On Optimization of Manufacturing of an Enhanced Swing Differential Colpitts Oscillator Based on Heterostructures to Increase Density of their Elements: Influence of Miss-Match Induced Stress

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***Abstract*— This paper presents an analytical approach to model linear and nonlinear mass and heat transport with space and time varying parameters. The approach gives a possibility to predict mass and heat transport in multilayer structures without crosslinking the solution on interfaces between layers in these structures. Based on the approach, we analyzed the possibility to increase the density of field-effect heterotransistors in an enhanced swing differential Colpitts oscillator. The results indicate that the increase in the density of these transistors could be obtained by manufacturing them in a heterostructure with specific configuration using dopant diffusion or ion implantation with future optimized annealing. We also found that the fulfillment of specific detected conditions gives a possibility to decrease the mismatch-induced stress, which was generated in layers of the considered heterostructure.**

***Index Terms*— Analytical Approach for Modelling; Enhanced Swing Differential Colpitts Oscillator; Increasing of Density of Elements.**

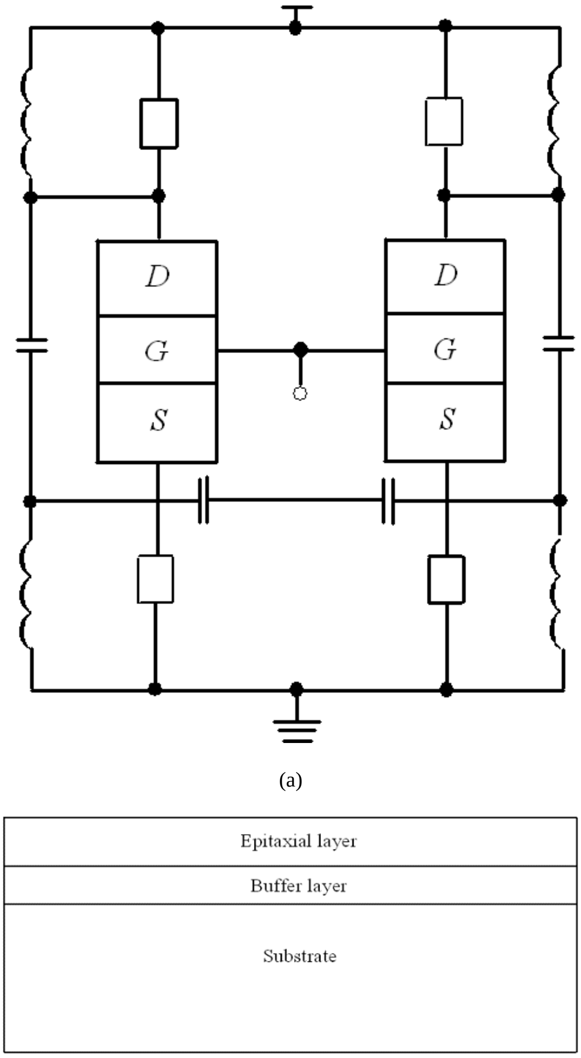
I. INTRODUCTION

At present, problems of solid state electronics (such as increasing of performance, reliability and density of elements of integrated circuits: diodes, field-effect and bipolar transistors) are intensively being solved [1-6]. There have been increasingly interests on the determination of materials with higher values of charge carrier's mobility [7-10] to increase the performance of these devices. One way to decrease dimensions of elements of integrated circuits is manufacturing them in thin film heterostructures [3-5, 11]. In this case, it is possible to use inhomogeneity of heterostructure, optimize doping of electronic materials [12] and develop epitaxial technology to improve these materials (including analysis of mismatch induced stress) [13-15]. An alternative approach to change the dimensions of integrated circuits are using laser and microwave types of annealing [16-18]. One of the approaches is to use inhomogeneity of distribution of temperature during annealing. The inhomogeneity leads to inhomogeneity of physical and technological parameters: diffusion coefficient of dopant and radiation defects (for ion type of doping), parameter of recombination of point radiation defects, due to Arrhenius law. However, the inhomogeneity of parameters is usually smoother in comparison with the analogous inhomogeneity in heterostructures.

In this the paper, we introduce an approach to manufacture field-effect transistors based on a thin film technology. The approach gives a possibility to decrease their dimensions by increasing their density framework to an enhanced swing differential Colpitts oscillator. We also consider the possibility to decrease mismatch-induced stress to decrease quantity of defects, generated due to the stress.

In this paper, we consider a heterostructure, which consists of a substrate and an epitaxial layer (see Figure 1). We also consider a buffer layer between the substrate and the epitaxial layer. The buffer layer is usually used as a layer with intermediate value of the lattice constant between lattice constants of the substrate and the epitaxial layer. The intermediate value of the lattice constant gives a possibility to decrease mismatch-induced stress in the substrate and the epitaxial layer. The considered epitaxial layer includes into itself several sections, manufactured by using another materials. These sections have been doped by diffusion or ion implantation to manufacture the required types of conductivity (*p* or *n*). These areas became sources, drains and gates (see Figure 1). After this doping, it is required annealing of dopant and/or radiation defects.

The main aim of this paper is to analyse the redistribution of dopant and radiation defects to determine conditions, which correspond to the decreasing of elements of the considered oscillator and at the same time to increase their density. At the same time, we consider a possibility to decrease mismatch-induced stress. We also introduce an analytical approach to model linear and nonlinear mass and heat transport with space and time varying parameters. The approach gives a possibility to predict mass and heat transport in multilayer structures without crosslinking of solution on interfaces between layers in these structures. In this situation, the introduced approach gives a possibility to make more adequate analysis of mass and heat transport in comparison with the recently introduced analytical approaches.



(a)

(b)

Figure 1: (a) Structure of the considered oscillator [24], and (b) Heterostructure with a substrate, epitaxial layers and buffer layer (view from side)

II. METHOD OF SOLUTION

We determined and analyzed spatio-temporal distribution of concentration of dopant in the considered heterostructure. We determined the distribution by solving the second Fick's law in the following form [1, 19-23].

$$\begin{aligned}\frac{\partial C(x,y,z,t)}{\partial t} &= \frac{\partial}{\partial x}\left[D\frac{\partial C(x,y,z,t)}{\partial x}\right]+ \\ &+\frac{\partial}{\partial y}\left[D\frac{\partial C(x,y,z,t)}{\partial y}\right]+\frac{\partial}{\partial z}\left[D\frac{\partial C(x,y,z,t)}{\partial z}\right]+ \\ &+\Omega\frac{\partial}{\partial x}\left[\frac{D_S}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}C(x,y,W,t)\,dW\right]+ \\ &+\Omega\frac{\partial}{\partial y}\left[\frac{D_S}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}C(x,y,W,t)\,dW\right]\end{aligned} \quad (1)$$

The boundary and initial conditions are:

$$\left.\frac{\partial C(x,y,z,t)}{\partial x}\right|_{x=0}=0;\ \left.\frac{\partial C(x,y,z,t)}{\partial x}\right|_{x=L_x}=0;$$
$$\left.\frac{\partial C(x,y,z,t)}{\partial y}\right|_{y=0}=0;\ \left.\frac{\partial C(x,y,z,t)}{\partial y}\right|_{x=L_y}=0;$$
$$\left.\frac{\partial C(x,y,z,t)}{\partial z}\right|_{z=0}=0;\ \left.\frac{\partial C(x,y,z,t)}{\partial z}\right|_{x=L_z}=0;$$
$$C(x,y,z,0)=f_C(x,y,z)$$

where: $C(x,y,z,t)$ = Spatio-temporal distribution of concentration of dopant
Ω = Atomic volume of dopant
∇_s = Symbol of surficial gradient $\int_0^{L_z}C(x,y,z,t)\,dz$ = Surficial concentration of dopant on interface between layers of heterostructure (in this situation we assume, that Z-axis is perpendicular to interface between layers of heterostructure)
$\mu_1(x,y,z,t)$ = Chemical potential due to the presence of mismatch-induced stress
D = Coefficients of volumetric diffusions
D_S = Coefficients of surficial diffusions

Values of dopant diffusions coefficients depend on the properties of the materials of heterostructure, speed of heating and cooling of materials during annealing, and spatio-temporal distribution of concentration of dopant. Dependences of dopant diffusions coefficients on parameters could be approximated by the following relations [21-23].

$$\begin{aligned}D_C &= D_L(x,y,z,T)\left[1+\xi\frac{C^{\gamma}(x,y,z,t)}{P^{\gamma}(x,y,z,T)}\right]\times \\ &\times\left[1+\varsigma_1\frac{V(x,y,z,t)}{V^*}+\varsigma_2\frac{V^2(x,y,z,t)}{(V^*)^2}\right] \\ D_S &= D_{SL}(x,y,z,T)\left[1+\xi\frac{C^{\gamma}(x,y,z,t)}{P^{\gamma}(x,y,z,T)}\right]\times \\ &\times\left[1+\varsigma_1\frac{V(x,y,z,t)}{V^*}+\varsigma_2\frac{V^2(x,y,z,t)}{(V^*)^2}\right]\end{aligned} \quad (2)$$

where: $D_L(x,y,z,T)$ = Spatial (due to accounting all layers of heterostruicture)
$D_{LS}(x,y,z,T)$ = Temperature (due to Arrhenius law) dependences of dopant diffusion coefficients
T = Temperature of annealing
$P(x,y,z,T)$ = Limit of solubility of dopant
γ = Parameter that depends on properties of materials and could be integer in the following interval $\gamma\in[1,3]$ [21]
$V(x,y,z,t)$ = Spatio-temporal distribution of concentration of radiation vacancies
V^* = Equilibrium distribution of vacancies

Concentrational dependence of dopant diffusion coefficient has been described in detail in [21]. Spatio-temporal distributions of concentration of point radiation defects have been determined by solving the following system of equations [19, 22-23].

$$\begin{aligned}
\frac{\partial I(x,y,z,t)}{\partial t} &= \frac{\partial}{\partial x}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial x}\right]+ \\
&+\frac{\partial}{\partial y}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial y}\right]+ \\
&+\frac{\partial}{\partial z}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial z}\right]- \\
&-k_{I,I}(x,y,z,T)\,I^2(x,y,z,t)- \\
&-k_{I,V}(x,y,z,T)I(x,y,z,t)V(x,y,z,t)+ \\
&+\Omega\frac{\partial}{\partial x}\left[\frac{D_{IS}}{kT}\nabla_S\mu(x,y,z,t)\int_0^{L_z}I(x,y,W,t)dW\right]+ \\
\frac{\partial V(x,y,z,t)}{\partial t} &= \frac{\partial}{\partial x}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial x}\right]+ \\
&+\frac{\partial}{\partial y}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial y}\right]+ \\
&+\frac{\partial}{\partial z}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial z}\right]- \\
&-k_{V,V}(x,y,z,T)\,V^2(x,y,z,t)- \\
&-k_{I,V}(x,y,z,T)\,I(x,y,z,t)V(x,y,z,t)+ \\
&+\Omega\frac{\partial}{\partial x}\left[\frac{D_{VS}}{kT}\nabla_S\mu(x,y,z,t)\int_0^{L_z}V(x,y,W,t)\,dW\right]+ \\
&+\Omega\frac{\partial}{\partial y}\left[\frac{D_{VS}}{kT}\nabla_S\mu(x,y,z,t)\int_0^{L_z}V(x,y,W,t)dW\right]
\end{aligned} \tag{3}$$

