coded vector is given by:

$$\mathbf{x} = \mathbf{p}_i s_j, \tag{2}$$

where the symbol s_j is drawn from an M-ary quadrature amplitude modulation (M-QAM) set with the power constraint $E(s_j s_j^*) = 1$, for $j = 1, \cdots, M$. In Equation (2), both i and j hold information and the total capacity is therefore equals to $\log_2(n_T M) = N + \log_2(M)$ bits per channel use. The received vector at the n_T selected receive antennas is given by:

$$\mathbf{y} = \hat{\mathbf{H}}\mathbf{p}_i s_j + \mathbf{n}, \tag{3}$$

where $\mathbf{n} \sim \mathrm{CN}(\mathbf{0}_{n_T \times 1}, \sigma_n^2 \mathbf{I}_{n_T})$. The data symbol and the receive antenna index are recovered using the maximum-likelihood estimator as detailed in [2].

Before introducing our proposed antenna selection algorithm, we first review the literature. The review on the optimal antenna subset selection and the greedy subset selection (fast RAS) algorithm due to Zheng [7] has been conducted.

A. Optimal selection.

The optimal antenna subset selection algorithm aims to maximise the scaling coefficient γ; that is:

$$\begin{aligned} A_{opt} &= \underset{A \in \{A_p,\ p=1,\cdots,P\}}{argmin} \gamma\{\mathbf{H}_A\}, \\ &= \underset{A \in \{A_p,\ p=1,\cdots,P\}}{argmin} \mathrm{Tr}\left\{\left(\mathbf{H}_A \mathbf{H}_A^H\right)^{-1}\right\}, \end{aligned} \tag{4}$$

where $P = C_{n_T}^{n_R}$ and $\mathbf{H}_A = [\mathbf{h}_{a_1}^T, \cdots, \mathbf{h}_{a_{n_T}}^T]^T$ with $A = \{a_1, \cdots, a_{n_T}\}$. The optimum antenna subset selection employs a brute-force search over the P possible subsets, which requires P matrix inversions in total.

B. Greedy selection

Zheng [7] proposed a greedy subset selection algorithm (fast RAS) that is based on Equation (4). In fast RAS, antennas are added one by one. This algorithm achieves good performance while reducing the computational complexity.

III. Proposed Antenna Selection Algorithm

Let the channel matrix after antenna selection $\mathbf{H}_A$ be factorised into a unitary matrix $\mathbf{Q}$ and an upper-triangular matrix R. Accordingly, the scaling factor in Equation (4) can be rewritten as Equation (5).

The equality in Equation (5) is satisfied if and only if $\mathbf{H}_A$ is orthogonal and, as a consequence, $\mathbf{R}$ is diagonal. In other words, the maximisation of the diagonal elements of the matrix $\mathbf{R}$ is quasi-equivalent to the maximisation of the singular values of $\mathbf{H}_A$, which is the goal of the optimum antenna subset selection.

$$\begin{aligned} \gamma &= \mathrm{Tr}\left\{\left(\mathbf{H}_A \mathbf{H}_A^H\right)^{-1}\right\} = \sum_{i=1}^{n_T} \frac{1}{\sigma_i^2(\mathbf{H}_A)} \\ &= \mathrm{Tr}\left\{\left(\mathbf{Q}\mathbf{R}\mathbf{R}^H\mathbf{Q}^H\right)^{-1}\right\}, \\ &= \mathrm{Tr}\left\{\left(\mathbf{R}^{-H}\mathbf{R}^{-1}\right)\right\}, \\ &= \sum_{i=1}^{n_T} \frac{1}{R_{i,i}^2} + \sum_{i=1}^{n_T}\sum_{j=i+1}^{n_T} \left\|(\mathbf{R}^{-1})_{i,j}\right\|^2 \\ &\geq \sum_{i=1}^{n_T} \frac{1}{R_{i,i}^2}. \end{aligned} \tag{5}$$

