Jacobi's Construction Method Codes for Zero Cross-Correlation OCDMA Systems

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***Abstract*— In this paper, we present a new code design for the Optical Code Division Multiple Access (OCDMA) system based on the Jacobi function. One of the major factors limiting the performance of OCDMA systems is known as Multiple Access Interference (MAI). A good code design provides a minimum cross-correlation, maximum number of users with minimum code length, and implementation flexibility to remove the MAI effectively. The use of finite field arithmetic's in the construction of our proposed code yield a zero cross-correlation (ZCC) which is the least desired value. Theoretical analysis and simulation results show that the proposed code outperforms the previously reported codes with zero maximum cross-correlation. Thus, the Multiple Access Interference (MAI) and Phase-Induced intensity noise (PIIN) effects are eliminated, improving the system's performance. The system can accommodate more simultaneous users than other codes at a standard acceptable bit error rate value ($\leq 10^{-9}$). Furthermore, the construction method offers flexibility in choosing between the number of users, the weight, and the code length.**

***Index Terms*— Jacobi's Function; Multiple Access Interface (MAI); Optical Code Division Multiple Access (OCDMA); Zero Cross-correlation (ZCC) Code.**

I. INTRODUCTION

Optical code-division multiple access (OCDMA) is one of the most important techniques in which each user in the communication channel is assigned a distinguishable and unique optical code at the transmitter [1][2]. The optical bandwidth is shared by several simultaneous users in the same time slot and frequency to increase the transmission capacity of a fiber optic.

This technique has several advantages besides the Wavelength Division Multiplexing (WDM) technique and Time Division Multiplexing (TDM), such as their asynchronous network access capability, flexibility, the simplicity of the network control, and higher security against unauthorized users [1]. In general, OCDMA systems can be classified into two categories: coherent and incoherent systems. The coherent OCDMA system uses the coherent property of light and implements bipolar encoding of the optical signal. In incoherent OCDMA systems, the presence or absence of light energy is used to represent binary numbers ''1'' and ''0'' known as unipolar codes. The performance of the OCDMA system is limited by shot noise, beat noise, thermal noise, dark current, and phase-induced intensity noise (PIIN). MAI noise is the main source of performance degradation in an incoherent OCDMA system due to the cross-correlation between multiple active users on a common channel. Thus, good code sequences and detection scheme design with high auto-correlation and zero cross-correlation properties is important to effectively eliminate the MAI effects [3][4].

In the literature, various coding schemes have been designed and implemented for the SAC-OCDMA systems, such as optical orthogonal codes, prime codes, Walsh-Hadamard code, double weight codes, and random diagonal (RD) codes [1],[4]-[9]. The construction of the code is limited by code length parameters, the increase in cross-correlation with the weight number, or the code length is too long. Consequently, the code design cannot be used for any number of simultaneous users or high data rates in SAC-OCDMA systems.

In this paper, we proposed a new construction method of Zero Cross-Correlation (ZCC) codes for OCDMA systems. The proposed new method allows an independent choice of the number of users, weight, and code length. The code cardinality is increased, and the cross-correlation is equal to zero. The rest of the paper is organized as follows: we will first describe the construction method for the proposed ZCC code mathematically. Then, code comparison with other codes from the references is introduced. Next, we validate the performance of the proposed ZCC code by using simulations under Optisystem software version 7. Finally, a conclusion is given to summarize our work.

II. CONSTRUCTION METHOD OF THE PROPOSED ZCC CODE

In this technique, a ZCC code sequence S_i can be constructed as follows: Let α be the root of a m^{th} degree primitive polynomial $f(x) \in GF(p)[x]$. All nonzero elements of $GF[p^m]$ are successive powers of α and the multiplicative order of α is $ord[\alpha] = p^m - 1$, such that: $\beta \in GF[p^m] = \{1, \beta, \beta^2, \dots, \beta^{p^m-2}\}$, where $q = p^m$ with p a prime and m an integer. The function is used in the finite field $GF[q]$ [10]. The nonzero element β is defined as $\beta = \gamma^i$ where $0 \leq i \leq q-2$

$$\text{For } i \leq j,\ \begin{cases} \gamma^i + \gamma^j = \gamma^i \gamma^{j-i} = \gamma^k \\ \text{where } k \equiv i + z_{j-i}\ (mod\ q-1) \end{cases} \quad (1)$$

For each element of the Jacobi's function z_j, we calculate:

$$\gamma^{z_j} = 1 + \gamma^j \quad (2)$$

We suppose that γ^i and γ^j and also their sum is both non zero. For the singular point when the result value is undetermined, we suppose that the value of z_j is equal to zero ($z_j = 0$). Clearly $GF[p^m]$ can thus be interpreted as a vector space over $GF(p)$. The set $\{1, \beta, \dots, \beta^{m-1}\}$ can be used as a basis for the vector space.

