Complexity Reduction of Turbo Equalization Using Cross-Entropy Stopping Criterion

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Abstract**—This article presents the application of the Cross-Entropy (CE) stopping criterion to Turbo Equalizer using the Maximum A Posteriori (MAP) algorithm. It is part of reducing computational complexities by decreasing the number of iterations of a turbo receiver. The proposed criterion CE initially for turbo codes is applicable for any type of turbo receiver. We consider here a turbo equalizer MAP of a severe channel that has five coefficients. The simulation results show that the resulting MAP-CE Turbo Equalizer provides the same performances as the Turbo Equalizer, which uses the absolute number of iterations.**

Index Terms**—Turbo Equalizer; Stopping Criterion; Average Number of Iterations; Cross-Entropy.**

I. INTRODUCTION

The Turbo principle has revolutionized digital communications since its invention [1]. Turbo codes performances have almost reached Shannon's theoretical limit. The decoding quality is improved when the number of iterations is increased. However, it is useless to continue the iterative processing when the frame is decoded. For this, several stopping criteria for decoding are proposed in the literature [2 - 13]. Among these criteria, the Cross-Entropy rule CE [2-3][13] guarantees acceptable performance in turbo decoding with reasonable complexity.

The Turbo principle can be applied to joint equalization and decoding to combat the InterSymbol Interference (ISI) created by the selectivity of channels,. The turbo receiver obtained is called Turbo Equalizer. It guarantees complete suppression of ISI after a given Signal-to-Noise Ratio (SNR) [14] [15].

It is necessary to stop processing after the total correction of the received frame to reduce the computational complexity of the MAP turbo equalizer [15]. So, is it possible to apply the Cross-Entropy CE criterion to the Turbo equalizer to reduce the number of iterations? Moreover, what is the degradation in performance compared to the Turbo equalizer using the total (Full) number of iterations?

This article presents answers to these questions and shows that the CE criterion behaves as a smart rule to stop turbo equalization. The article is structured as follows: In section II, the MAP turbo equalizer is presented. Section III presents the principle of the Cross-Entropy CE criterion. Section IV describes the system model, and section V presents the simulation results. The article ends with a conclusion.

II. MAP TURBO EQUALIZER

Authors in [14] and [15] show that the Turbo principle is applicable in joint equalization and decoding. This process achieves ideal channel performance with complete cancellation of InterSymbol Interference ISI. Over the iterations, an exchange of information is done between the equalizer and the MAP decoder [16]. This exchange improves the estimation of the detected and decoded symbols.

A turbo equalizer is a receiver adapted to a transmitter consisting of a channel encoder followed by a modulator, as shown in Figure 1. They are separated by an interleaver (BICM system: Bit Interleaved Coded Modulation). The information bits *u(k)* are encoded, interleaved (*c(n)*), mapped into Binary Phase Shift Keying BPSK symbols (*d(n)*) and then transmitted through a frequency selective channel.

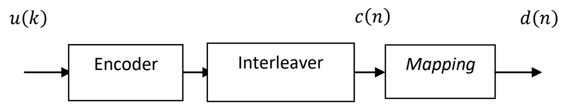

Figure 1: Model of a BICM system

The scheme of a MAP Turbo equalizer is represented in Figure 2.

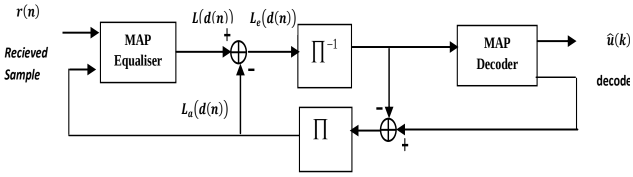

Figure 2: MAP Turbo equalizer

The equalizer calculates the Log-Likelihood Ratio LLRs of the transmitted symbols and then provides these measurements to the MAP decoder. It calculates the LLRs of the emitted symbols [16]:

$$L\big(d(n)\big) = \log\left(\frac{prob(d(n) = +1\,/R)}{prob(d(n) = -1\,/R)}\right) \tag{1}$$

R is the received frame.

$$R = \{r_1\,, r_2\,, \dots\dots\dots, r_N\} \tag{2}$$

N is the length of the frame.