The accuracy of the proposed QRAS algorithm can be further analysed in terms of the orthogonality deficiency measure frequently used in lattice basis reduction. The orthogonality deficiency measure is given by:

$$\begin{aligned} \Omega(\mathbf{H}_A) &= 1 - \frac{\det\left(\mathbf{H}_A^H \mathbf{H}_A\right)}{\prod_{i=1}^{n_T} \|\mathbf{h}_i\|^2} \\ &= 1 - \frac{\det\left(\mathbf{R}^H \mathbf{R}\right)}{\prod_{i=1}^{n_T} \|\mathbf{h}_i\|^2}, \\ &= 1 - \frac{\prod_{i=1}^{n_T} R_{i,i}^2}{\prod_{i=1}^{n_T} \|\mathbf{h}_i\|^2}, \end{aligned} \tag{6}$$

where $(\Omega(\mathbf{H}_A) \geq 0)$ with the equality satisfied in case of orthogonal $\mathbf{H}_A$. Minimising $\Omega(\mathbf{H}_A)$, hence obtaining a more orthogonal $\mathbf{H}_A$ with minimised scaling factor γ, is equivalent to maximising the following:

$$\frac{\prod_{i=1}^{n_T} R_{i,i}^2}{\prod_{i=1}^{n_T} \|\mathbf{h}_i\|^2},$$

Note that $\|\mathbf{r}_i\|^2 = \|\mathbf{h}_i\|^2$ for $i = 1, \cdots, n_T$. That is,

$$\|\mathbf{h}_i\|^2 = R_{i,i}^2 + \sum_{j=1}^{i+1} \left\|R_{j,i}\right\|^2.$$

In each iteration, the column with the maximum square Euclidean norm, which maximises the diagonal element of the matrix $\mathbf{R}$ is selected. Thus, the off-diagonal elements of $\mathbf{R}$ are consequently minimised, resulting in the minimisation of the orthogonality deficiency measured in Equation (6), and hence improving the scaling factor γ. This analysis is consistent with the idea of matrix diagonalisation and block-diagonalisation using Jacobi algorithms—c.f. Equation (2.1) in [10] and the subsequent discussion.

Our QRAS algorithm utilising the ideas highlighted above is shown in Figure 1, and is described as follows:

- At each iteration, the antenna corresponding to the column of $\mathbf{Q}$ with the maximum Euclidean norm is selected.
- The remaining columns of $\mathbf{Q}$ are orthogonalised with respect to the selected column (Lines 15-16).

- The norms of the remaining columns of **Q** are updated in Line 17. Note that $\mathbf{norm}_j$ is either kept constant, if $\mathbf{q}_i$ and $\mathbf{q}_j$ are orthogonal, or is reduced, if these columns are not orthogonal, which is the more probable case.
- In case of large n_R, at each iteration, the m columns with the least norms are excluded in order to reduce the complexity. This is motivated by the fact that the smallest norms vanish as the matrix dimension increases, which is the same behaviour of the minimum singular values since $\lim_{n\to\infty}\mathbf{G}^H\mathbf{G} = n\mathbf{I}_n$ (see also Theorem 5.1 in [9]).
- This process is repeated (n_T-1) times because the orthogonalisation part of the proposed algorithm is not required in the n_T-th iteration.

Input: $\mathbf{R} = 0_{n_R}$, $\mathbf{Q} = \mathbf{H}^T$, $S = [1,2,\cdots,n_R]$, n_T, n_R, m

