Design and Implementation of Two-Wheeled Robot Control Using MRAC

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***Abstract*— The control strategy for a two-wheeled robot is still continuously researched and developed due to its natural behavior (unstable and non-linear). In this research, the Model Reference Adaptive Control (MRAC) using the Lyapunov stability theorem was applied in a two-wheeled robot. MRAC was used to design adaptive controllers that work on the principle of adjusting controller parameters so that the actual output traces the output of the reference model, which has the same reference input. The adjusting mechanism was built to ensure stability and convergence from the adaptation errors. The verification experiment showed that with the adaptation gain $\gamma_1, \gamma_2, \gamma_3$ are 1, 0.005, and 0.001, respectively, the response could follow the reference model with a rising time and settling time 0.27 s and 0.87 s, respectively.**

***Index Terms*—Two-Wheeled robot; MRAC; Kalman Filter.**

I. INTRODUCTION

An inverted pendulum is naturally dynamic, unstable, and prone to disturbance due to environmental conditions or robot load. One of the inverted pendulum applications is a two-wheeled robot. The two-wheeled robot is a robot with an inverted pendulum model that is placed on a two-wheeled cart. The input and output of a two-wheeled robot varies depending on the design, but the mechanics driven by the system are always the same [1]. A two-wheeled inverted pendulum robot has a working principle as an inverted pendulum and it is an unstable and non-linear robotic system. The dynamics of this system have become a research base to test various types of control methods.

The balancing control of the two-wheeled robot has become a popular topic in the scientific community. Many authors have studied these robots for the purpose of finding the perfect mathematical model and characteristic structure. Exploration, search and rescue, materials handling, and entertainment are some examples of two-wheeled robot applications [2].

Many researchers have studied the efficiency of control algorithms to solve balancing problems in two-wheeled robots or inverted pendulums. For example, [3]-[4] used a PID controller to balance the two-wheeled robot and kept it in a still upright position. They found the effectiveness of the controller designed in performing stabilization and trajectory.

In [5], aiming to maintain system stability, they used the PID tunning method, where the correct parameter tunning process took a long time. The sliding mode control method has been applied in [6]-[7]. [8] used a proportional-feedback controller, where control parameters were obtained using a self-tunning algorithm. Research in [9] built a self-balancing robot by applying the PID control method; however, a rigid mechanical design and a control method that is unable to adapt to uncertainty made the system response to oscillate, causing less stability.

While in [10], two different methods, namely Linear Quadratic Gaussian (LQG) and Model Predictive Control (MPC) were compared and applied to a two-wheeled robot. The results showed that the LQG control generates better performance. From the various control methods that have been mentioned, in general, there are deficiencies when using classical feedback controllers as the accuracy of mobile robot movement has not been fully achieved. This is due to the dynamic uncertainties caused by environmental changes [11]. In this regard, an approach using adaptive control has been proposed as a solution to overcome the dynamic uncertainty and disturbances in the two-wheeled robot. This controller is adaptive to the environment so that it can modify the response behavior to change the process parameters.

The two-wheeled robot model and design of the Model Reference Adaptive Control (MRAC) are discussed in this paper. The MRAC is used to design an adaptive controller that works on the principle of adjusting controller parameters so that the actual output tracks the output of the reference model, which has the same reference input. A mathematical approach, such as Lyapunov's theorem is employed to develop an adjusting mechanism to ensure stability and convergence from adaptation errors.

II. MODEL DESCRIPTION OF TWO-WHEELED ROBOT

A two-wheeled robot model is built based on an inverted pendulum system with an analysis based on the force acting on it. The modeling process on a two-wheeled robot begins from a simplification that is represented as an inverted pendulum, which can be seen in Figure 1 and Figure 2. The two-wheeled robot consists of three parts: a cart, pendulum, and case (a place to store a microcontroller, motor, and other devices required by the system).