The boundary and initial conditions are:

$$\begin{aligned}
&\left.\frac{\partial I(x,y,z,t)}{\partial x}\right|_{x=0}=0;\ \left.\frac{\partial I(x,y,z,t)}{\partial x}\right|_{x=L_x}=0; \\
&\left.\frac{\partial I(x,y,z,t)}{\partial y}\right|_{y=0}=0;\ \left.\frac{\partial I(x,y,z,t)}{\partial y}\right|_{y=L_y}=0; \\
&\left.\frac{\partial I(x,y,z,t)}{\partial z}\right|_{z=0}=0;\ \left.\frac{\partial I(x,y,z,t)}{\partial z}\right|_{z=L_z}=0; \\
&\left.\frac{\partial V(x,y,z,t)}{\partial x}\right|_{x=0}=0;\ \left.\frac{\partial V(x,y,z,t)}{\partial x}\right|_{x=L_x}=0; \\
&\left.\frac{\partial V(x,y,z,t)}{\partial y}\right|_{y=0}=0;\ \left.\frac{\partial V(x,y,z,t)}{\partial y}\right|_{y=L_y}=0; \\
&\left.\frac{\partial V(x,y,z,t)}{\partial z}\right|_{z=0}=0;\ \left.\frac{\partial V(x,y,z,t)}{\partial z}\right|_{z=L_z}=0; \\
&I(x,y,z,0)=f_I(x,y,z);\ V(x,y,z,0)=f_V(x,y,z).
\end{aligned} \tag{4}$$

where: I (x,y,z,t) = Spatio-temporal distribution of concentration of radiation interstitials

I^* = Equilibrium distribution of interstitials

$D_I(x,y,z,T)$ = Coefficients of volumetric diffusion of intersitials

$D_V(x,y,z,T)$ = Coefficients of surficial diffusions of interstitials

$D_{IS}(x,y,z,T)$ = Coefficients of volumetric diffusion of vacancies

$D_{VS}(x,y,z,T)$ = Coefficients of surficial diffusions of vacancies

$V^2(x,y,z,t)$ = Terms for generation of divacancies,and

$I^2(x,y,z,t)$ = Terms for generation of diinterstitials, (see, for example, [23] and appropriate references in this book)

$k_{I,V}(x,y,z,T)$, $k_{I,I}(x,y,z,T)$ and $k_{V,V}(x,y,z,T)$ = Parameters of recombination of point radiation defects and generation of their complexes

Spatio-temporal distributions of divacancies Φ_V (x,y,z,t) and diinterstitials Φ_I (x,y,z,t) could be determined by solving the following system of equations [19, 22-23].

$$\begin{aligned}
\frac{\partial \Phi_I(x,y,z,t)}{\partial t} &= k_{I,I}(x,y,z,T)\,I^2(x,y,z,t)+ \\
&+\frac{\partial}{\partial x}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial x}\right]+ \\
&+\frac{\partial}{\partial y}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial y}\right]+ \\
&+\frac{\partial}{\partial z}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial z}\right]+ \\
&+\Omega\frac{\partial}{\partial x}\left[\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}\Phi_I(x,y,W,t)dW\right]+ \\
&+\Omega\frac{\partial}{\partial y}\left[\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}\Phi_I(x,y,W,t)dW\right]+ \\
&+k_I(x,y,z,T)\,I(x,y,z,t) \\
\frac{\partial \Phi_V(x,y,z,t)}{\partial t} &= k_{V,V}(x,y,z,T)V^2(x,y,z,t)+ \\
&+\frac{\partial}{\partial x}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial x}\right]+ \\
&+\frac{\partial}{\partial y}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial y}\right]+ \\
&+\frac{\partial}{\partial z}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial z}\right]+ \\
&+\Omega\frac{\partial}{\partial x}\left[\frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}\Phi_V(x,y,W,t)dW\right]+ \\
&+\Omega\frac{\partial}{\partial y}\left[\frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}\Phi_V(x,y,W,t)dW\right]+ \\
&+k_V(x,y,z,T)V(x,y,z,t)
\end{aligned} \tag{5}$$

The boundary and initial conditions are:

$$\left.\frac{\partial \Phi_I(x,y,z,t)}{\partial x}\right|_{x=0}=0\,;\ \left.\frac{\partial I(x,y,z,t)}{\partial x}\right|_{x=L_x}=0\,;$$
$$\left.\frac{\partial I(x,y,z,t)}{\partial y}\right|_{y=0}=0\,;\ \left.\frac{\partial I(x,y,z,t)}{\partial y}\right|_{y=L_y}=0\,;$$
$$\left.\frac{\partial \Phi_I(x,y,z,t)}{\partial z}\right|_{z=0}=0\,;\ \left.\frac{\partial I(x,y,z,t)}{\partial z}\right|_{z=L_z}=0\,;$$
$$\left.\frac{\partial \Phi_V(x,y,z,t)}{\partial x}\right|_{x=0}=0\,;\ \left.\frac{\partial V(x,y,z,t)}{\partial x}\right|_{x=L_x}=0\,; \qquad (6)$$
$$\left.\frac{\partial V(x,y,z,t)}{\partial y}\right|_{y=0}=0\,;\ \left.\frac{\partial V(x,y,z,t)}{\partial y}\right|_{y=L_y}=0\,;$$
$$\left.\frac{\partial V(x,y,z,t)}{\partial z}\right|_{z=0}=0\,;\ \left.\frac{\partial \Phi_V(x,y,z,t)}{\partial z}\right|_{z=L_z}=0\,;$$
$$\Phi_I(x,y,z,0)=f_{\Phi I}(x,y,z);$$
$$\Phi_V(x,y,z,0)=f_{\Phi V}(x,y,z)$$

where: $D_{\Phi I}(x,y,z,T)$, $D_{\Phi V}(x,y,z,T)$, $D_{\Phi IS}(x,y,z,T)$ and $D_{\Phi VS}(x,y,z,T)$ = Coefficients of volumetric and surficial diffusions of complexes of radiation defects
$k_I(x,y,z,T)$ and $k_V(x,y,z,T)$ = Parameters of decay of complexes of radiation defects

Chemical potential μ_1 in Equation (1) could be determined by the following relation [19].

$$\mu_1=E(z)\Omega\sigma_{ij}\left[u_{ij}(x,y,z,t)+u_{ji}(x,y,z,t)\right]/2 \qquad (7)$$

where: $E(z)$ = Young modulus
σ_{ij} = Stress tensor
$u_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)$ = Deformation tensor
u_i, u_j = Components $u_x(x,y,z,t)$, $u_y(x,y,z,t)$ and $u_z(x,y,z,t)$ of the displacement vector $\vec{u}(x,y,z,t)$
x_i, x_j = Coordinate x, y, z

The Equation (3) could be transformed to the following form.

$$\mu(x,y,z,t)=\left[\frac{\partial u_i(x,y,z,t)}{\partial x_j}+\frac{\partial u_j(x,y,z,t)}{\partial x_i}\right]\times$$
$$\times\left\{\frac{1}{2}\left[\frac{\partial u_i(x,y,z,t)}{\partial x_j}+\frac{\partial u_j(x,y,z,t)}{\partial x_i}\right]-\varepsilon_0\delta_{ij}+\right.$$
$$+\frac{\sigma(z)\delta_{ij}}{1-2\sigma(z)}\left[\frac{\partial u_k(x,y,z,t)}{\partial x_k}-3\varepsilon_0\right]-K(z)\times$$
$$\left.\times\beta(z)\left[T(x,y,z,t)-T_0\right]\delta_{ij}\right\}\Omega E(z)/2,$$

where: σ = Poisson coefficient
$\varepsilon_0=(a_s-a_{EL})/a_{EL}$ is = Mismatch parameter
a_s, a_{EL} = Lattice distances of the substrate and the epitaxial layer
K = Modulus of uniform compression
β = Coefficient of thermal expansion
T_r = Equilibrium temperature, which coincide (for our case) with room temperature

Components of displacement vector could be obtained by solution of the following equations [25].

$$\begin{cases}\rho(z)\frac{\partial^2 u_x(x,y,z,t)}{\partial t^2}=\frac{\partial \sigma_{xx}(x,y,z,t)}{\partial x}+\\ +\frac{\partial \sigma_{xy}(x,y,z,t)}{\partial y}+\frac{\partial \sigma_{xz}(x,y,z,t)}{\partial z}\\ \rho(z)\frac{\partial^2 u_y(x,y,z,t)}{\partial t^2}=\frac{\partial \sigma_{yx}(x,y,z,t)}{\partial x}+\\ +\frac{\partial \sigma_{yy}(x,y,z,t)}{\partial y}+\frac{\partial \sigma_{yz}(x,y,z,t)}{\partial z}\\ \rho(z)\frac{\partial^2 u_z(x,y,z,t)}{\partial t^2}=\frac{\partial \sigma_{zx}(x,y,z,t)}{\partial x}+\\ +\frac{\partial \sigma_{zy}(x,y,z,t)}{\partial y}+\frac{\partial \sigma_{zz}(x,y,z,t)}{\partial z}\end{cases}$$

where $\sigma_{ij}=\frac{E(z)}{2[1+\sigma(z)]}\left[\frac{\partial u_i(x,y,z,t)}{\partial x_j}+\right.$
$$\left.+\frac{\partial u_j(x,y,z,t)}{\partial x_i}-\frac{\delta_{ij}}{3}\frac{\partial u_k(x,y,z,t)}{\partial x_k}\right]+K(z)\delta_{ij}\times$$
$$\times\frac{\partial u_k(x,y,z,t)}{\partial x_k}-\beta(z)\,K(z)\left[T(x,y,z,t)-T_r\right]$$
$\rho(z)$ = Density of materials of heterostructure
δ_{ij} = Kronecker symbol

Taking into account the relation for σ_{ij}, the last system of equation could be written as.