1: **for** $i = 1$ to n_R **do**
2: $\quad \mathbf{norm}_i = \|\mathbf{q}_i\|^2$
3: **end for**
4: **for** $i = 1$ to n_T **do**
5: $\quad k = \arg\max_{j=i,\cdots,n_R-m(i-1)} (\mathbf{norm}_j)$
6: $\quad$ Exchange columns i and k_i in S
7: $\quad$ **if** $i = n_T$ **then**
8: $\quad\quad A = S_1^{n_T}$
9: $\quad\quad$ **break**
10: $\quad$ **end if**
11: $\quad$ Exchange columns i and k_i in **Q**, **R** and **norm**
12: $\quad R_{i,i} = |\mathbf{q}_i|$
13: $\quad \mathbf{q}_i = \mathbf{q}_i / R_{i,i}$
14: $\quad$ **for** $j = i+1$ to $n_R - m(i-1)$ **do**
15: $\quad\quad R_{i,j} = \mathbf{q}_i^H \cdot \mathbf{q}_j$
16: $\quad\quad \mathbf{q}_j = \mathbf{q}_j - R_{i,j}\mathbf{q}_i$
17: $\quad\quad \mathbf{norm}_j = \mathbf{norm}_j - R_{i,j}^2$
18: $\quad$ **end for**
19: $\quad$ **for** $k = 1$ to m **do**
20: $\quad\quad k_i = \arg\min_{j=i+1,\cdots,n_R} (\mathbf{norm}_j)$
21: $\quad\quad \mathbf{Q} = \mathbf{Q}^{-k_i}$
22: $\quad\quad \mathbf{R} = \mathbf{R}^{-k_i}$
23: $\quad\quad \mathbf{norm} = \mathbf{norm}^{-k_i}$
24: $\quad$ **end for**
25: **end for**
26: $A = S_1^{n_T}$

Output: A

Figure 1: Pseudo-code of the proposed QR decomposition-based antenna selection algorithm (QRAS)

We can clearly see that under-determined systems ($n_T > n_R$) are treated similarly: **H** is assigned to **Q** and necessary changes in qras are performed.

Figure 2 depicts the flowchart of the proposed algorithm. Note that the orthogonalisation of the i-th column of the matrix **Q** is followed by the computation of the i-th row of the matrix **R** in an iterative fashion as depicted in the flowchart.

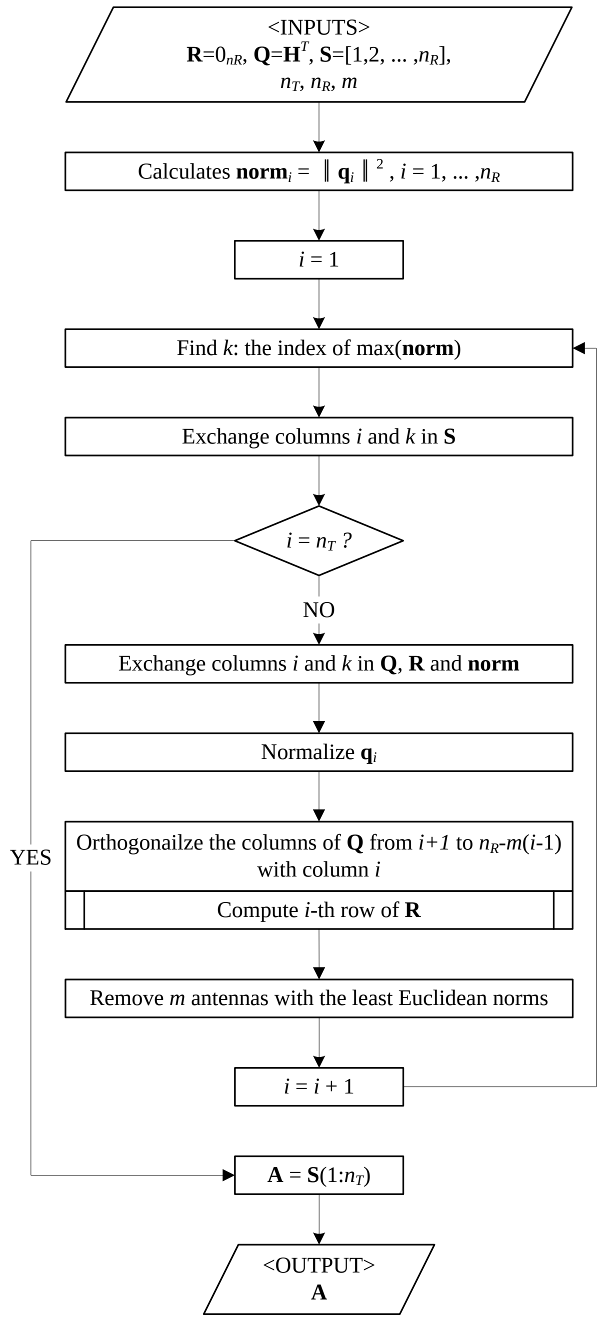

Figure 2: Flowchart of the proposed QR decomposition-based antenna selection algorithm (QRAS)

IV. Computational Complexity

In this section, we analyse the computational complexity of QRAS. In this analysis, we consider that a complex multiplication requires four real multiplications and two real additions, while a complex addition requires two real additions. **Fast RAS.** The complexity of the fast RAS is computed using Equation (11) and (12) in [7].