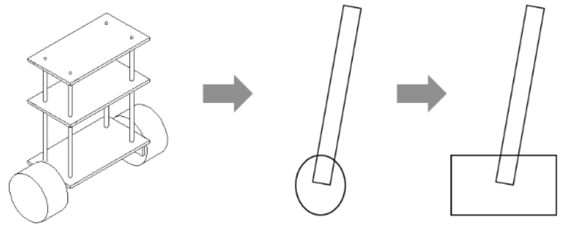

Figure 1: Illustration of simplified two-wheeled robot

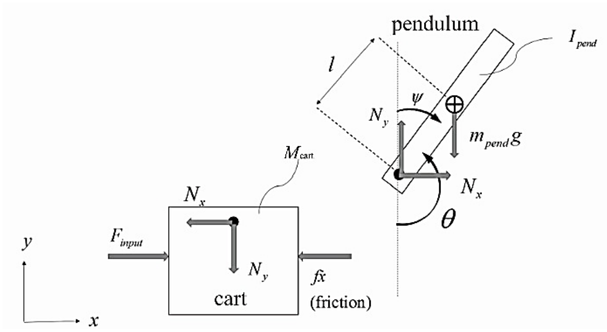

Figure 2: Inverted pendulum forces

The two-wheeled robot mathematical model is based on the Newton-Euler equation. An analysis has been performed based on the forces that affect it. As shown in Figure 2, the pendulum and the wheel were analyzed separately so that this modeling will produce two non-linear dynamic movements: the movement of the cart and the pendulum. The two-wheeled robot system modeling has been explained in [12]. The position, speed, and acceleration of the cart are denoted by $(x, \dot{x}, \ddot{x})$ while the position, speed, and angular acceleration pendulum are denoted by $(\phi, \dot{\phi}, \ddot{\phi})$. The state-space equation of the two-wheeled robot is shown in Equations (1) and (2).

$$\begin{bmatrix}\dot{x}\\ \ddot{x}\\ \dot{\phi}\\ \ddot{\phi}\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ 0 & \frac{-(I+ml^2)b}{I(M+m)+Mml^2} & \frac{m^2gl^2}{I(M+m)+Mml^2} & 0\\ 0 & 0 & 0 & 1\\ 0 & \frac{-mlb}{I(M+m)Mml^2} & \frac{mgl(M+m)}{I(M+m)+Mml^2} & 0\end{bmatrix}\begin{bmatrix}x\\ \dot{x}\\ \phi\\ \dot{\phi}\end{bmatrix} + \begin{bmatrix}0\\ \frac{I+ml^2}{I(M+m)+Mml^2}\\ 0\\ \frac{ml^2}{I(M+m)+Mml^2}\end{bmatrix}u \quad (1)$$

$$y = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}x\\ \dot{x}\\ \phi\\ \dot{\phi}\end{bmatrix} + \begin{bmatrix}0\\ 0\end{bmatrix}u \quad (2)$$

In this paper, the position and velocity carts $(x, \dot{x})$ are not used; thus, the state space can be simplified as Equations (3) and (4).

$$\begin{bmatrix}\dot{\phi}\\ \ddot{\phi}\end{bmatrix} = \begin{bmatrix}0 & 1\\ \frac{mgl(M+m)}{I(M+m)+Mml^2} & 0\end{bmatrix}\begin{bmatrix}\phi\\ \dot{\phi}\end{bmatrix} + \begin{bmatrix}0\\ \frac{ml^2}{I(M+m)+Mml^2}\end{bmatrix}u \quad (3)$$

$$y = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}\phi\\ \dot{\phi}\end{bmatrix} \quad (4)$$

where M is the mass of the cart, m is the mass of the pendulum, I is the moment of inertia, g is the gravity and l is the length of the pendulum's equilibrium point. The parameters of the plant can be seen in Table 1.

Table 1
Two-Wheeled Robot Parameters

Symbol	Parameter	Value
I	Moment of inertia	0.003639165 Kgm^2
M	Mass of the cart	0.285 Kg
m	Mass Pendulum	0.285 Kg
l	The length of the pendulum's equilibrium point	0.0565 m
g	Gravity	9.8 m/s^2

III. Model Reference Adaptive Control (MRAC)

The two-wheeled robot is a non-linear, unstable, and uncertain system. The strategy of implementing controllers at this plant is a challenge itself. The controller type is determined from the purpose of the design control goals. Adaptive controllers can be a solution to certain control problems that involve understanding the character of the plant as well as other requirements for plant performance. The Model Reference Adaptive Control (MRAC) was used in this research. MRAC is a control system with the desired specifications given in the form of a reference model, which is illustrated on the block diagram (Figure 3). The ideal behavior of the reference model should be achieved for adaptive control systems.