Based on these assumptions, we can determine the number of user's as:

$$K \leq \left\lfloor \frac{q-1}{w} \right\rfloor \quad (3)$$

Where: $(q-1)$ = Code length of the proposed ZCC code
w = Code weight

Then, the set of $(q-1)$ elements is partitioned into K sub-sets. Each constructed sub-set has w elements corresponding to a specific user. The ZCC sequence is constructed as:

$$S_i = \begin{cases} (z_0, z_1, \dots, z_{(i*w+k)}, \dots, z_{(i+1)*w-1}) \\ 0 \leq i \leq K-1 \;\; and \;\; 0 \leq k \leq w-1 \end{cases} \quad (4)$$

Finally, the i^{th} binary code of the ZCC code with length $(q-1)$ is deduced by replacing each number element from the ZCC sequence with ''1''. The resulting $(K \times (q-1))$ matrix of the ZCC code is given as follows:

$$C_{ZCC} = \begin{bmatrix} c_{0,0} & \cdots & c_{0,(q-1)} \\ \vdots & \ddots & \vdots \\ c_{(K-1),0} & \cdots & c_{(K-1),q-1} \end{bmatrix}_{(K \times (q-1))} \quad (5)$$

An example of ZCC code matrix generated by the Jacobi's function is derived below with $q = 8$, $w = 2$ and $K = \left\lfloor \frac{8-1}{2} \right\rfloor = 3$:

Consider $\alpha \in GF(2^3) \equiv GF(2)$ to be a root of a primitive polynomial: $f(x) = x^3 + x + 1$, which is equivalent to $\beta^3 = \beta + 1$ in the $GF(2)$ finite field. So, we can obtain all the polynomial representations of γ^i with $(\gamma = z_j)$ as follows: $\beta + 1,\ \beta^2 + \beta, \beta^3 + \beta^2 + \beta, \beta^2 + 1$.

The Jacobi's elements to the base $\gamma = \beta$ are obtained using Equation (2), leading to the set of z_j vector elements: (0, 3, 6, 1, 5, 4, 2). The undetermined value for j=0 is set equal to zero ($z_j = 0$). Then, the elements of the resulting vector are partitioned according to the number of users and weight given by Equation (3). In this case, the number of users $K = 3$ and the code weight is w = 2, $(K > w)$ resulting in either the set of pairs: $\{(0,3), (6,1), (5,4)\}$ or $\{(3,6), (1,5), (4,2)\}$.

Finally, the pairs (0, 3), (6, 1), (5, 4) corresponding to the weight code of each user are replaced by the ''1' in their positions resulting in the following ZCC sequence code:

$$C_{ZCC} = \begin{bmatrix} 1\,0\;\,1\,0\,0\,0 \\ 0\,1\;\,0\,0\,0\,1 \\ 0\,0\;\,0\,1\;\,1\,0 \end{bmatrix} \quad (6)$$

As we can see, the number of columns in the matrix code is truncated to be equal to the length of the code $(L = K.w)$ where the unused column is ignored. The number of users and weight is too related to the code length. If the number of users increases, the weight code is decreased.

The number of users can be further increased using the mapping matrix below for $i = 3$:

$$ZCC(i=3) = \begin{bmatrix} Zeros(2K \times 2L) & ZCC(2) \\ ZCC(2) & Zeros(2K \times 2L) \end{bmatrix} \quad (7)$$

Where K denotes the number of users, $L = (K.w)$ is the code length, and Zeros(.) denotes a matrix of zeros. $ZCC(1)$ is the basic matrix of the proposed code with the dimension $(K \times (K.w)) = K \times L$. From that mapping, the length of the code increases as the number of users increases. The number of users is determined by the number of rows in the newly generated matrix as follows: $K_{Total} = K.2^{i-1}$ with K is given by equation (3), and the code length which represents the number of columns in the newly generated matrix is given by: $L_{Total} = L.2^{i-1}$.

III. Code Comparison

In this section, the performance of the proposed ZCC code method is compared to that of other available optical codes. All the mathematical relationships between parameter codes are shown in Table 1.