$$\rho(z)\frac{\partial^2 u_x(x,y,z,t)}{\partial t^2}=\left\{K(z)+\frac{5E(z)}{6[1+\sigma(z)]}\right\}\times$$
$$\times\frac{\partial^2 u_x(x,y,z,t)}{\partial x^2}+\left\{K(z)-\frac{E(z)}{3[1+\sigma(z)]}\right\}\times$$
$$\times\frac{\partial^2 u_y(x,y,z,t)}{\partial x\partial y}+\frac{E(z)}{2[1+\sigma(z)]}\left[\frac{\partial^2 u_y(x,y,z,t)}{\partial y^2}+\right.$$
$$\left.+\frac{\partial^2 u_z(x,y,z,t)}{\partial z^2}\right]+\left[K(z)+\frac{E(z)}{3[1+\sigma(z)]}\right]\times$$
$$\times\frac{\partial^2 u_z(x,y,z,t)}{\partial x\partial z}-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial x} \qquad (8)$$
$$\rho(z)\frac{\partial^2 u_y(x,y,z,t)}{\partial t^2}=\frac{E(z)}{2}\left[\frac{\partial^2 u_y(x,y,z,t)}{\partial x^2}+\right.$$
$$\left.+\frac{\partial^2 u_x(x,y,z,t)}{\partial x\partial y}\right]\frac{1}{1+\sigma(z)}-\beta(z)\frac{\partial T(x,y,z,t)}{\partial y}\times$$
$$\times K(z)+\frac{\partial}{\partial z}\left\{\left[\frac{\partial u_y(x,y,z,t)}{\partial z}+\frac{\partial u_z(x,y,z,t)}{\partial y}\right]\times\right.$$

$$\times \frac{E(z)}{2[1+\sigma(z)]}\Bigg\}+\left\{\frac{5E(z)}{12[1+\sigma(z)]}+K(z)\right\}\times \times\frac{\partial^2 u_y(x,y,z,t)}{\partial y^2}+\left\{K(z)-\frac{E(z)}{6[1+\sigma(z)]}\right\}\times \times\frac{\partial^2 u_y(x,y,z,t)}{\partial y\partial z}+K(z)\frac{\partial^2 u_y(x,y,z,t)}{\partial x\partial y}$$

$$\rho(z)\frac{\partial^2 u_z(x,y,z,t)}{\partial t^2}=\left[\frac{\partial^2 u_z(x,y,z,t)}{\partial x^2}+\frac{\partial^2 u_z(x,y,z,t)}{\partial y^2}+\frac{\partial^2 u_x(x,y,z,t)}{\partial x\partial z}+\frac{\partial^2 u_y(x,y,z,t)}{\partial y\partial z}\right]\times \times\frac{E(z)}{2[1+\sigma(z)]}+\frac{\partial}{\partial z}\left\{K(z)\left[\frac{\partial u_x(x,y,z,t)}{\partial x}+\frac{\partial u_y(x,y,z,t)}{\partial y}+\frac{\partial u_x(x,y,z,t)}{\partial z}\right]\right\}-K(z)\times \times\beta(z)\frac{\partial T(x,y,z,t)}{\partial z}+\frac{1}{6}\frac{\partial}{\partial z}\left\{\frac{E(z)}{1+\sigma(z)}\times \left[6\frac{\partial u_z(x,y,z,t)}{\partial z}-\frac{\partial u_x(x,y,z,t)}{\partial x}-\frac{\partial u_y(x,y,z,t)}{\partial y}-\frac{\partial u_z(x,y,z,t)}{\partial z}\right]\right\}$$

Conditions for the system of Equation (8) could be written in the form.

$$\frac{\partial\,\vec u(0,y,z,t)}{\partial x}=0;\ \frac{\partial\vec u(L_x,y,z,t)}{\partial x}=0;$$
$$\frac{\partial\,\vec u(x,0,z,t)}{\partial y}=0;\ \frac{\partial\,\vec u(x,L_y,z,t)}{\partial y}=0;$$
$$\frac{\partial\,\vec u(x,y,0,t)}{\partial z}=0;\ \frac{\partial\,\vec u(x,y,L_z,t)}{\partial z}=0;$$
$$\vec u(x,y,z,0)=\vec u_0;\ \vec u(x,y,z,\infty)=\vec u_0$$

We determined spatio-temporal distributions of concentrations of dopant and radiation defects by solving the Equations (1), (3) and (5) framework standard method of averaging of function corrections [26]. Previously, we transformed the Equations (1), (3) and (5) to the following forms taking into account the initial distributions of the considered concentrations.

$$\frac{\partial C(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D\frac{\partial C(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D\frac{\partial C(x,y,z,t)}{\partial y}\right]+\frac{\partial}{\partial z}\left[D\frac{\partial C(x,y,z,t)}{\partial z}\right]+\Omega\frac{\partial}{\partial x}\left[\frac{D_S}{kT}\nabla_S\mu\,(x,y,z,t)\int_0^{L_z}C(x,y,W,t)\,dW\right]+\Omega\frac{\partial}{\partial y}\left[\frac{D_S}{kT}\nabla_S\mu\,(x,y,z,t)\int_0^{L_z}C(x,y,W,t)\,dW\right]+f_C(x,y,z)\delta(t) \tag{1a}$$

$$\frac{\partial I(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial y}\right]+\frac{\partial}{\partial z}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial z}\right]+\Omega\frac{\partial}{\partial x}\left[\frac{D_{IS}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}I(x,y,W,t)\,dW\right]+\Omega\frac{\partial}{\partial y}\left[\frac{D_{IS}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}I(x,y,W,t)\,dW\right]-k_{I,I}(x,y,z,T)I^2(x,y,z,t)+f_I(x,y,z)\delta(t)-k_{I,V}(x,y,z,T)I(x,y,z,t)V(x,y,z,t) \tag{3a}$$

$$\frac{\partial V(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial y}\right]+\frac{\partial}{\partial z}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial z}\right]+\Omega\frac{\partial}{\partial x}\left[\frac{D_{VS}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}V(x,y,W,t)\,dW\right]+\Omega\frac{\partial}{\partial y}\left[\frac{D_{VS}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}V(x,y,W,t)\,dW\right]-k_{V,V}(x,y,z,T)V^2(x,y,z,t)+f_V(x,y,z)\delta(t)-k_{I,V}(x,y,z,T)I(x,y,z,t)V(x,y,z,t) \tag{5a}$$

$$\frac{\partial\Phi_I(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial\Phi_I(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial\Phi_I(x,y,z,t)}{\partial y}\right]+\frac{\partial}{\partial z}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial\Phi_I(x,y,z,t)}{\partial z}\right]+\Omega\frac{\partial}{\partial x}\left[\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}\Phi_I(x,y,W,t)\,dW\right]+\Omega\frac{\partial}{\partial y}\left[\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}\Phi_I(x,y,W,t)\,dW\right]+k_I(x,y,z,T)I(x,y,z,t)+f_{\Phi_I}(x,y,z)\delta(t)+k_{I,I}(x,y,z,T)\,I^2(x,y,z,t)$$

$$\frac{\partial\Phi_V(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial\Phi_V(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial\Phi_V(x,y,z,t)}{\partial y}\right]+\frac{\partial}{\partial z}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial\Phi_V(x,y,z,t)}{\partial z}\right]+\Omega\frac{\partial}{\partial x}\left[\frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}\Phi_V(x,y,W,t)\,dW\right]+$$

$$+\Omega\frac{\partial}{\partial y}\left[\frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}\Phi_V(x,y,W,t)dW\right]+ +k_V(x,y,z,T)V(x,y,z,t)+f_{\Phi_V}(x,y,z)\delta(t)+ +k_{V,V}(x,y,z,T)V^2(x,y,z,t)$$

Further, we replaced the concentrations of dopant and radiation defects in right sides of the Equations (1*a*), (3*a*) and (5*a*) on their yet to be known average values $\alpha_{1\rho}$. In this situation, we obtained the equations for the first-order approximations of the required concentrations in the following forms.

$$\frac{\partial C_1(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[z\frac{D_S}{kT}\nabla_S\mu_1(x,y,z,t)\right]\times \times\alpha_{1C}\Omega+\alpha_{1C}\Omega\frac{\partial}{\partial y}\left[z\frac{D_S}{kT}\nabla_S\mu_1(x,y,z,t)\right]+ +f_C(x,y,z)\delta(t) \tag{1b}$$

$$\frac{\partial I_1(x,y,z,t)}{\partial t}=\Omega\frac{\partial}{\partial x}\left[\frac{D_{IS}}{kT}\nabla_S\mu(x,y,z,t)\right]\times \times\alpha_{1I}z+\alpha_{1I}z\Omega\frac{\partial}{\partial y}\left[\frac{D_{IS}}{kT}\nabla_S\mu(x,y,z,t)\right]+ +f_I(x,y,z)\delta(t)-\alpha_{1I}\alpha_{1V}k_{I,V}(x,y,z,T)- -\alpha_{1I}^2k_{I,I}(x,y,z,T) \tag{3b}$$

$$\frac{\partial V_1(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[\frac{D_{VS}}{kT}\nabla_S\mu_1(x,y,z,t)\right]\times \times\alpha_{1V}z\Omega+\alpha_{1V}\Omega\frac{\partial}{\partial y}\left[z\frac{D_{VS}}{kT}\nabla_S\mu_1(x,y,z,t)\right]+ +f_V(x,y,z)\delta(t)-\alpha_{1I}\alpha_{1V}k_{I,V}(x,y,z,T)- -\alpha_{1V}^2k_{V,V}(x,y,z,T)$$

$$\frac{\partial\Phi_{1I}(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(x,y,z,t)\right]\times \times\alpha_{1\Phi_I}z\Omega+\alpha_{1\Phi_I}z\Omega\frac{\partial}{\partial y}\left[\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(x,y,z,t)\right]+ +f_{\Phi_I}(x,y,z)\delta(t)+k_I(x,y,z,T)I(x,y,z,t)+ +k_{I,I}(x,y,z,T)I^2(x,y,z,t)$$

$$\frac{\partial\Phi_{1V}(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[\frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(x,y,z,t)\right]\times \times\alpha_{1\Phi_V}z\Omega+\alpha_{1\Phi_V}z\Omega\frac{\partial}{\partial y}\left[\frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(x,y,z,t)\right]+ +f_{\Phi_V}(x,y,z)\delta(t)+k_V(x,y,z,T)V(x,y,z,t)+ +k_{V,V}(x,y,z,T)V^2(x,y,z,t)$$

The integration of the left and right sides of the Equations (1*b*), (3*b*) and (5*b*) on time gives us the possibility to obtain relations for the above approximation in the final form.

$$C_1(x,y,z,t)=\alpha_{1C}\Omega\frac{\partial}{\partial x}\int_0^t D_{SL}(x,y,z,T)\frac{z}{kT}\times \tag{1c}$$

$$\times\left[1+\varsigma_1\frac{V(x,y,z,\tau)}{V^*}+\varsigma_2\frac{V^2(x,y,z,\tau)}{(V^*)^2}\right]\times \times\nabla_S\mu_1(x,y,z,\tau)\left[1+\frac{\xi_S\alpha_{1C}^\gamma}{P^\gamma(x,y,z,T)}\right]d\tau\Bigg\}+ +\alpha_{1C}\frac{\partial}{\partial y}\int_0^t D_{SL}(x,y,z,T)\left[1+\frac{\xi_S\alpha_{1C}^\gamma}{P^\gamma(x,y,z,T)}\right]+ \times\left[1+\varsigma_1\frac{V(x,y,z,\tau)}{V^*}+\varsigma_2\frac{V^2(x,y,z,\tau)}{(V^*)^2}\right]\times \times\nabla_S\mu_1(x,y,z,\tau)\frac{\Omega z}{kT}d\tau+f_C(x,y,z)$$

$$I_1(x,y,z,t)=\alpha_{1I}z\Omega\frac{\partial}{\partial x}\int_0^t\frac{D_{IS}}{kT}\nabla_S\mu_1(x,y,z,\tau)d\tau+ +f_I(x,y,z)+\alpha_{1I}\frac{\partial}{\partial y}\int_0^t\frac{D_{IS}}{kT}\nabla_S\mu_1(x,y,z,\tau)d\tau\times \times z\Omega-\alpha_{1I}\alpha_{1V}\int_0^t k_{I,V}(x,y,z,T)d\tau- -\alpha_{1I}^2\int_0^t k_{I,I}(x,y,z,T)d\tau \tag{3c}$$