There are two approaches to creating MRAC, the MIT rule, and the Lyapunov stability theory. Based on [12]-[13], the approach with Lyapunov's theory has low difficulty and physical realization is comparatively more feasible than the MIT rule.

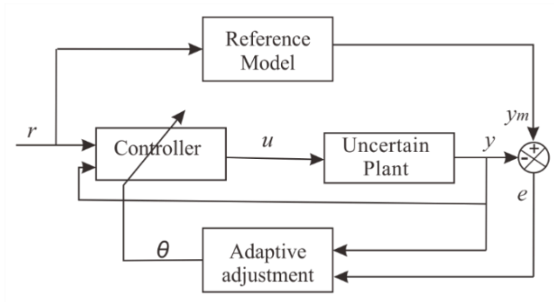

Figure 2: Block diagram of MRAC [14]

MRAC consists of two loops: the first loop is for the feedback control and the second loop is for the parameter adaptive adjustment controller. The reference model provides information on how the process output should respond to the desired command signal. Output reference model and plant were compared, and the error between them was given as feedback through an adaptive adjustment loop. Controller parameters were updated such as minimizing errors to zero. The nonlinear reference model can also be used to formulate the reference model from the LTI model, but it requires more complex design techniques. The general equation for the control signal (u) in MRAC is shown in Equation (5).

$$u = k_x(t)x + k_r(t)r \quad (5)$$

with $k_x(t)x$ and $k_r(t)r$ is adjustable gain control.

The error equation (6) is defined as

$$e = y_m - y \quad (6)$$

where y is the actual plant output and y_m is the reference model signal output, which only depends on the reference signal (r).

Lyapunov stability theorem can be used to describe the parameter adjusting algorithm in MRAC. Therefore, the MRAC design in this research used the Lyapunov stability concept. In a linear system, the Lyapunov candidate function is given by $V(x) = x^T P x$ where $P \in R^{n \times n}$ is a symmetric matrix. The system is said to be asymptotically stable if the function $V(x): R^n \to R$ fulfills the Lyapunov stability constraints as follows [15]:

Theorem 1: $V(x = 0) = 0$; always fulfill for $V(x)$ if the equilibrium point of the system is at the origin, then Lyapunov's function at the equilibrium point is locally positive definite.

Theorem 2: $V(x) > 0, \forall x \neq 0$; global asymptotic system when there is a definite positive P matrix, or it can be written $P > 0$.

Theorem 3: $\dot{V}(x) < 0, \forall x \neq 0$; where $\dot{V}(x)$ is expressed as Equation (7)

$$\dot{V}(x) = \dot{x}^T Px + x^T P\dot{x} = (Ax + Bu)^T Px + x^T P(Ax + Bu) \quad (7)$$

by choosing $u = 0$, the stability analysis of the open-loop system becomes Equation (8):

$$\dot{V}(x) = x^T[A^T P + PA]x < 0 \quad (8)$$

then Theorem 3 is satisfied only if it fulfills the quadratic Equation (9):

$$A^T P + PA < 0 \quad (9)$$

if the input $u = -Kx$, then Theorem 3 is satisfied only if the linear system fulfills the following Equation (10):

$$(A - BK)^T P + P(A - BK) = -Q \quad (10)$$

IV. Control System Design

A. *The Block Diagram*

The block diagram of the two-wheeled robot control system can be seen in Figure 4. A two-wheeled robot block consists of one driver and two DC motors. The output from the plant is the position and the angle of the plant. The output is read by the Gyro sensor, the reading process will be converted from radians to degrees.

To remove noise from the output of the two-wheeled robot, the signal must be firstly filtered, then the results of filtering are processed to be compared with a reference model and become an error signal. The error value and filtering results are included in the adaptive law. The output of the adaptive law is the controller parameters (θ_1, θ_2, and θ_3). The controller parameters will be compared with the desired reference (r) on the controller block. The two-wheeled robot will move according to the control signal generated by the controller so that it can be stable.

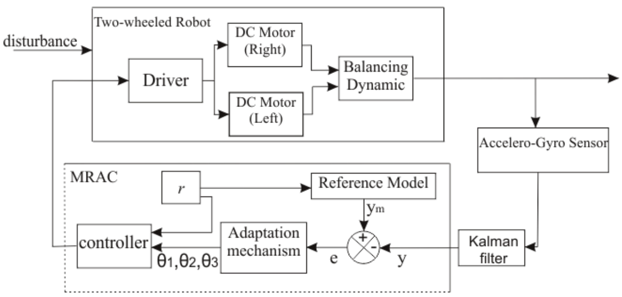

Figure 4: Block diagram of two-wheeled robot control system

B. *Model Reference Design*

The reference model is made based on the desired system specifications as shown in Table 2.