Table 1
Expressions parameters of the proposed code and some other optical codes

Codes	No. of users	Code length	λ
MDW [11]	n	$3K + 8/3\,[sin(K.\pi/3)]^2$	$\lambda = 1$
Prime code [5]	$w = p$	p^2	$\lambda = 2$
ZCC [12]-[13]	$2^i.(w+1)$	$2^i.w.(w+1)$	$\lambda = 0$
ZCC [14]-[16]	2^i	$2^i.w$	$\lambda = 0$
Hadamard [15]	$(2^i - 1)$	2^i	$\lambda = 1$
Proposed ZCC	$K.2^{i-1}$	$L.2^{i-1}$	$\lambda = 0$

From Table 1, we can conclude that our proposed code gives the minimum cross-correlation, which is the same as the ZCC code references [12]-[14],[16]-[19]. MDW and Walsh-Hadamard methods have a unit cross-correlation ($\lambda = 1$), and the prime code has a value of two ($\lambda = 2$). Furthermore, using the same number of users as is shown in Table II, we can conclude that the prime code (PC) gives the largest code length and weight while the proposed Jacobi's ZCC code has the same length as those of ZCC references. Walsh-Hadamard gives the minimum code length and high weight but a unity cross-correlation value.

Table 2
The proposed code in comparison with some other optical codes for $K_{Tot} =$ 30 users

Codes	No. of users	Code length	Weight
MDW [11]-	30	90	$w = 4$
[21]	30	900	$w = 31$
Prime Code [5]	30	120	$w = 4$
ZCC [11]-[12]	30	120	$w = 4$
ZCC [13]-[20]	30	32	$w = 16$
Hadamard [15]	30	120	$w = 4$
Proposed ZCC			

Consequently, the proposed method provides the best cross-correlation value with more flexibility in choosing an increased number of users and the weight. The code length increases with the same factor of proportionality of the number of users but is still too short compared to other optical methods. The proposed ZCC code's construction method is less complicated and can be generated using pre-stored tables. The code length of the proposed method is calculated using Equation (3) for the basic code length of $L = q - 1 \geq K \times W$. Then, for a total number of users, $K_{Total} = 30$, the code length L_{Total} obtained is equal to $L = 120$.

IV. PERFORMANCE ANALYSIS AND SIMULATION RESULTS

The performance of the proposed ZCC code for OCDMA systems using direct detection is simulated in OptiSystem 7.0. The software simulates the transmitters, optical fiber, and receivers in the physical layer of optical fiber networks. In this simulation, a conventional signal mode fiber with attenuation of $0.2\, dB/km$ and dispersion coefficient of $16.75\, ps/nm/km$ are used. The main parameters used in the simulation setup are presented in Table 3. Each chip has a spectral width of 0.8 nm. The bit rates variation is from $622\, MHz$ to $2\, GHz$. The performance of the system was characterized by referring to the BER and the eye patterns.

Table 3
Parameters used in the simulation

Parameter	Value
Operating wavelengths	$[1478.8\, nm, 1487.6\, nm]$
Bit rate	variable
Signal format	NRZ
Fiber attenuation	$0.2\, dB/km$
Fiber dispersion	$16.75\, ps/nm/km$
Fiber length	variable
Number of users	3

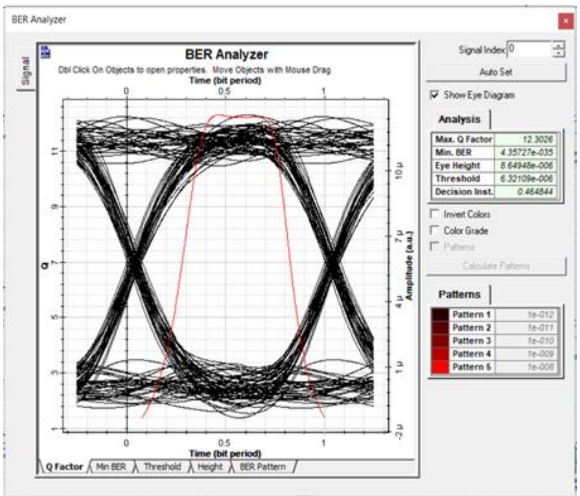

Figure 1: Eye diagram of the proposed ZCC code at 1.5GHz user's data rate, three users, and code weight 4 through a fiber length distance of 20 Km.