Using the MAP algorithm, this measure can be estimated by [16],

$$L\big(d(n)\big) = \log\left(\frac{\sum_{(\grave{m},m)/d(n)=+1} \alpha_{\grave{m}}(n-1)\, \gamma_{(\grave{m},m)}(n)\, \beta_m(n)}{\sum_{(\grave{m},m)/d(n)=-1} \alpha_{\grave{m}}(n-1)\, \gamma_{(\grave{m},m)}(n)\, \beta_m(n)}\right) \quad (3)$$

where $\gamma_{(\grave{m},m)}(n)$ are the probabilities of the channel trellis transitions between the state $\grave{m}$ and m at time n. $\alpha_{\grave{m}}(n)$ are the forward probabilities of the MAP algorithm. $\beta_m(n)$ are its backward probabilities [16].

$L\big(d(n)\big)$ can be written as in Equation (4) [15],

$$L\big(d(n)\big) = L_e\big(d(n)\big) + L_a\big(d(n)\big) \quad (4)$$

where $L_a\big(d(n)\big)$ is the intrinsic information (a priori probability) of the $d(n)$ symbols provided by the decoder. To determine the extrinsic information (a posteriori probability), it suffices to extract (Figure 2):

$$L_e\big(d(n)\big) = L\big(d(n)\big) - L_a\big(d(n)\big) \quad (5)$$

This information is applied to the input of the decoder after deinterleaving.

The decoder estimates the LLRs of the information bits *u(k)* by the same MAP algorithm and can therefore apply a decision. In addition, it provides the LLRs of the coded bits *d(n)*, which represent the extrinsic information, and therefore, they can be used as the a priori information $L_a\big(d(n)\big)$ by the subsequent iteration (Figure 2).

III. Cross-Entropy Criterion

The Cross-Entropy criterion uses the LLRs at the outputs of two processings to provide information on the end of decoding [2] [13]. Let us consider the Turbo receiver shown in Figure 3.

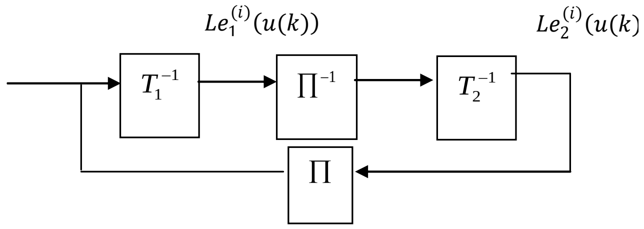

Figure 3: Turbo receiver

T_1^{-1} and T_2^{-1} are the reverse processings of those performed at emission. ∏ and ∏-1 represent the associated interleaver and deinterleaver, respectively. Let $Le_m^{(i)}(u(k))$ be the extrinsic information of the bits $u(k)$ provided by the m^{th} treatment at iteration (*i*).

The Cross-Entropy of the iteration (*i*) is given by Equation (6);

$$CE(i) \approx \sum_n \frac{\left|\Delta Le_2^{(i)}\big(u(k)\big)\right|^2}{e^{\left|L_1^{(i)}(u(k))\right|}} \quad (6)$$

where,

$$\Delta Le_2^{(i)}\big(u(k)\big) = Le_2^{(i)}\big(u(k)\big) - Le_2^{(i-1)}\big(u(k)\big) \quad (7)$$

That is the difference between the extrinsic information of two successive iterations.

Equation (7) is also equivalent to Equation (8).

$$\Delta Le_2^{(i)}\big(u(k)\big) = L_2^{(i)}\big(u(k)\big) - L_1^{(i)}\big(u(k)\big) \quad (8)$$

$L_1^{(i)}\big(u(k)\big)$ is the LLR of the information bits $u(k)$ calculated by the first processing T_1^{-1}.

The frame is correct if:

$$CE(i) < \varepsilon \quad (9)$$

with

$$10^{-2}\, CE(1) \le \varepsilon \le 10^{-4}\, CE(1) \quad (10)$$

$CE(1)$ is the Cross-Entropy of the first iteration.