Tabel 2
The specification for model reference design

Symbol	Parameters	Value
T_s	Settling time for 2% standard	1 s
M_p	Overshoot	0-10%

According to Table 2, Equations (11) and (12) were used to obtain both damping ratio (ζ) and natural frequency (ω_n). These parameters were substituted to Equation (13) to obtain the reference model as stated in Equation (14).

$$\%M_p = e^{\left(-\frac{\zeta}{\sqrt{1-\zeta^2}}\right)\pi} \times 100\% \quad (11)$$

$$T_s = \frac{4}{\zeta\omega_n} \quad (12)$$

The reference model can be defined in nonlinear or linear equations form. In this research, reference model design was performed using a second-order system approach, defined as Equation (13).

$$\frac{V_{out}}{V_{in}} = \frac{{\omega_n}^2}{s^2 + 2\zeta\omega_n s + {\omega_n}^2} \quad (13)$$

$$\frac{V_{out}}{V_{in}} = \frac{45.83}{s^2 + 7.98s + 45.83} \quad (14)$$

The reference model state equation is derived by Equations (15) and (16),

$$\dot{x}_m = A_m x_m + B_m r \quad (15)$$

where, $x_m = \begin{bmatrix} y_m \\ \dot{y}_m \end{bmatrix}$

$$A_m = \begin{bmatrix} 0 & 1 \\ -{\omega_n}^2 & -2\zeta\omega_n \end{bmatrix} \quad B_m = \begin{bmatrix} 0 \\ {\omega_n}^2 \end{bmatrix} \quad (16)$$

with A_m and B_m are the state matrix and input matrix of the model reference, respectively. The value ζ = 0.59 and ${\omega_n}^2$= 45.83 is obtained from Equation (11) and (12), substituted into Equation (16), then the values of A_m and B_m are obtained, Equation (17),

$$A_m = \begin{bmatrix} 0 & 1 \\ -45.83 & -7.98 \end{bmatrix} \quad B_m = \begin{bmatrix} 0 \\ 45.83 \end{bmatrix}. \quad (17)$$

C. *Controller (MRAC) Design*

MRAC is an adaptive controller where the desired specifications are formed in the reference model so that the actual plant response can follow the reference model output, where the reference model has the same value as the input reference. Generally, the MRAC design on a two-wheeled robot uses a state-space model. The derivative of the error equation and the selection of the Lyapunov function is used to update the parameters so that the error will be zero [13]. The first step in the controller design is to define the state equation of the two-wheeled robot, as seen in Equations (18) and (19) , which then define the control signal.

1. State-space model

$$\dot{x} = Ax + Br$$
$$y = Cx \quad (18)$$

where $x = \begin{bmatrix} \phi \\ \dot{\phi} \end{bmatrix}$,

substituting the plant parameter values in Table 1 into Equations (3) and (4), then it gives Equation (19),

$$A = \begin{bmatrix} 0 & 1 \\ 38.54 & 0 \end{bmatrix}$$
$$B = \begin{bmatrix} 0 \\ 6.9 \end{bmatrix} \quad (19)$$
$$C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

where *A* is a state matrix, *B* is an input matrix, and *C* is an output matrix of the two-wheeled robot.

2. Control signal is usually represented by the number of adjustable controller parameters (θ). Due to the plant specifications and reference model (second-order system model), the controller for the two-wheeled robot has three adjustment parameters, (θ_1, θ_2, and θ_3) [16]. Usually, these three control parameters depend on the adaptation gain (γ) which can change the control algorithm of the adaptation mechanism. The form of the control signal (u) is as seen in Equation (20):

$$u = \theta_1 r - \theta_2 y - \theta_3 \dot{y} \quad (20)$$

subtitute Equation (20) into (18), then its gives Equation (21),

$$\dot{x} = A_{ct} x + B_{ct} r \quad (21)$$

with $x = [y \quad \dot{y}]^T$,

$$A_{ct} = \begin{bmatrix} 0 & 1 \\ 38.54\,\theta_2 & 6.9\theta_3 \end{bmatrix};\ B_{ct} = \begin{bmatrix} 0 \\ 6.9\theta_1 \end{bmatrix} \quad (22)$$

A_{ct} is a new state matrix, B_{ct} is a new input matrix of the plant.