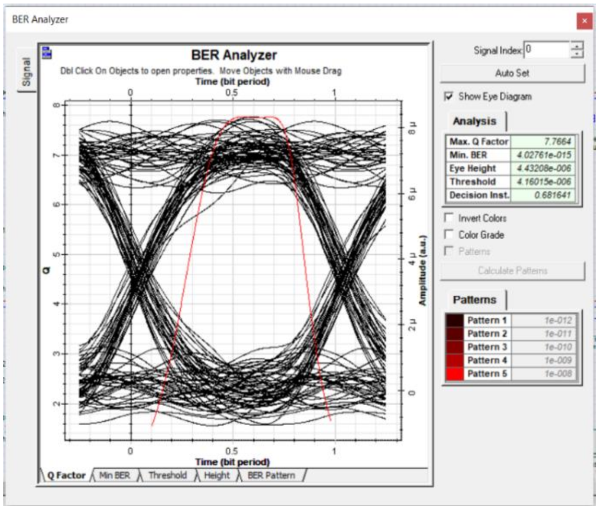

Figure 2: Eye diagram of the proposed ZCC code at 1.5GHz user's data rate, three users, and code weight 4 through a fiber length distance of 30 Km.

Figure 1 and Figure 2 show the eye diagram and BER values of the proposed ZCC code in the OCDMA system at a data rate of $1.5\, GHz$ and fiber length distances of $20\, Km$ and $30\, Km$, respectively. The number of users was chosen to be equal to three, and the code weight value $W = 4$.

From Figure 1 and Figure 2, the eyes diagram clearly illustrates that the proposed ZCC code system gives a good performance, even with a large fiber distance, having a large eye-opening for practical BER applications. The more the eye closes, the more difficult it is to distinguish between the ones and zeros of the signal. The height of the eye-opening at the specified sampling time shows the noise margin or immunity to noise. The BER values of 4.36×10^{-35} and 4.027×10^{-15} were obtained for fiber distance lengths of $20\, Km$ and $30\, Km$, respectively. These BER values are much lower than the basic required 10^{-9} value for system performance. The corresponding fill factors (Q) were 12.303 and 7.77 (> 6).

Bit Error Rate (BER) expression can be estimated, from Signal to Noise Ratio (SNR), using Gaussian approximations as [15],[22][23]:

$$BER = \frac{1}{2}\sqrt{\frac{SNR}{8}} \tag{8}$$

The SNR expression for SAC-OCDMA systems with direct detection is given by [24]:

$$SNR = \frac{\left(\frac{\mathcal{R}\ P_{sr}\ W}{L}\right)^2}{2\ e\ B\ \mathcal{R}\ P_{sr}\ W/L\ +\ 4\ \frac{K_b\ T_n\ B}{R_L}} \tag{9}$$

Where:
- $\mathcal{R}$ = Responsivity of the photodiode
- B = Electrical equivalent noise bandwidth of the receiver
- K_b = Boltzmann Constant
- R_L = Load resistance
- T_b = Temperature of noise at the receiver
- P_{rc} = Effective power at the receiver
- e = Charge of Electron

Performance comparison between Prime code, Hadamard, ZCC of references, and our proposed ZCC code with direct detection is made by using the above formulas as in Equations (8) and (9), respectively. The parameters used in our analysis are shown in Table 4.

Table 4
Parameters used in Matlab calculations

Character	Parameter	Value
$\mathcal{R}$	Responsivity of the photodiode	1
B	Electrical equivalent noise bandwidth of	$311\, MHz$
K_b	the receiver	$\lambda = 0$
R_L	Boltzmann constant	$1030\, \Omega$
T_b	Resistance load	$300\, K$
P_{rc}	Temperature of noise at the receiver	$-10\, dBm$
e	Effective power at the receiver	$1.6\ 10^{-19} C$
	Charge of Electron	

The BER is evaluated taking into account shot noise and thermal noise only. The BER versus the total number of active users simulated by using the Matlab software is shown in Figure 3.

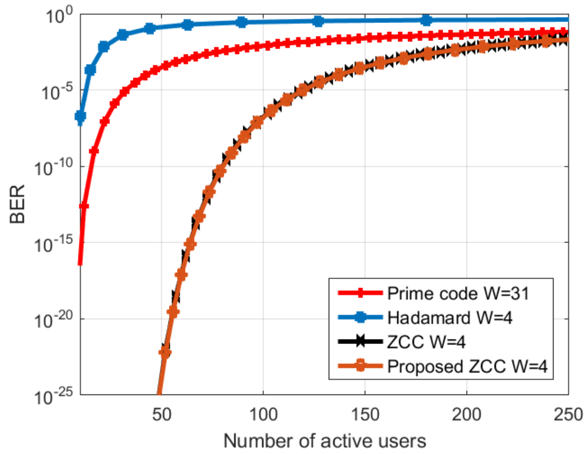

Figure 3: BER versus number of active users when $P_{sr} = -10\ dBm$.