$$V_1(x,y,z,t)=\alpha_{1V}z\Omega\frac{\partial}{\partial x}\int_0^t\frac{D_{IS}}{kT}\nabla_S\mu_1(x,y,z,\tau)d\tau+ +\alpha_{1V}z\Omega\frac{\partial}{\partial y}\int_0^t\frac{D_{IS}}{kT}\nabla_S\mu_1(x,y,z,\tau)d\tau+ +f_V(x,y,z)-\alpha_{1V}^2\int_0^t k_{V,V}(x,y,z,T)d\tau- -\alpha_{1I}\alpha_{1V}\int_0^t k_{I,V}(x,y,z,T)d\tau$$

$$\Phi_{1I}(x,y,z,t)=\frac{\partial}{\partial x}\int_0^t\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(x,y,z,\tau)d\tau\times \times\alpha_{1\Phi_I}z\Omega+\Omega\frac{\partial}{\partial x}\int_0^t\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(x,y,z,\tau)d\tau\times \times\alpha_{1\Phi_I}z+\int_0^t k_I(x,y,z,T)I(x,y,z,\tau)d\tau+ +f_{\Phi_I}(x,y,z)+\int_0^t k_{I,I}(x,y,z,T)I^2(x,y,z,\tau)d\tau \tag{5c}$$

$$\Phi_{1V}(x,y,z,t)=\frac{\partial}{\partial x}\int_0^t\frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(x,y,z,\tau)d\tau\times \times\alpha_{1\Phi_V}z\Omega+\Omega\frac{\partial}{\partial x}\int_0^t\frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(x,y,z,\tau)d\tau\times \times\alpha_{1\Phi_V}z+\int_0^t k_V(x,y,z,T)V(x,y,z,\tau)d\tau+ +f_{\Phi_V}(x,y,z)+\int_0^t k_{V,V}(x,y,z,T)V^2(x,y,z,\tau)d\tau$$

We determined the average values of the first-order approximations of concentrations of dopant and radiation defects by the following standard relation [26].

$$\alpha_{1\rho}=\frac{1}{\Theta L_xL_yL_z}\times \tag{9}$$

$$\times\int_0^\Theta\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}\rho_1(x,y,z,t)\,d\,z\,d\,y\,d\,x\,d\,t$$

Substitution of the relations (1*c*), (3*c*) and (5*c*) into relation (9) gives us the possibility to obtain the required average values in the following form.

$$\alpha_{1C}=\frac{1}{L_xL_yL_z}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}f_C(x,y,z)\,d\,z\,d\,y\,d\,x,$$

$$\alpha_{1I}=\sqrt{\frac{(a_3+A)^2}{4a_4^2}-4\left(B+\frac{\Theta a_3B+\Theta^2L_xL_yL_za_1}{a_4}\right)}-\frac{a_3+A}{4a_4}$$

$$\alpha_{1V}=\frac{1}{S_{IV00}}\left[-\alpha_{1I}S_{II00}-\Theta L_xL_yL_z+\frac{\Theta}{\alpha_{1I}}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}f_I(x,y,z)\,d\,z\,d\,y\,d\,x\right]$$

where: $S_{\rho\rho\,ij}=\int_0^\Theta(\Theta-t)\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}k_{\rho,\rho}(x,y,z,T)\times I_1^i(x,y,z,t)V_1^j(x,y,z,t)\,d\,z\,d\,y\,d\,x\,d\,t$

$$a_4=S_{II00}\times\left(S_{IV00}^2-S_{II00}S_{VV00}\right)$$

$$a_3=S_{IV00}S_{II00}+S_{IV00}^2-S_{II00}S_{VV00}$$

$$a_2=\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}f_V(x,y,z)\,d\,z\,d\,y\,d\,x\times S_{IV00}S_{IV00}^2+S_{II00}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}f_I(x,y,z)\,d\,z\,d\,y\,d\,x\times 2S_{VV00}+S_{IV00}\Theta L_x^2L_y^2L_z^2-\Theta L_x^2L_y^2L_z^2S_{VV00}-S_{IV00}^2\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}f_I(x,y,z)\,d\,z\,d\,y\,d\,x$$

$$a_0=S_{VV00}\times\left[\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}f_I(x,y,z)\,d\,z\,d\,y\,d\,x\right]^2$$

$$B=\frac{\Theta a_2}{6a_4}+\sqrt[3]{\sqrt{q^2+p^3}-q}-\sqrt[3]{\sqrt{q^2+p^3}+q}$$

$$a_1=S_{IV00}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}f_I(x,y,z)\,d\,z\,d\,y\,d\,x$$

$$A=\sqrt{8y+\Theta^2\frac{a_3^2}{a_4^2}-4\Theta\frac{a_2}{a_4}},\qquad q=\frac{\Theta^3a_2}{24a_4^2}\times\left(4a_0-\Theta L_xL_yL_z\frac{a_1a_3}{a_4}\right)-\left(4\Theta a_2-\Theta^2\frac{a_3^2}{a_4}\right)\times\frac{a_0\Theta^2}{8a_4^2}-\frac{\Theta^3a_2^3}{54a_4^3}-L_x^2L_y^2L_z^2\frac{\Theta^4a_1^2}{8a_4^2}$$

$$p=-\frac{\Theta a_2}{18a_4}+\Theta^2\frac{4a_0a_4-\Theta L_xL_yL_za_1a_3}{12a_4^2}$$

$$\alpha_{1\Phi_I}=\frac{R_{I1}}{\Theta L_xL_yL_z}+\frac{S_{II20}}{\Theta L_xL_yL_z}+\frac{1}{L_xL_yL_z}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}f_{\Phi_I}(x,y,z)\,d\,z\,d\,y\,d\,x$$

$$\alpha_{1\Phi_V}=\frac{R_{V1}}{\Theta L_xL_yL_z}+\frac{S_{VV20}}{\Theta L_xL_yL_z}+\frac{1}{L_xL_yL_z}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}f_{\Phi_V}(x,y,z)\,d\,z\,d\,y\,d\,x$$

where: $R_{\rho i}=\int_0^\Theta(\Theta-t)\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}k_I(x,y,z,T)\times I_1^i(x,y,z,t)\,d\,z\,d\,y\,d\,x\,d\,t$.

We determined the approximations of the second and higher orders of concentrations of dopant and radiation defects framework standard iterative procedure of the method of averaging of function corrections [26]. For the procedure to determine the approximations of the *n*-th order of concentrations of dopant and radiation defects, we replaced the required concentrations in the Equations (1*c*), (3*c*), (5*c*) on the following sum $\alpha_{n\rho}+\rho_{n-1}(x,y,z,t)$. The replacement leads to the following transformation of the appropriate equations.

$$\frac{\partial C_2(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left(\left\{1+\xi\frac{[\alpha_{2C}+C_1(x,y,z,t)]^\gamma}{P^\gamma(x,y,z,T)}\right\}\times\left[1+\varsigma_1\frac{V(x,y,z,t)}{V^*}+\varsigma_2\frac{V^2(x,y,z,t)}{(V^*)^2}\right]\times D_L(x,y,z,T)\frac{\partial C_1(x,y,z,t)}{\partial x}\right)+\frac{\partial}{\partial y}\left(\left[1+\varsigma_1\frac{V(x,y,z,t)}{V^*}+\varsigma_2\frac{V^2(x,y,z,t)}{(V^*)^2}\right]\times D_L(x,y,z,T)\left\{1+\xi\frac{[\alpha_{2C}+C_1(x,y,z,t)]^\gamma}{P^\gamma(x,y,z,T)}\right\}\times\frac{\partial C_1(x,y,z,t)}{\partial y}\right)+\frac{\partial}{\partial z}\left(D_L(x,y,z,T)\frac{\partial C_1(x,y,z,t)}{\partial z}\times\left[1+\varsigma_1\frac{V(x,y,z,t)}{V^*}+\varsigma_2\frac{V^2(x,y,z,t)}{(V^*)^2}\right]\times\left\{1+\xi\frac{[\alpha_{2C}+C_1(x,y,z,t)]^\gamma}{P^\gamma(x,y,z,T)}\right\}\right)+f_C(x,y,z)\delta(t)+\Omega\frac{\partial}{\partial x}\{\nabla_S\mu_1(x,y,z,t)\times \tag{1d}$$

$$\times \frac{D_S}{kT}\int_0^{L_z}\left[\alpha_{2C}+C(x,y,W,t)\right]dW\Bigg\}+ +\Omega\frac{\partial}{\partial y}\left\{\frac{D_S}{kT}\nabla_S\mu_1(x,y,z,t)\times \times\int_0^{L_z}\left[\alpha_{2C}+C(x,y,W,t)\right]dW\right\}$$

$$\frac{\partial I_2(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_I(x,y,z,T)\frac{\partial I_1(x,y,z,t)}{\partial x}\right]+ +\frac{\partial}{\partial y}\left[D_I(x,y,z,T)\frac{\partial I_1(x,y,z,t)}{\partial y}\right]+ -k_{I,I}(x,y,z,T)\left[\alpha_{1I}+I_1(x,y,z,t)\right]^2- -k_{I,V}(x,y,z,T)\left[\alpha_{1I}+I_1(x,y,z,t)\right]\times \times\left[\alpha_{1V}+V_1(x,y,z,t)\right]+\Omega\frac{\partial}{\partial x}\left\{\nabla_S\mu(x,y,z,t)\times \times\int_0^{L_z}\left[\alpha_{2I}+I_1(x,y,W,t)\right]dW\frac{D_{IS}}{kT}\right\}+\Omega\frac{\partial}{\partial y}\left\{\frac{D_{IS}}{kT}\times \times\nabla_S\mu(x,y,z,t)\int_0^{L_z}\left[\alpha_{2I}+I_1(x,y,W,t)\right]dW\right\} \quad (3d)$$

$$\frac{\partial V_2(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_V(x,y,z,T)\frac{\partial V_1(x,y,z,t)}{\partial x}\right]+ +\frac{\partial}{\partial y}\left[D_V(x,y,z,T)\frac{\partial V_1(x,y,z,t)}{\partial y}\right]+ +\frac{\partial}{\partial z}\left[D_V(x,y,z,T)\frac{\partial V_1(x,y,z,t)}{\partial z}\right]- -k_{V,V}(x,y,z,T)\left[\alpha_{1V}+V_1(x,y,z,t)\right]^2- -\left[\alpha_{1I}+I_1(x,y,z,t)\right]\left[\alpha_{1V}+V_1(x,y,z,t)\right]\times \times k_{I,V}(x,y,z,T)+\Omega\frac{\partial}{\partial x}\left\{\nabla_S\mu(x,y,z,t)\frac{D_{VS}}{kT}\times \times\int_0^{L_z}\left[\alpha_{2V}+V_1(x,y,W,t)\right]dW\right\}+\Omega\frac{\partial}{\partial y}\left\{\frac{D_{VS}}{kT}\times \times\nabla_S\mu(x,y,z,t)\int_0^{L_z}\left[\alpha_{2V}+V_1(x,y,W,t)\right]dW\right\} \quad (5d)$$