Table 1
Computational Complexity of the Proposed QRAS for m = 0 versus those of the Fast RAS and the Optimal Search Algorithm for Several Scenarios.

(n_T, n_R)	Optimal $(N_{opt}^{RM}, N_{opt}^{RA})$	Fast RAS $(N_{fast}^{RM}, N_{fast}^{RA})$	QRAS $(N_{qras}^{RM}, N_{qras}^{RA})$	ε_{opt}^{qras}%	$\varepsilon_{fast}^{qras}$%
(4, 6)	(4560, 1770)	(2164, 1586)	(507, 339)	(92.566, 80.848)	(76.571, 68.033)
(4, 8)	(21280, 8260)	(3396, 2490)	(727, 491)	(94.693, 94.056)	(78.593, 70.803)
(4, 16)	(553280, 214760)	(8324, 6106)	(1607, 1099)	(99.801, 99.488)	(80.694, 73.682)
(8, 12)	(871200, 385110)	(8324, 6106)	(4119, 2917)	(99.665, 99.243)	(90.180, 87.571)
(8, 16)	(22651200, 10012860)	(67928, 53716)	(6031, 4293)	(99.981, 99.957)	(91.122, 88.772)
(8, 20)	(221707200, 98004660)	(93912, 74292)	(7943, 5669)	(99.997, 99.994)	(91.542, 89.308)

A. *Optimal selection*

Due to the special structure of the matrix in Equation (4), the computation of $\left(\mathbf{H}_A\mathbf{H}_A^H\right)$ requires $6n_T^2 - 4n_T$ real multiplications and $6n_T^2 - 7n_T + 2$ real additions. The matrix inversion requires $2n_T^3 + 6n_T^2$ real multiplications and $n_T^3 - n_T^2$ real additions as it is performed using the Cholesky decomposition due to the Hermitian structure of the matrix $\left(\mathbf{H}_A\mathbf{H}_A^H\right)$. The total computational complexity of the optimal search algorithm is given by the following:

$$N_{opt}^{RM} = C_{n_T}^{n_R} \cdot (2n_T^3 + 12n_T^2 - 4n_T),$$
$$N_{opt}^{RA} = C_{n_T}^{n_R} \cdot (n_T^3 + 5n_T^2 - 7n_T + 2),$$

with the superscripts *RM* and *RA* standing for real multiplication and real addition, respectively.

B. *QRAS Algorithm*

The computational complexity of the proposed QRAS algorithm is given by:

$$N_{qras}^{RM} = 8n_R n_T^2 - 4n_T^2 - 4n_T n_R + 7n_T^2 - 2n_R - 2n_T - 1$$
$$- m \cdot (4n_T^3 - 11n_T^2 + 5n_T + 2),$$
$$N_{qras}^{RA} = \frac{11}{2}n_T^2 + 6n_R n_T^2 - 5n_T n_R - 3n_T^3 - \frac{7}{2}n_T + 1$$
$$- m \cdot (3n_T^3 + \frac{15}{2}n_T - \frac{19}{2}n_T^2 - 1).$$

Table 1 lists the computational complexities of the proposed QRAS, the fast RAS, and the optimal search algorithms for several (n_T, n_R) scenarios. In Table 1, $\varepsilon_{fast}^{qras}$ and ε_{opt}^{qras} are the saving factors achieved by the proposed QRAS algorithm with respect to the fast RAS and the optimal search algorithm, respectively. For n_T = 8 and n_R = 12, the proposed QRAS algorithm with $m = 0$, i.e., at each iteration a single antenna is chosen and none is removed, performs only about 11% and 0.55% of the computations required by the fast RAS and the optimal search algorithm, respectively. Furthermore, the computational complexity of the proposed algorithm for $m = 1$ is reduced by 102 real multiplications and 69 real additions when n_T = 4 and 1386 real multiplications and 987 real additions when n_T = 8. This reduction is doubled when m = 2.