For the Turbo Equalizer, the processing T_1^{-1} is a MAP equalizer. T_2^{-1} is a MAP decoder. In this case, T_2^{-1} can provide the extrinsic information $Le_2^{(i)}(u(k))$ of the information bits $u(k)$. While T_1^{-1} is responsible for estimating the LLRs $L(d(n))$ of the coded and mapped symbols $d(n)$ but cannot have information on the $u(k)$ bits. For this, we apply the Cross-Entropy CE criterion using the symbols $d(n)$ since the MAP decoder can also estimate their LLR. In this case, the criterion becomes:

For each processed frame, calculate the Cross-Entropy of the symbols $d(n)$ by:

$$CE(i) = \sum_n \frac{\left|\Delta Le_2^{(i)}\big(d(n)\big)\right|^2}{e^{\left|L_1^{(i)}(d(n))\right|}} \quad (11)$$

where;

$$\Delta Le_2^{(i)}\big(d(n)\big) = Le_2^{(i)}\big(d(n)\big) - Le_2^{(i-1)}\big(d(n)\big) \quad (12)$$

The frame is correct if;

$$CE(i) < \varepsilon \quad (13)$$

with always:

$$10^{-2}\, CE(1) \le \varepsilon \le 10^{-4}\, CE(1) \quad (14)$$

We choose;

$$\varepsilon = 10^{-3}\, CE(1) \quad (15)$$

IV. System Model

The model of the used system consists of a Non-Systematic Convolutional encoder (NSC) of polynomials [23,35], followed by a pseudo-random S-Random interleaver of size 5120. The information bits $u(k)$ are encoded with this encoder, interleaved, mapped using a BPSK constellation (*d(n)*) and then transmitted through a selective Gaussian channel. Its impulse response is:

$$h(n) = 0.227\,\delta(n) + 0.46\,\delta(n-1) + 0.688\,\delta(n-2) + 0.46\,\delta(n-3) + 0.227\,\delta(n-4) \quad (16)$$

It is called the 'Proakis C channel'. Its frequency response is shown in Figure 4. It is a complex channel to equalize (Attenuation of − 60 dB in Medium frequencies MF).

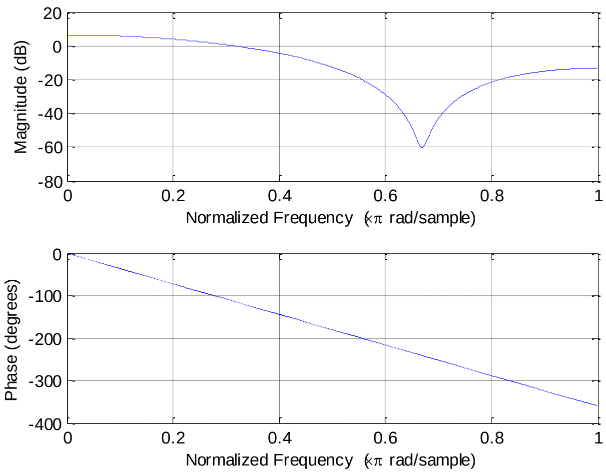

Figure 4: Frequency response of the Proakis C channel

The receiver is a MAP Turbo equalizer. It means, both the equalizer and decoder use the MAP algorithm. The two are separated by an interleaver and a deinterleaver (Figure 2). This Turbo equalizer uses a maximum of 10 iterations and driven by the Cross-Entropy criterion. The turbo processing is stopped at iteration (*i*) when the criterion is verified. That is, when

$$\mathrm{CE(i)} < 10^{-3}\,\mathrm{CE(1)} \tag{17}$$

The number of transmitted frames is 3000.

V. Simulation Results

The simulation results show that the MAP turbo equalizer controlled by the Cross-Entropy stopping criterion ensures the same performances of the turbo equalizer using the full number of iterations (full, here is ten iterations). Their Bit Error Rates BER (Figure 5) and Frame Error Rates FER (Figure 6) are the same.

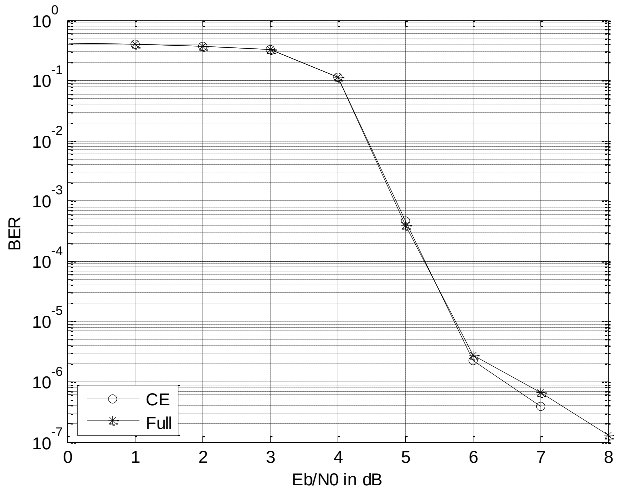

Figure 5: Bit Error Rate (BER) of MAP-CE and MAP-Full turbo equalizers.