$$\frac{\partial\Phi_{2I}(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial\Phi_{1I}(x,y,z,t)}{\partial x}\right]+ +\frac{\partial}{\partial y}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial\Phi_{1I}(x,y,z,t)}{\partial y}\right]+ +\frac{\partial}{\partial z}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial\Phi_{1I}(x,y,z,t)}{\partial z}\right]+ +f_{\Phi_I}(x,y,z)\delta(t)+\Omega\frac{\partial}{\partial x}\left\{\frac{D_{\Phi_I S}}{kT}\nabla_S\mu(x,y,z,t)\times \times\int_0^{L_z}\left[\alpha_{2\Phi_I}+\Phi_{1I}(x,y,W,t)\right]dW\right\}+ +\Omega\frac{\partial}{\partial y}\left\{\frac{D_{\Phi_I S}}{kT}\nabla_S\mu(x,y,z,t)\times \times\int_0^{L_z}\left[\alpha_{2\Phi_I}+\Phi_{1I}(x,y,W,t)\right]dW\right\}+k_{I,I}(x,y,z,T)\times \times I^2(x,y,z,t)+k_I(x,y,z,T)I(x,y,z,t)$$

$$\frac{\partial\Phi_{2V}(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial\Phi_{1V}(x,y,z,t)}{\partial x}\right]+ +\frac{\partial}{\partial y}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial\Phi_{1V}(x,y,z,t)}{\partial y}\right]+ +\frac{\partial}{\partial z}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial\Phi_{1V}(x,y,z,t)}{\partial z}\right]+ +f_{\Phi_V}(x,y,z)\delta(t)+\Omega\frac{\partial}{\partial x}\left\{\frac{D_{\Phi_V S}}{kT}\nabla_S\mu(x,y,z,t)\times \times\int_0^{L_z}\left[\alpha_{2\Phi_V}+\Phi_{1V}(x,y,W,t)\right]dW\right\}+ +\Omega\frac{\partial}{\partial y}\left\{\frac{D_{\Phi_V S}}{kT}\nabla_S\mu(x,y,z,t)\times \times\int_0^{L_z}\left[\alpha_{2\Phi_V}+\Phi_{1V}(x,y,W,t)\right]dW\right\}+k_{V,V}(x,y,z,T)\times \times V^2(x,y,z,t)+k_V(x,y,z,T)V(x,y,z,t)$$

The integration of the left and the right sides of Equations (1*d*), (3*d*) and (5*d*) gives us the possibility to obtain relations for the required concentrations in the final form.

$$C_2(x,y,z,t)=\frac{\partial}{\partial x}\int_0^t\left\{1+\xi\frac{\left[\alpha_{2C}+C_1(x,y,z,\tau)\right]^\gamma}{P^\gamma(x,y,z,T)}\right\}\times \times\left[1+\varsigma_1\frac{V(x,y,z,\tau)}{V^*}+\varsigma_2\frac{V^2(x,y,z,\tau)}{(V^*)^2}\right]\times \times D_L(x,y,z,T)\frac{\partial C_1(x,y,z,\tau)}{\partial x}d\tau+ +\frac{\partial}{\partial y}\int_0^t\left[1+\varsigma_1\frac{V(x,y,z,\tau)}{V^*}+\varsigma_2\frac{V^2(x,y,z,\tau)}{(V^*)^2}\right]\times \times D_L(x,y,z,T)\left\{1+\xi\frac{\left[\alpha_{2C}+C_1(x,y,z,\tau)\right]^\gamma}{P^\gamma(x,y,z,T)}\right\}\times \times\frac{\partial C_1(x,y,z,\tau)}{\partial y}d\tau+\frac{\partial}{\partial z}\int_0^t\frac{\partial C_1(x,y,z,\tau)}{\partial z}\times \times\left[1+\varsigma_1\frac{V(x,y,z,\tau)}{V^*}+\varsigma_2\frac{V^2(x,y,z,\tau)}{(V^*)^2}\right]\times \times\left\{1+\xi\frac{\left[\alpha_{2C}+C_1(x,y,z,\tau)\right]^\gamma}{P^\gamma(x,y,z,T)}\right\}\times \times D_L(x,y,z,T)d\tau+f_C(x,y,z)+\Omega\frac{\partial}{\partial x}\int_0^t\frac{D_S}{kT}\times \times\nabla_S\mu(x,y,z,\tau)\int_0^{L_z}\left[\alpha_{2C}+C_1(x,y,W,\tau)\right]dW\,d\tau+ +\Omega\frac{\partial}{\partial y}\int_0^t\frac{D_S}{kT}\int_0^{L_z}\left[\alpha_{2C}+C_1(x,y,W,\tau)\right]dW\times \times\nabla_S\mu(x,y,z,\tau)\,d\tau+f_I(x,y,z) \quad (1e)$$

$$I_2(x,y,z,t)=\frac{\partial}{\partial x}\int_0^t D_I(x,y,z,T)\frac{\partial I_1(x,y,z,\tau)}{\partial x}d\tau+\frac{\partial}{\partial y}\int_0^t D_I(x,y,z,T)\frac{\partial I_1(x,y,z,\tau)}{\partial y}d\tau+\frac{\partial}{\partial z}\int_0^t D_I(x,y,z,T)\frac{\partial I_1(x,y,z,\tau)}{\partial z}d\tau-\int_0^t k_{I,I}(x,y,z,T)\left[\alpha_{2I}+I_1(x,y,z,\tau)\right]^2 d\tau-\int_0^t\left[\alpha_{2I}+I_1(x,y,z,\tau)\right]\left[\alpha_{2V}+V_1(x,y,z,\tau)\right]\times k_{I,V}(x,y,z,T)\,d\tau+\frac{\partial}{\partial x}\int_0^t\nabla_S\mu(x,y,z,\tau)\times\Omega\frac{D_{IS}}{kT}\int_0^{L_z}\left[\alpha_{2I}+I_1(x,y,W,\tau)\right]dW\,d\tau+\frac{\partial}{\partial y}\int_0^t\nabla_S\mu(x,y,z,\tau)\int_0^{L_z}\left[\alpha_{2I}+I_1(x,y,W,\tau)\right]\times\Omega\frac{D_{IS}}{kT}d\,W\,d\tau \tag{3e}$$

$$V_2(x,y,z,t)=\frac{\partial}{\partial x}\int_0^t D_V(x,y,z,T)\frac{\partial V_1(x,y,z,\tau)}{\partial x}d\tau+\frac{\partial}{\partial y}\int_0^t D_V(x,y,z,T)\frac{\partial V_1(x,y,z,\tau)}{\partial y}d\tau+\frac{\partial}{\partial z}\int_0^t D_V(x,y,z,T)\frac{\partial V_1(x,y,z,\tau)}{\partial z}d\tau-\int_0^t k_{V,V}(x,y,z,T)\left[\alpha_{2V}+V_1(x,y,z,\tau)\right]^2 d\tau-\int_0^t\left[\alpha_{2I}+I_1(x,y,z,\tau)\right]\left[\alpha_{2V}+V_1(x,y,z,\tau)\right]\times k_{I,V}(x,y,z,T)\,d\tau+\frac{\partial}{\partial x}\int_0^t\nabla_S\mu(x,y,z,\tau)\times\Omega\frac{D_{VS}}{kT}\int_0^{L_z}\left[\alpha_{2V}+V_1(x,y,W,\tau)\right]dW\,d\tau+\frac{\partial}{\partial y}\int_0^t\nabla_S\mu(x,y,z,\tau)\int_0^{L_z}\left[\alpha_{2V}+V_1(x,y,W,\tau)\right]\times\Omega\frac{D_{VS}}{kT}d\,W\,d\tau+f_V(x,y,z)$$

$$\Phi_{2I}(x,y,z,t)=\frac{\partial}{\partial x}\int_0^t\frac{\partial\Phi_{1I}(x,y,z,\tau)}{\partial x}\times D_{\Phi_I}(x,y,z,T)\,d\tau+\frac{\partial}{\partial y}\int_0^t\frac{\partial\Phi_{1I}(x,y,z,\tau)}{\partial y}\times D_{\Phi_I}(x,y,z,T)d\tau+\frac{\partial}{\partial z}\int_0^t\frac{\partial\Phi_{1I}(x,y,z,\tau)}{\partial z}\times D_{\Phi_I}(x,y,z,T)\,d\tau+\Omega\frac{\partial}{\partial x}\int_0^t\nabla_S\mu(x,y,z,\tau)\times\frac{D_{\Phi_I S}}{kT}\int_0^{L_z}\left[\alpha_{2\Phi_I}+\Phi_{1I}(x,y,W,\tau)\right]dW\,d\tau+\Omega\frac{\partial}{\partial y}\int_0^t\frac{D_{\Phi_I S}}{kT}\int_0^{L_z}\left[\alpha_{2\Phi_I}+\Phi_{1I}(x,y,W,\tau)\right]dW\times\nabla_S\mu(x,y,z,\tau)\,d\tau+\int_0^t k_{I,I}(x,y,z,T)\times I^2(x,y,z,\tau)d\tau+f_{\Phi_I}(x,y,z)+\int_0^t k_I(x,y,z,T)I(x,y,z,\tau)\,d\tau \tag{5e}$$

$$\Phi_{2V}(x,y,z,t)=\frac{\partial}{\partial x}\int_0^t\frac{\partial\Phi_{1V}(x,y,z,\tau)}{\partial x}\times D_{\Phi_V}(x,y,z,T)\,d\tau+\frac{\partial}{\partial y}\int_0^t\frac{\partial\Phi_{1V}(x,y,z,\tau)}{\partial y}\times D_{\Phi_V}(x,y,z,T)d\tau+\frac{\partial}{\partial z}\int_0^t\frac{\partial\Phi_{1V}(x,y,z,\tau)}{\partial z}\times D_{\Phi_V}(x,y,z,T)\,d\tau+\Omega\frac{\partial}{\partial x}\int_0^t\nabla_S\mu(x,y,z,\tau)\times\frac{D_{\Phi_V S}}{kT}\int_0^{L_z}\left[\alpha_{2\Phi_V}+\Phi_{1V}(x,y,W,\tau)\right]dW\,d\tau+\Omega\frac{\partial}{\partial y}\int_0^t\frac{D_{\Phi_V S}}{kT}\int_0^{L_z}\left[\alpha_{2\Phi_V}+\Phi_{1V}(x,y,W,\tau)\right]dW\times\nabla_S\mu(x,y,z,\tau)\,d\tau+\int_0^t k_{V,V}(x,y,z,T)\times V^2(x,y,z,\tau)d\tau+f_{\Phi_V}(x,y,z)+\int_0^t k_V(x,y,z,T)V(x,y,z,\tau)\,d\tau$$

The average values of the second-order approximations of the required approximations are derived by using the following standard relation [26].

$$\alpha_{2\rho}=\frac{1}{\Theta L_x L_y L_z}\int_0^{\Theta}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}\left[\rho_2(x,y,z,t)-\rho_1(x,y,z,t)\right]d\,z\,d\,y\,d\,x\,d\,t \tag{10}$$

The substitution of the relations (1*e*), (3*e*), (5*e*) into relation (10) gives us the possibility to obtain relations for the required average values $\alpha_{2\rho}$.

$$\alpha_{2V}=\sqrt{\frac{(b_3+E)^2}{4b_4^2}-4\left(F+\frac{\Theta a_3F+\Theta^2L_xL_yL_zb_1}{b_4}\right)}-\frac{b_3+E}{4b_4}$$