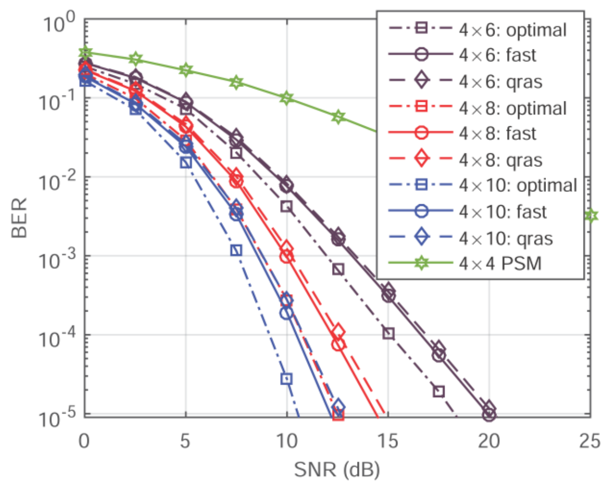

Figure 3: BER performances of the QRAS algorithm for $m = 0$, the fast RAS algorithm, and the optimal search algorithm for n_T = 4 and n_R = 6, 8, and 10.

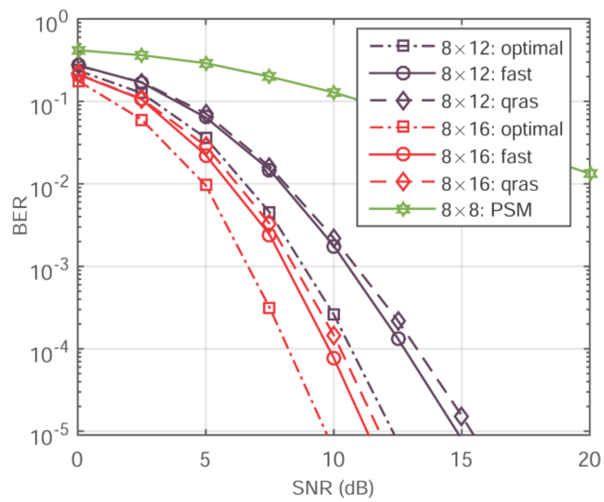

Figure 4: BER performances of the QRAS algorithm for $m = 0$, the fast RAS algorithm, and the optimal search algorithm for n_T = 8 and n_R = 12 and 16.

V. SIMULATION RESULTS

In this Section, we assume that the transmitter has perfect knowledge of the channel state information (CSI) and that it employs quadrature phase shift keying (QPSK) modulation. The receiver has a scaling factor γ, necessary for the maximum-likelihood receiver, but not CSI.

Figure 3 and Figure 4 depict the performance of the proposed QRAS algorithm for m = 0, the fast RAS algorithm, and the optimal search algorithm for $n_T = 4$ and $n_T = 8$, respectively, and for several values of n_R. Both the QRAS and the fast RAS algorithms achieve a quasi-optimal diversity order, as manifested by the parallel BER curves of these algorithms to that of the optimal selection algorithm, with a tolerable degradation in the bit-error rate (BER), given the huge reduction in the computational complexity that our proposed algorithm achievements. From Figure 3, we can conclude that the proposed algorithm lags the performance of the fast algorithm by 0.2, 0.35, and 0.35dB at a target BER of 10^{-4} for n_T = 4 and n_R = 6, 8, and 10, respectively. The degradation is less than 0.5dB in case of n_T = 8 and n_R = 12 and 16, as shown in Figure 4.