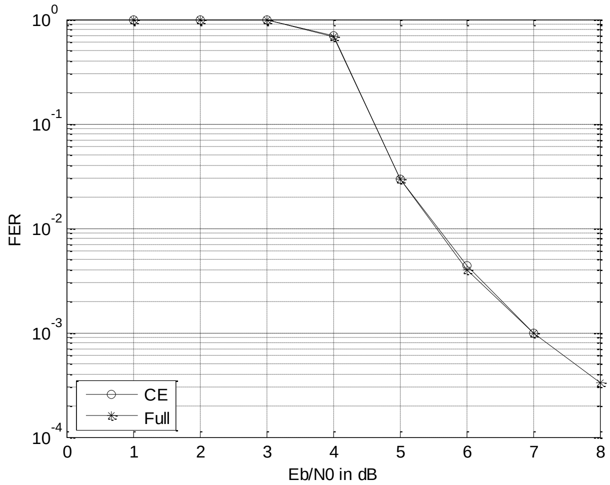

Figure 6: Frame Error Rate (FER) of MAP-CE and MAP-Full turbo equalizers

Figure 7 plots the average number of iterations of the MAP turbo equalizer coupled to the Cross-Entropy criterion. In high SNRs, the criterion stopped decoding at the right time. The average number of iterations is reduced from 10 iterations to 3 iterations at SNR = 8 dB. In addition, the behavior of the CE criterion applied to turbo equalization is considered intelligent (smart) in low SNR because it has stopped the turbo processing when the interference is strong; subsequently, the frames will be undecodable. At SNR = 1 dB, it uses four iterations only. The maximum number of iterations reached 10 for SNR = 4 dB, which is the trigger point of the Turbo process of this channel. This is the point at which performance improves. This phenomenon is interesting. We have called it ‘the phenomenon 8 eight’ because it describes the East Indo-Arabic number ٨ ≡ 8.

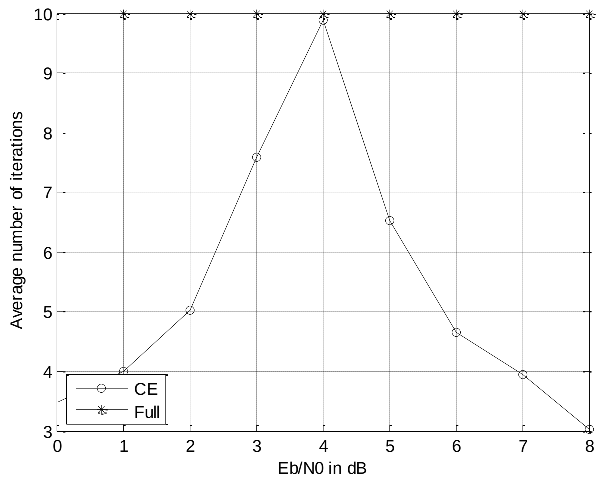

Figure 7: Average number of iterations of the MAP Turbo equalizer using the CE criterion

VI. Conclusion

This article shows that the Cross-Entropy criterion is an ideal rule for stopping the iterative processing of the MAP Turbo Equalizer. The MAP-CE Turbo Equalizer provides the same performances as the Full Turbo Equalizer. In addition, the Cross-Entropy criterion stops the Turbo processing at the right time and minimizing the number of iterations, and therefore the computational complexity. The turbo equalizer controlled by the Cross-Entropy stopping criterion ensures the same BER and FER of the turbo equalizer using a full number of iterations. The average number of iterations is reduced from 10 iterations to 3 iterations at high SNR. According to the Eight 8 phenomenon, the stopping process shows a maximum use of iterations at a trigger point. At strong SNRs (corrected frames) and low SNRs (undecodable frames), the criterion saves the computation complexity as much as possible.

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