α_{2C}=0, $\alpha_{2\Phi I}$=0, $\alpha_{2\Phi V}$=0

$$\alpha_{2I}=\frac{1}{S_{IV01}+\alpha_{2V}S_{IV00}}\left[C_V-\alpha_{2V}^2S_{VV00}-S_{VV02}-\alpha_{2V}\left(2S_{VV01}+S_{IV10}+\Theta L_xL_yL_z\right)-S_{IV11}\right]$$

where: $b_4=\frac{S_{IV00}^2S_{VV00}}{\Theta L_xL_yL_z}-\frac{S_{VV00}^2S_{II00}}{\Theta L_xL_yL_z}$

$$b_3=-\frac{S_{II00}}{\Theta L_x}\times\left(2S_{VV01}+S_{IV10}+\Theta L_xL_yL_z\right)\frac{S_{VV00}}{L_yL_z}+\frac{S_{IV00}S_{VV00}}{\Theta L_xL_yL_z}\times$$

$$\times\left(S_{IV01}+2S_{II10}+S_{IV01}+\Theta L_xL_yL_z\right)+\frac{S_{IV00}^2}{\Theta L_xL_yL_z}\times$$

$$\times\left(2S_{VV01}+S_{IV10}+\Theta L_xL_yL_z\right)-\frac{S_{IV00}^2S_{IV10}}{\Theta^3L_x^3L_y^3L_z^3}$$

$$b_2=\frac{S_{II00}S_{VV00}}{\Theta L_xL_yL_z}\left(S_{VV02}+S_{IV11}+C_V\right)-\left(S_{IV10}-\right.$$

$$\left.-2S_{VV01}+\Theta L_xL_yL_z\right)^2+\left(\Theta L_xL_yL_z+2S_{II10}+\right.$$

$$\left.+S_{IV01}\right)\frac{S_{IV01}S_{VV00}}{\Theta L_xL_yL_z}+S_{IV00}\frac{\left(2S_{IV01}+2S_{II10}+\right.}{\Theta L_xL_yL_z}$$

$$\left.+S_{IV01}+\Theta L_xL_yL_z\right)\left(2S_{VV01}+S_{IV10}+L_xL_yL_z\times\right.$$

$$\left.\times\Theta\right)-\frac{S_{IV00}^2}{\Theta L_xL_yL_z}\left(C_V-S_{VV02}-S_{IV11}\right)+\frac{S_{IV00}^2}{\Theta^2L_x^2}\times$$

$$\times\frac{C_I}{L_y^2L_z^2}-2S_{IV01}\frac{S_{IV10}S_{IV00}}{\Theta L_xL_yL_z}$$

$$b_1=\frac{S_{IV01}}{\Theta L_xL_yL_z}\times$$

$$\times\left(2S_{VV01}+S_{IV10}+\Theta LL_yL_z\right)\left(2S_{II10}+S_{IV01}+\right.$$

$$\left.+\Theta L_xL_yL_z\right)+\frac{S_{IV11}+S_{VV02}+C_V}{\Theta L_xL_yL_z}\left(2S_{VV01}+S_{IV10}+\right.$$

$$\left.+\Theta L_xL_yL_z\right)S_{II00}-\frac{S_{IV10}S_{IV01}^2}{\Theta L_xL_yL_z}+\frac{S_{IV00}}{\Theta L_xL_yL_z}\times$$

$$\times\left(3S_{IV01}+2S_{II10}+\Theta L_xL_yL_z\right)\left(S_{VV02}+S_{IV11}-\right.$$

$$\left.-C_V\right)+2C_IS_{IV00}S_{IV01}$$

$$b_0=\frac{\left(S_{IV00}+S_{VV02}\right)^2}{\Theta L_xL_yL_z}\times$$

$$\times S_{II00}-S_{IV01}\frac{C_V-S_{VV02}-S_{IV11}}{\Theta L_xL_yL_z}\left(2S_{II10}+L_xL_y\times\right.$$

$$\left.\times L_z\Theta+2S_{II10}+S_{IV01}\right)-\frac{C_V-S_{VV02}-S_{IV11}}{\Theta L_xL_yL_z}\times$$

$$\times S_{IV01}\left(\Theta L_xL_yL_z+2S_{II10}+S_{IV01}\right)+2C_IS_{IV01}^2$$

$$C_I=\frac{\alpha_{1I}\alpha_{1V}S_{IV00}}{\Theta L_xL_yL_z}+\frac{\alpha_{1I}^2S_{II00}}{\Theta L_xL_yL_z}-\frac{S_{II20}S_{II20}}{\Theta L_xL_yL_z}-$$

$$-\frac{S_{IV11}}{\Theta L_xL_yL_z}$$

$$E=\sqrt{8y+\Theta^2\frac{a_3^2}{a_4^2}-4\Theta\frac{a_2}{a_4}}$$

$$F=\frac{\Theta a_2}{6a_4}+\sqrt[3]{\sqrt{r^2+s^3}-r}-\sqrt[3]{\sqrt{r^2+s^3}+r}$$

$$C_V=\alpha_{1I}\alpha_{1V}S_{IV00}+\alpha_{1V}^2S_{VV00}-S_{VV02}-S_{IV11}$$

$$r=\frac{\Theta^3b_2}{24b_4^2}\left(4b_0-\Theta L_xL_yL_z\frac{b_1b_3}{b_4}\right)-\frac{\Theta^3b_2^3}{54b_4^3}-$$

$$-b_0\frac{\Theta^2}{8b_4^2}\left(4\Theta b_2-\Theta^2\frac{b_3^2}{b_4}\right)-L_x^2L_y^2L_z^2\frac{\Theta^4b_1^2}{8b_4^2}$$

$$s=\Theta^2\frac{4b_0b_4-\Theta L_xL_yL_zb_1b_3}{12b_4^2}-\frac{\Theta b_2}{18b_4}$$

Further, we determined the solutions of Equations (8), that is the components of displacement vector. To determine the first-order approximations of the considered components framework method of averaging of function corrections, we replaced the required functions in the right sides of the equations by the yet to be known average values α_i. The substitution leads to the following result.

$$\rho(z)\frac{\partial^2u_{1x}(x,y,z,t)}{\partial t^2}=-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial x};$$

$$\rho(z)\frac{\partial^2u_{1y}(x,y,z,t)}{\partial t^2}=-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial y};$$

$$\rho(z)\frac{\partial^2u_{1z}(x,y,z,t)}{\partial t^2}=-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial z}$$

The integration of the left and the right sides of the above relations on time t leads to the following result.

$$u_{1x}(x,y,z,t)=\frac{\partial}{\partial x}\int_0^t\int_0^\vartheta T(x,y,z,\tau)\,d\tau\,d\vartheta\times$$

$$\times K(z)\frac{\beta(z)}{\rho(z)}+u_{0x}-\frac{\partial}{\partial x}\int_0^\infty\int_0^\vartheta T(x,y,z,\tau)\,d\tau\,d\vartheta\times$$

$$\times K(z)\beta(z)/\rho(z)$$

$$u_{1y}(x,y,z,t)=u_{0y}\frac{\partial}{\partial y}\int_0^t\int_0^\vartheta T(x,y,z,\tau)\,d\tau\,d\vartheta\times$$

$$\times K(z)\frac{\beta(z)}{\rho(z)}-K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial y}\int_0^\infty\int_0^\vartheta T(x,y,z,\tau)\,d\tau\,d\vartheta$$

$$u_{1z}(x,y,z,t)=u_{0z}+\frac{\partial}{\partial z}\int_0^t\int_0^\vartheta T(x,y,z,\tau)\,d\tau\,d\vartheta\times$$

$$\times K(z)\frac{\beta(z)}{\rho(z)}-K(z)\frac{\partial}{\partial z}\int_0^\infty\int_0^\vartheta T(x,y,z,\tau)\,d\tau\,d\vartheta\times$$

$$\times\beta(z)/\rho(z)$$

The approximations of the second and higher orders of components of displacement vector could be determined by using the standard replacement of the required components on the following sums $\alpha_i+u_i(x,y,z,t)$ [26]. The replacement leads to the following result.

$$\rho(z)\frac{\partial^2u_{2x}(x,y,z,t)}{\partial t^2}=\left\{K(z)+\frac{5E(z)}{6[1+\sigma(z)]}\right\}\times$$

$$\times\frac{\partial^2u_{1x}(x,y,z,t)}{\partial x^2}+\left\{K(z)-\frac{E(z)}{3[1+\sigma(z)]}\right\}\times$$

$$\times\frac{\partial^2u_{1y}(x,y,z,t)}{\partial x\partial y}+\frac{E(z)}{2[1+\sigma(z)]}\left[\frac{\partial^2u_{1y}(x,y,z,t)}{\partial y^2}+\right.$$

$$\left.+\frac{\partial^2u_{1z}(x,y,z,t)}{\partial z^2}\right]-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial x}+$$

$$+\left\{K(z)+\frac{E(z)}{3[1+\sigma(z)]}\right\}\frac{\partial^2u_{1z}(x,y,z,t)}{\partial x\partial z}$$

$$
\begin{aligned}
&\rho(z)\frac{\partial^2 u_{2y}(x,y,z,t)}{\partial t^2}=\Bigg[\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial x^2}+\\
&+\frac{\partial^2 u_{1x}(x,y,z,t)}{\partial x\partial y}\Bigg]\frac{E(z)}{2[1+\sigma(z)]}-\frac{\partial T(x,y,z,t)}{\partial y}\times\\
&\times K(z)\beta(z)+\frac{1}{2}\frac{\partial}{\partial z}\Bigg\{\frac{E(z)}{[1+\sigma(z)]}\Bigg[\frac{\partial u_{1y}(x,y,z,t)}{\partial z}+\\
&+\frac{\partial u_{1z}(x,y,z,t)}{\partial y}\Bigg]\Bigg\}+\Bigg\{\frac{5E(z)}{12[1+\sigma(z)]}+K(z)\Bigg\}\times\\
&\times\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial y^2}+\Bigg\{K(z)-\frac{E(z)}{6[1+\sigma(z)]}\Bigg\}\times\\
&\times\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial y\partial z}+K(z)\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial x\partial y}
\end{aligned}
$$

$$
\begin{aligned}
&\rho(z)\frac{\partial^2 u_{2z}(x,y,z,t)}{\partial t^2}=\Bigg[\frac{\partial^2 u_{1z}(x,y,z,t)}{\partial x^2}+\\
&+\frac{\partial^2 u_{1z}(x,y,z,t)}{\partial y^2}+\frac{\partial^2 u_{1x}(x,y,z,t)}{\partial x\partial z}\\
&+\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial y\partial z}\Bigg]\frac{E(z)}{2[1+\sigma(z)]}+\\
&+\frac{\partial}{\partial z}\Bigg\{K(z)\Bigg[\frac{\partial u_{1x}(x,y,z,t)}{\partial x}+\frac{\partial u_{1y}(x,y,z,t)}{\partial y}+\\
&+\frac{\partial u_{1x}(x,y,z,t)}{\partial z}\Bigg]\Bigg\}+\frac{\partial}{\partial z}\Bigg\{\frac{E(z)}{6[1+\sigma(z)]}\times\\
&\times\Bigg[6\frac{\partial u_{1z}(x,y,z,t)}{\partial z}-\frac{\partial u_{1x}(x,y,z,t)}{\partial x}-\\
&-\frac{\partial u_{1y}(x,y,z,t)}{\partial y}-\frac{\partial u_{1z}(x,y,z,t)}{\partial z}\Bigg]\Bigg\}-\\
&-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial z}
\end{aligned}
$$