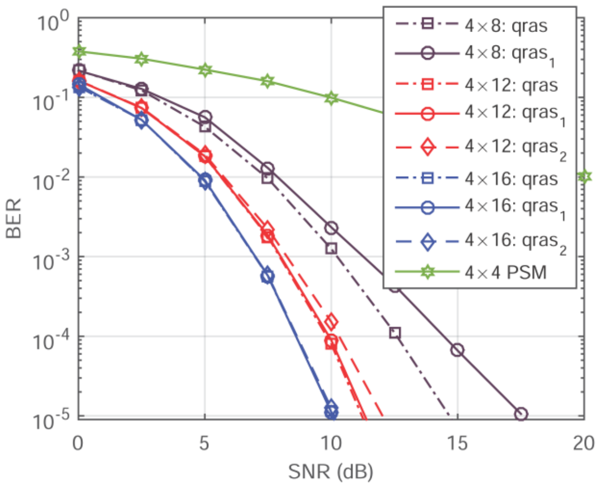

Figure 5: BER performances of the proposed QRAS algorithm for m = 0, 1 and 2, n_T = 4, and n_R = 8, 12 and 16.

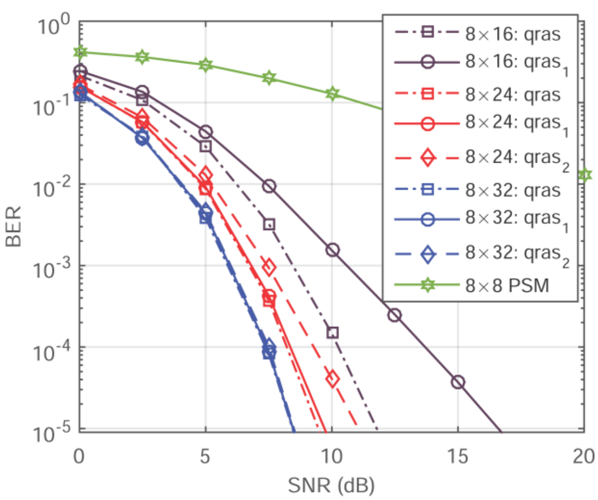

Figure 6: BER performances of the proposed QRAS algorithm for for m = 0, 1 and 2, n_T = 8, and n_R = 16, 24 and 32.

Figure 5 and Figure 6 depict the BER of the proposed algorithm for m = 0, referred to as qras, m = 1, referred to as qras$_1$, and m = 2, referred to as qras$_2$ for n_T = 4 and n_T = 8, respectively, and for several values of n_R. Based on Figure 1, qras$_1$ can be applied iff $n_R \geq 2n_T$ and qras$_2$ can be applied iff $n_R \geq 3n_T$. As n_R increases beyond this limit, the BER performances of qras$_1$ and qras$_2$ converge to that of qras. The number of excluded antennas per iteration, m, is selected as a trade-off between the computational complexity and BER performance. In this paper, we analysed the performance for m = 0, 1, 2, but more than 2 antennas can be removed per iteration as long as n_T antenna selection iterations are still possible, which means that n_R should be greater than or equal to $(m+1)\, n_T$. This applies to the case of massive MIMO systems, where a relatively small set of antennas is selected from a large set of available antennas. In order to reduce the computational complexity, more than 2 antennas can be removed at each iteration, leading to a tremendous reduction in the complexity of the proposed algorithm. The only drawback of the proposed algorithm is that its performance is slightly degraded as compared to the algorithm proposed in [7], while achieving considerable reduction in the computational complexity.

VI. CONCLUSION

In this paper, we proposed an efficient antenna selection algorithm based on the Gram-Schmidt orthogonalisation method. Instead of performing several matrix inversions, as in the optimal and the fast RAS algorithms, our algorithm successively minimises the optimisation function, which is a sum of scalars. In each iteration, and in addition to selecting one antenna, the proposed QRAS algorithm can also remove one or two antennas, leading to further reduction in the computational complexity. For instance, when n_T = 8 and n_R = 16, the proposed algorithm requires only a few hundredths and 10% of the computational complexities of the optimal search method and the fast RAS algorithm, respectively.

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