The Lyapunov method is used to state the adjustment mechanism, where MRAC must be able to guarantee the stability and convergence of adaptation errors. Therefore, the differential equation of adaptation error is as follows:

$$\dot{x}_e = A_m x_e + (A_m - A_{ct})x + (B_m - B_{ct})r \quad (23)$$

where $x_e = [e \quad \dot{e}]^T$, and the error signal is denoted in Equation (6).

The main problem in Lyapunov's is determining the positive definite function of the function $V(t, x_e)$. Based on Lyapunov's Theorem 2, the quadratic function of V is given in Equation (24) as follows:

$$V = \frac{1}{2}\left\{ \begin{array}{c} \gamma x_e{}^T P x_e + tr\{(A_{ct} - A_m)^T (A_{ct} - A_m)\} + \\ tr\{(B_{ct} - B_m)^T (B_{ct} - B_m)\} \end{array} \right\} \quad (24)$$

This function will be zero if the error is zero and the controller parameters (θ_1, θ_2, and θ_3) match the desired values. The derivative of Equation (24) is given in Equations (25), (26), and (27) [14]:

$$\dot{V} = \frac{1}{2}\{2\gamma x_e{}^T P (A_m x_e + (A_{ct} - A_m)x + (B_{ct} - B_m)r) + 2tr\{(A_{ct} - A_m)^T (A_{ct} - A_m)\} + 2tr\{(B_{ct} - B_m)^T (B_{ct} - B_m)\} \quad (25)$$

$$\dot{V} = -\frac{1}{2}\gamma x_e{}^T Q x_e + tr\{(A_{ct} - A_m)^T (\dot{A_{ct}} + \gamma P x_e x^T)\} + tr\{(B_{ct} - B_m)^T (\dot{B_{ct}} + \gamma P x_e r^T)\} \quad (26)$$

where,

$$A_m{}^T P + P A_m = -Q \quad (27)$$

Q is an identity matrix and $\dot{V}$ is a negative definite. Therefore, the solution is asymptotically stable if we choose the adaptation laws for closed-loop systems, given in Equations (28) and (29) as follows [16]:

$$\frac{dA_{ct}}{dt} = -\gamma P x_e x^T \quad (28)$$

$$\frac{dB_{ct}}{dt} = -\gamma P x_e r^T \quad (29)$$

by substituting the identity matrix (Q) into Equation (27), the matrix solution *P* is obtained as seen in Equation (30).

$$P = \begin{bmatrix} \frac{1}{\omega_n}(\zeta + (1 + \omega_n{}^2)/4\zeta & \frac{1}{2\omega_n{}^2} \\ \frac{1}{2\omega_n{}^2} & \frac{1}{4\zeta\omega_n}(1 + 1/\omega_n{}^2) \end{bmatrix} \quad (30)$$

The controller parameters are obtained as

$$\dot{\theta_1} = \gamma_1 [P_{12} e + P_{22} \dot{e}] r \quad (31)$$
$$\dot{\theta_2} = -\gamma_2 [P_{12} e + P_{22} \dot{e}] y \quad (32)$$
$$\dot{\theta_3} = -\gamma_3 [P_{12} e + P_{22} \dot{e}] \dot{y} \quad (33)$$

where P_{12} and P_{22} are the elements of the matrix *P* and γ_1, γ_2, γ_3 are the adaptive gains.

V. Methodology of Experiment

A quantitative approach is used in this research. The first experiment was performed by modeling a two-wheeled robot plant. The system state-space matrix is obtained as stated in Equation (3)-(4). If the robot parameter values in Table 1 are substituted into equation (3), then the robot state space is stated in equation (19), then the design of the reference model is as in Equation (17).

The Accelero-Gyro sensor (MPU6050) was used as feedback from the robot position. Kalman Filter has been designed to minimize measurement noise. The Lyapunov stability analysis method was used to find the stability of the system represented by matrix solution *P* (Equation 30), and the control parameters were obtained in Equations (31)-(33).