The integration of the left and right sides of the above relations on time t leads to the following result.

$$
\begin{aligned}
&u_{2x}(x,y,z,t)=\frac{1}{\rho(z)}\Bigg\{K(z)+\frac{5E(z)}{6[1+\sigma(z)]}\Bigg\}\times\\
&\times\frac{\partial^2}{\partial x^2}\int_0^t\int_0^\vartheta u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta+\frac{1}{\rho(z)}\Bigg\{K(z)-\\
&-\frac{E(z)}{3[1+\sigma(z)]}\Bigg\}\frac{\partial^2}{\partial x\partial y}\int_0^t\int_0^\vartheta u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta+\\
&+\frac{E(z)}{2[1+\sigma(z)]}\Bigg[\frac{\partial^2}{\partial y^2}\int_0^t\int_0^\vartheta u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta+\\
&+\frac{\partial^2}{\partial z^2}\int_0^t\int_0^\vartheta u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta\Bigg]\frac{1}{\rho(z)}+\frac{1}{\rho(z)}\times\\
&\times\frac{\partial^2}{\partial x\partial z}\int_0^t\int_0^\vartheta u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta\Bigg\{K(z)+\\
&+\frac{E(z)}{3[1+\sigma(z)]}\Bigg\}-\frac{\partial}{\partial x}\int_0^t\int_0^\vartheta T(x,y,z,\tau)\,d\tau\,d\vartheta\times\\
&\times K(z)\frac{\beta(z)}{\rho(z)}-\frac{\partial^2}{\partial x^2}\int_0^\infty\int_0^\vartheta u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta\times\\
&\times\frac{1}{\rho(z)}\Bigg\{K(z)+\frac{5E(z)}{6[1+\sigma(z)]}\Bigg\}+\Bigg\{\frac{E(z)}{3[1+\sigma(z)]}-\\
&-K(z)\Bigg\}\frac{1}{\rho(z)}\frac{\partial^2}{\partial x\partial y}\int_0^\infty\int_0^\vartheta u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta-\\
&-\frac{E(z)}{2[1+\sigma(z)]}\Bigg[\frac{\partial^2}{\partial y^2}\int_0^\infty\int_0^\vartheta u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta+\\
&+\frac{\partial^2}{\partial z^2}\int_0^\infty\int_0^\vartheta u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta\Bigg]\frac{1}{\rho(z)}-\frac{1}{\rho(z)}\times\\
&\times\frac{\partial^2}{\partial x\partial z}\int_0^\infty\int_0^\vartheta u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta\Bigg\{\frac{E(z)}{3[1+\sigma(z)]}+\\
&+K(z)\Bigg\}+u_{0x}+\frac{\partial}{\partial x}\int_0^\infty\int_0^\vartheta T(x,y,z,\tau)\,d\tau\,d\vartheta\times\\
&\times K(z)\,\beta(z)/\rho(z)
\end{aligned}
$$

$$
\begin{aligned}
&u_{2y}(x,y,z,t)=\Bigg[\frac{\partial^2}{\partial x^2}\int_0^t\int_0^\vartheta u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta+\\
&+\frac{\partial^2}{\partial x\partial y}\int_0^t\int_0^\vartheta u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta\Bigg]\frac{E(z)}{2[1+\sigma(z)]}\times\\
&\times\frac{1}{\rho(z)}+\frac{K(z)}{\rho(z)}\frac{\partial^2}{\partial x\partial y}\int_0^t\int_0^\vartheta u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta+\\
&+\frac{1}{\rho(z)}\frac{\partial^2}{\partial y^2}\int_0^t\int_0^\vartheta u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta\Bigg\{K(z)+\\
&+\frac{5E(z)}{12[1+\sigma(z)]}\Bigg\}+\frac{\partial}{\partial z}\Bigg\{\Bigg[\frac{\partial}{\partial z}\int_0^t\int_0^\vartheta u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta+\\
&+\frac{\partial}{\partial y}\int_0^t\int_0^\vartheta u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta\Bigg]\Bigg\}\frac{E(z)}{2\rho(z)[1+\sigma(z)]}-\\
&-K(z)\frac{\beta(z)}{\rho(z)}\int_0^t\int_0^\vartheta T(x,y,z,\tau)\,d\tau\,d\vartheta-\Bigg\{\frac{E(z)}{6[1+\sigma(z)]}-\\
&-K(z)\Bigg\}\frac{1}{\rho(z)}\frac{\partial^2}{\partial y\partial z}\int_0^t\int_0^\vartheta u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta-\\
&-\frac{E(z)}{2\rho(z)}\Bigg[\frac{\partial^2}{\partial x^2}\int_0^\infty\int_0^\vartheta u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta+\\
&+\frac{\partial^2}{\partial x\partial y}\int_0^\infty\int_0^\vartheta u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta\Bigg]\frac{1}{1+\sigma(z)}-\\
&-K(z)\frac{\beta(z)}{\rho(z)}\int_0^\infty\int_0^\vartheta T(x,y,z,\tau)\,d\tau\,d\vartheta-\frac{K(z)}{\rho(z)}\times\\
&\times\frac{\partial^2}{\partial x\partial y}\int_0^\infty\int_0^\vartheta u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta-\frac{1}{\rho(z)}\times\\
&\times\frac{\partial^2}{\partial y^2}\int_0^\infty\int_0^\vartheta u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta\Bigg\{\frac{5E(z)}{12[1+\sigma(z)]}+\\
&+K(z)\Bigg\}-\frac{\partial}{\partial z}\Bigg\{\Bigg[\frac{\partial}{\partial z}\int_0^\infty\int_0^\vartheta u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta+
\end{aligned}
$$

$$+\frac{\partial}{\partial y}\int_0^{\infty}\int_0^{\vartheta}u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta\Bigg]\frac{E(z)}{2\rho(z)[1+\sigma(z)]}\Bigg\}+$$

$$+u_{0y}-\frac{1}{\rho(z)}\frac{\partial^2}{\partial y\,\partial z}\int_0^{\infty}\int_0^{\vartheta}u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta\times$$

$$\times\left\{K(z)-\frac{E(z)}{6[1+\sigma(z)]}\right\}$$

$$u_z(x,y,z,t)=\Bigg[\frac{\partial^2}{\partial x^2}\int_0^{\infty}\int_0^{\vartheta}u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta+$$

$$+\frac{\partial^2}{\partial y^2}\int_0^{\infty}\int_0^{\vartheta}u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta+$$

$$+\frac{\partial^2}{\partial x\,\partial z}\int_0^{\infty}\int_0^{\vartheta}u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta+$$

$$+\frac{\partial^2}{\partial y\,\partial z}\int_0^{\infty}\int_0^{\vartheta}u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta\Bigg]\frac{E(z)}{2\,[1+\sigma(z)]}\times$$

$$\times\frac{1}{\rho(z)}+\frac{\partial}{\partial z}\Bigg\{\Bigg[\frac{\partial}{\partial x}\int_0^{\infty}\int_0^{\vartheta}u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta+$$

$$+\frac{\partial}{\partial z}\int_0^{\infty}\int_0^{\vartheta}u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta\Bigg]K(z)\Bigg\}\frac{1}{\rho(z)}+$$

$$+\frac{\partial}{\partial z}\Bigg\{\frac{E(z)}{\rho(z)\,[1+\sigma(z)]}\Bigg[6\frac{\partial}{\partial z}\int_0^{\infty}\int_0^{\vartheta}u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta-$$

$$-\frac{\partial}{\partial x}\int_0^{\infty}\int_0^{\vartheta}u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta-$$

$$-\frac{\partial}{\partial y}\int_0^{\infty}\int_0^{\vartheta}u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta-$$

$$-\frac{\partial}{\partial z}\int_0^{\infty}\int_0^{\vartheta}u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta\Bigg]\Bigg\}+u_{0z}-$$

$$-K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial z}\int_0^{\infty}\int_0^{\vartheta}T(x,y,z,\tau)\,d\tau\,d\vartheta$$

In this paper we determined the concentration of dopant, concentrations of radiation defects and components of displacement vector by using the second-order approximation framework method of averaging of function corrections. This approximation is usually a good approximation to make qualitative analysis and to obtain some quantitative results. All obtained results have been checked by making comparison with results of the numerical simulations. It should be noted that the checking shows essentially higher calculation speed when using the analytical simulation in comparison with the numerical simulation.

III. DISCUSSION

In this section, we analyzed the dynamics of redistributions of dopant and radiation defects during the annealing and under the influence of mismatch-induced stress. Typical distributions of concentrations of dopant in heterostructures are presented on Figures 2 and 3 for diffusion and ion types of doping, respectively. These distributions have been calculated for the case, when the value of dopant diffusion coefficient in the epitaxial layer is larger than the substrate. The figures show that inhomogeneity of heterostructure gives us the possibility to increase the sharpness of *p-n-* junctions. At the same time, one can find increasing homogeneity of dopant distribution in doped part of epitaxial layer. Increasing of sharpness of *p-n*-junctions gives us the possibility to decrease their switching time. The second effect leads to decreasing local heating of materials during functioning of *p-n*-junction or decreasing of dimensions of the *p-n*-junction for fixed maximal value of local overheat. However, the for the framework of manufacturing the field-effect transistors, it is necessary to optimize annealing of dopant and/or radiation defects. The reason of this optimization is as follows: If annealing time is small, the dopant did not achieve any interfaces between materials of heterostructure. In this situation, one cannot find any modifications of distribution of concentration of dopant. If annealing time is large, the distribution of concentration of dopant is too homogenous. We have optimized annealing time using the framework introduced recently [15,25-32]. For the criterion, we approximated the real distribution of concentration of dopant by step-wise function (see Figures. 4 and 5). Further, we determined the optimal values of annealing time by minimizing the following mean-squared error.