As we can conclude from Figure 3, our proposed ZCC construction method gives identical performance to the ZCC reported codes and outperforms the other methods. Ninety (90) users can be supported with our method with approximately a BER of 9×10^{-10}. In reference [18], a total number of seventy (70) active users are accommodated by the proposed ZCC method. The results are a consequence of the code cross-correlation properties that eliminate the effect of multiple access interference (MAI) and phase-induced intensity noise (PIIN), which have an important impact on system performance degradation.

Figure 4 and Figure 5 show the BER and the Fill Factor (Q) against the fiber length at different data rates: $622\ MHz$, $1\ GHz$, $1.5\ GHz$, and $2\ GHz$, respectively. As we can see, the average BER value increases with the increase of the transmission length while the Fill Factor (Q) value decreases. Moreover, at system performance BER of 10^{-9}, the maximum transmission distance to be reached for the proposed scheme without amplification is approximately about $44\ Km$ at data rates of $622\ MHz$, $40\ Km$ at data rates of $1\ GHz$, $35\ Km$ at data rates of $1.5\ GHz$ and $30\ Km$ at data rates of $2\ GHz$.

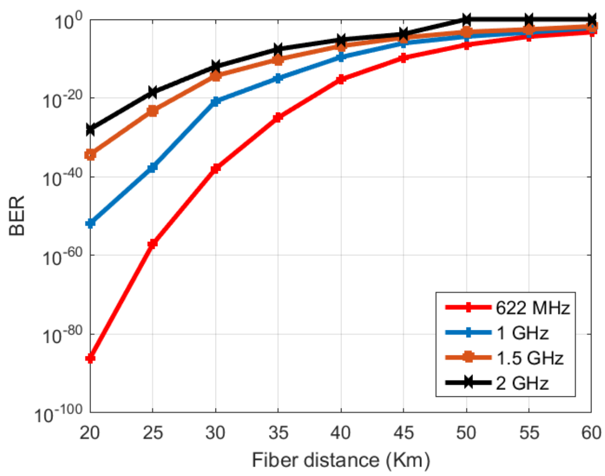

Figure 4: BER and Fill Factor (Q) versus fiber distance for the proposed ZCC code at different user's data rates of 1GHz, 1.5 GHz, and 2 GHz.

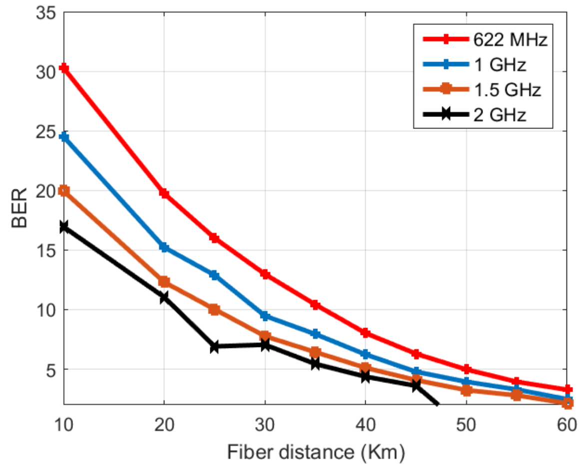

Figure 5: Fill Factor (Q) versus fiber distance for the proposed ZCC code at different user's data rates of 1GHz, 1.5 GHz, and 2 GHz.

The maximum distance to be reached is further reduced when the user data bit is increased. This reduction in performance is caused by the higher optical attenuation in the longer fiber span.

As a result, the system's performance is best with low BER values when the distance or user data rate used is well chosen. Using Equation (10), we can find the maximum fiber distance given as [25]:

$$P_{imax}(dB) = \alpha \times L + P_r(dB) \tag{10}$$

Where: P_{imax} = the maximum input power,
α = the attenuation,
L = the transmission distances
P_r= the minimum receiver power.

For the OCDMA system, a typical dynamic range of the receiver is between -7 dBm and -28 dBm [25]. As shown in Figure 4 and Figure 5, the system's performance is best with low BER values when the distance or user data rate used is well chosen.

V. Conclusion

In this paper, a new code construction method is proposed for OCDMA systems using zero cross-correlation (ZCC) property based on the so-called Jacobi's function. The code proposed is very simple, with high flexibility in the choice of the number of users and the weight code values. The design of the code is made simple using a pre-stored table for codes.

The direct decoding technique is exploited since there is no overlapping occurrence in the code construction, which reduces the number of filters used at the receiver. Thus, the overall cost and complexity of the system are reduced.

Simulation and theoretical analysis show that our proposed construction code gives the system a good performance (BER, Q) compared to other reported codes. The zero cross-correlation value efficiently eliminates the MAI and PIIN effects and increases the BER performance of the system. Furthermore, the proposed code can support more active users simultaneously at high data rates and longer distances.

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