The results of the MRAC design were tested through simulation, and the results were analyzed to obtain the best control parameters. The final experiment step is implementing the MRAC algorithm into the embedded system in this study, the Arduino-Uno as the controller board. The hardware setup shows in Figure 5.

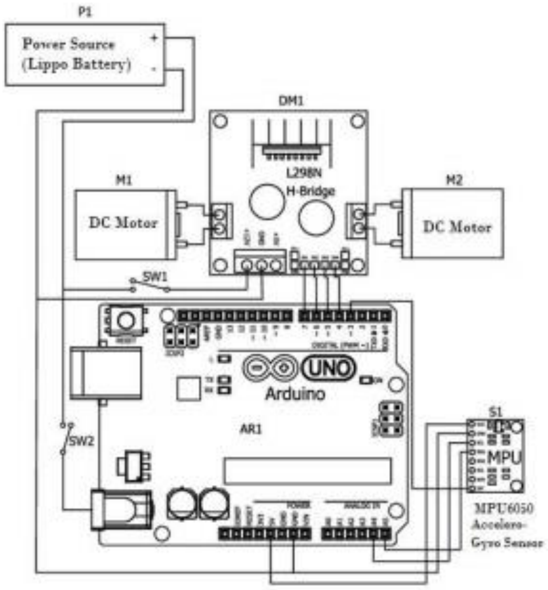

Figure 5: The Hardware Setup [12]

VI. Results and Discussion

This section shows the results of the research, which consist of the hardware realization of two-wheeled robots and the implementation of constructed control structures in the plants. The two-wheeled robot plant, as shown in Figure 6, consists of two wheels, two DC motors, and an embedded controller above it.

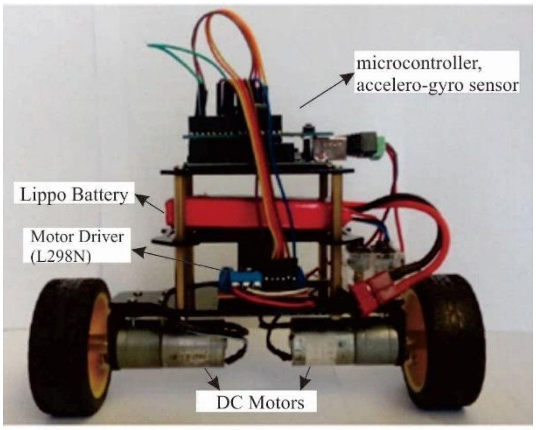

Figure 6: Two-wheeled robot

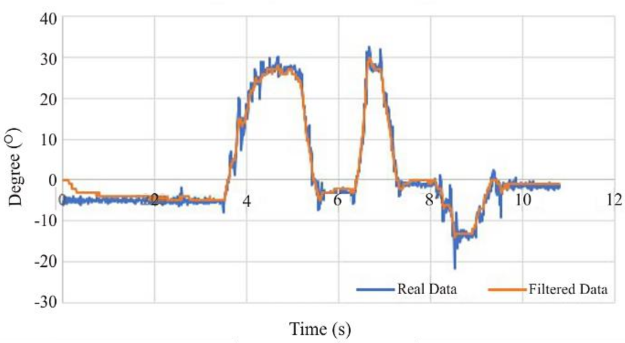

Figure 7: Response using Kalman Filter

The two-wheeled robot is built using an Accelero-Gyro sensor as a feedback robot position that is above the robot. Therefore, Kalman Filter is added to solve the noise problem in the feedback sensor. Based on the explanation in [12], matrix Qacc = {0.4; 0.01; 0.001}, Qgyro = {0.2; 0.03; 0.003}, and the matrix value R = {1; 50; 100} were used in this research. The system response using the filter is shown in Figure 7.

As shown in Equations (31) - (33), the adaptive gain that affects the angular position of the pendulum is γ_2, Therefore, the controller test was performed by setting the adaptive gain value γ_1, γ_3 as constant, while γ_2 varies. The first controller test was performed by selecting the adaptive gain value of γ_1 = 1, γ_2 = 0.005, and γ_3 = 0.001. The test results are shown in Figure 8.