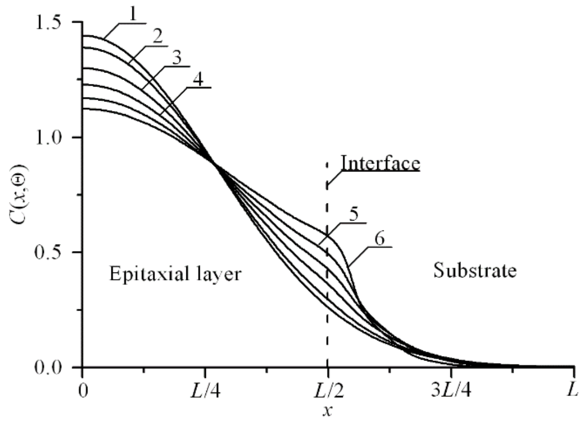


Figure 2: Distributions of concentration of infused dopant in heterostructure from Figure 1 in direction, which is perpendicular to the interface between epitaxial layer substrate. Increasing the number of curve corresponds to the increasing of difference between the values of dopant diffusion coefficient in the layers of heterostructure under the condition, when the value of dopant diffusion coefficient in epitaxial layer is larger than the value of dopant diffusion coefficient in substrate

$$U=\frac{1}{L_xL_yL_z}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}\left[C(x,y,z,\Theta)-\psi(x,y,z)\right]dz\,dy\,dx \tag{15}$$

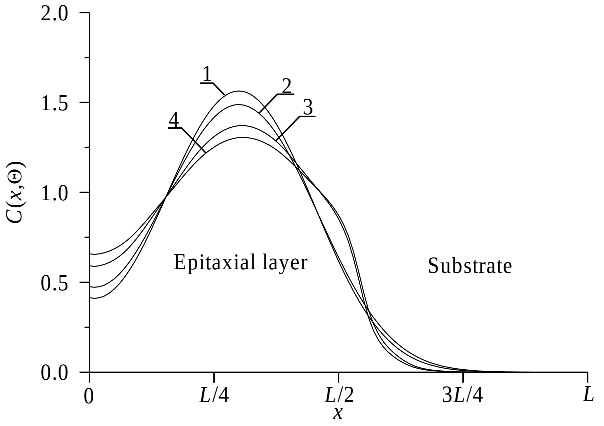


Figure 3: Distributions of concentration of implanted dopant in heterostructure from Figure 1 in direction, which is perpendicular to interface between epitaxial layer substrate. Curves 1 and 3 correspond to annealing time $\Theta = 0.0048(L_x^2+L_y^2+L_z^2)/D_0$. Curves 2 and 4 correspond to annealing time $\Theta = 0.0057(L_x^2+L_y^2+L_z^2)/D_0$. Curves 1 and 2 corresponds to homogenous sample. Curves 3 and 4 correspond to heterostructure under the condition, when the value of dopant diffusion coefficient in epitaxial layer is larger than the value of dopant diffusion coefficient in substrate

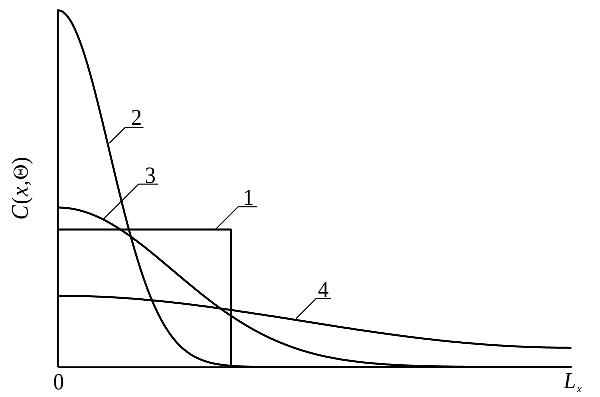


Figure 4: Spatial distributions of dopant in heterostructure after dopant infusion. Curve 1 is an idealized distribution of dopant. Curves 2-4 are real distributions of dopant for different values of annealing time. Increasing in the number of curve corresponds to the increasing of annealing time.

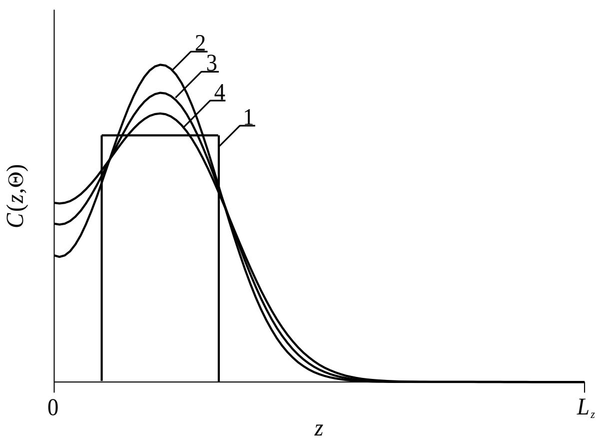


Figure 5: Spatial distributions of dopant in heterostructure after ion implantation. Curve 1 is an idealized distribution of dopant. Curves 2-4 are real distributions of dopant for different values of annealing time. Increasing in the number of curve corresponds to the increasing of annealing time.

The $\psi(x,y,z)$ is the approximation function. Dependences of optimal values of annealing time on parameters are presented on Figures 6 and 7 for diffusion and ion types of doping, respectively. It should be noted that it is necessary to anneal radiation defects after ion implantation. One could find spreading of concentration of distribution of dopant during this annealing. In the ideal case, distribution of dopant achieves appropriate interfaces between materials of heterostructure during annealing of radiation defects. If dopant did not achieve any interfaces during annealing of radiation defects, it is practicably to the additional anneal of the dopant. In this situation, optimal value of additional annealing time of implanted dopant is smaller than annealing time of infused dopant.

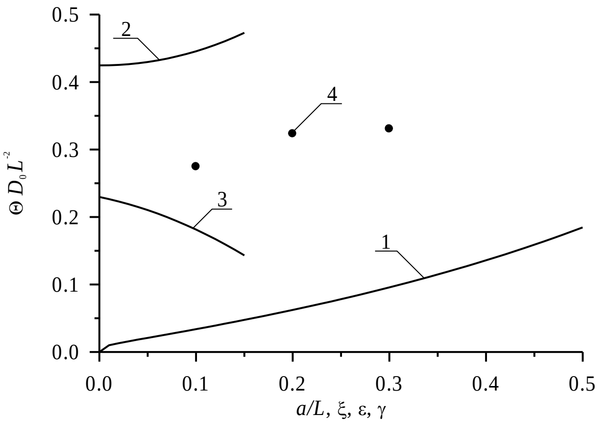


Figure 6: Dependences of dimensionless optimal annealing time for doping by diffusion, which have been obtained by minimization of mean-squared error on several parameters. Curve 1 is the dependence of dimensionless optimal annealing time on the relation a/L and $\xi = \gamma = 0$ for equal to each other values of dopant diffusion coefficient in all parts of heterostructure. Curve 2 is the dependence of dimensionless optimal annealing time on value of parameter ε for a/L=1/2 and $\xi = \gamma = 0$. Curve 3 is the dependence of dimensionless optimal annealing time on value of parameter ξ for a/L=1/2 and $\varepsilon = \gamma = 0$. Curve 4 is the dependence of dimensionless optimal annealing time on value of parameter γ for a/L=1/2 and $\varepsilon = \xi = 0$

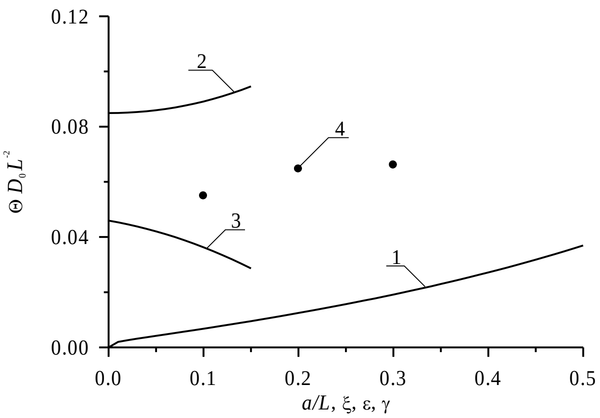


Figure 7: Dependences of dimensionless optimal annealing time for doping by ion implantation, which have been obtained by minimization of mean-squared error on several parameters. Curve 1 is the dependence of dimensionless optimal annealing time on the relation a/L and $\xi = \gamma = 0$ for equal to each other values of dopant diffusion coefficient in all parts of heterostructure. Curve 2 is the dependence of dimensionless optimal annealing time on value of parameter ε for a/L=1/2 and $\xi = \gamma = 0$. Curve 3 is the dependence of dimensionless optimal annealing time on value of parameter ξ for a/L=1/2 and $\varepsilon = \gamma = 0$. Curve 4 is the dependence of dimensionless optimal annealing time on value of parameter γ for a/L=1/2 and $\varepsilon = \xi = 0$

Further, we analyzed the influence of relaxation of mechanical stress on distribution of dopant in doped areas of heterostructure. Under the following condition $\varepsilon_0 < 0$, one can find compression of distribution of concentration of dopant near interface between materials of heterostructure. Contrary (at $\varepsilon_0 > 0$), one can find the spreading of distribution of concentration of dopant in this area. The changing of distribution of concentration of dopant could be at least partially compensated by using laser annealing [29]. This type of annealing gives us the possibility to accelerate diffusion of dopant and another process in annealed area due to inhomogenous distribution of temperature and Arrhenius law. The relaxation of mismatch-induced stress in heterostructure could lead to changing of optimal values of annealing time. Mismatch-induced stress could be used to increase the density of elements of integrated circuits. On the other hand, it could lead to the generation of dislocations of the discrepancy. Figure 8 shows the distributions of the component of displacement vector, which is perpendicular to the interface between layers of heterostructure.

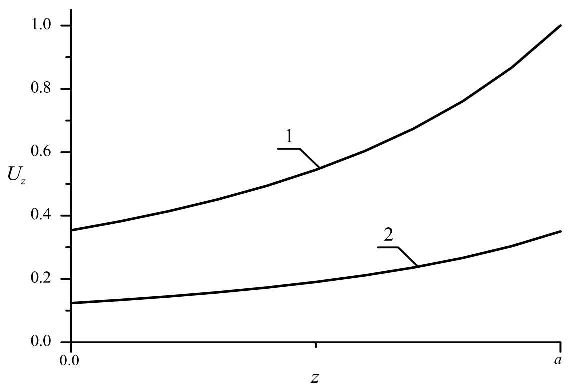


Figure 8: Normalized dependences of component u_z of displacement vector on coordinate z for nonporous (curve 1) and porous (curve 2) epitaxial layers

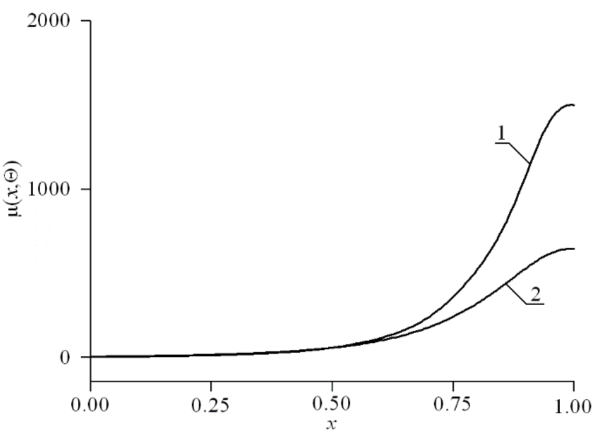


Figure 9: Normalized distributions of charge carrier mobility in the considered heterostructure. Curve 1 corresponds to the heterostructure, which has been considered in Figure 1. Curve 2 correspond to a homogenous material with averaged parameters of heterostructure from Figure 1

IV. Conclusion

In this paper, we modelled the redistribution of infused and implanted dopants, taking into account the relaxation mismatch-induced stress during manufacturing the field-effect heterotransistors framework that enhances the swing differential Colpitts oscillator. We formulated recommendations for optimization of annealing to decrease dimensions of transistors and to increase their density. We formulated recommendations to decrease mismatch-induced stress. Analytical approach to model diffusion and ion types of doping taking into account the concurrent changing of parameters in space and time has been introduced. At the same time, the approach gives us the possibility to take into account nonlinearity of considered processes.

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