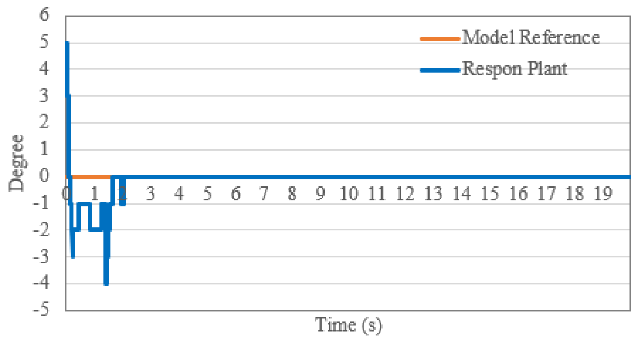

Figure 8: The response with γ_1 = 1, γ_2= 0.005, and γ_3= 0.001

Figure 8 shows that the response can reach settling conditions at 2.1 seconds, with a maximum undershoot of -4 °.

Furthermore, the value of γ_2 changed to 0.1, while the adaptive gain γ_1, γ_3 have a fixed value, 1 and 0.001, respectively. The system response in Figure 9 shows that the two-wheeled robot takes a long time to stabilize, which is 5 seconds with a maximum overshoot of 5 ° of rod tilt.

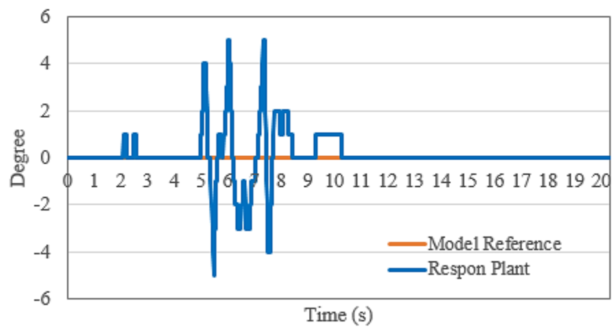

Figure 9: The response with γ_1 = 1, γ_2 = 0.1, and γ_3 = 0.001

Lyapunov stability theory is used to obtain the solution matrix of *P* (as stated in Equation 30) so that the controller parameters of MRAC are formulated as in Equations (31) - (33). The system response performance is also affected by the change of adaptive gain values ($\gamma_1, \gamma_2, \gamma_3$). Where the value of adaptive gain γ_1 affects the reference model, adaptive gain γ_2 affects the angular position, and adaptive gain $\gamma_3$affects the angular velocity of the plant. It can be seen from Figure 8 – 9 that the smaller the adaptive gain value γ_2, the faster the plant reaches a stable position.

To see the robustness of the system, the test was performed by giving external disturbance (the plant is forced by hand) to the system with a slope of 6˚. The system response with external disturbance is shown in Figure 10, where the system takes 4.72 seconds to stabilize at the origin point.

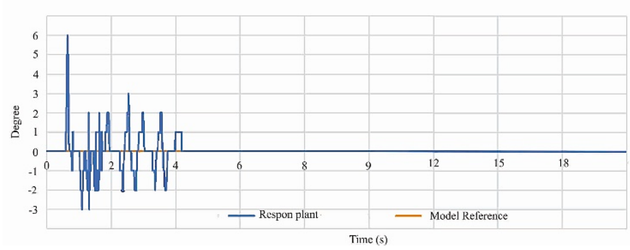

Figure 10: Response against the disturbance

VII. CONCLUSION

The design of the Model Reference Adaptive Control (MRAC) using the Lyapunov stability theory is discussed in this paper. The model reference is determined based on the desired transient response of the two-wheeled robot. In this research, the desired overshoot and settling time are 0-10% and 1 s, respectively. The MARC design process begins by comparing the two-wheeled robot output with the reference model output. Lyapunov stability theorem is used to obtain a control signal ($u = \theta_1 r - \theta_2 y - \theta_3 \dot{y}$) that can make a zero error signal (asymptotic stable). To get the adjustment parameters (θ_1, θ_2, and θ_3), it is necessary to tune the adaptation gain ($\gamma_1, \gamma_2, \gamma_3$).

The experiment results show that for adaptation gain $\gamma_1, \gamma_2, \gamma_3$ are 1, 0.005 and 0.001, respectively, the response could follow the reference model with rise time and settling time 0.27 s and 0.87 s, respectively. The proposed control method can also overcome the presence of external disturbance.

ACKNOWLEDGMENT

We would like to thank P3M Politeknik Negeri Bandung for the research grant "Penelitian Mandiri 2021"

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