Balancing Control of Two-Wheeled Robot by Using Linear Quadratic Gaussian (LQG)

Feriyonika and Asep Hidayat
Electrical Engineering, Politeknik Negeri Bandung, Indonesia.
feriyonika@gmail.com

***Abstract*—The Optimal Control method such as Linear Quadratic Regulator (LQR) deals with both the qualities of the response and its consumed power. In such a system, LQR faces a problem with the feedback sensor, which contains a lot of noise. Therefore, this issue can be solved by combining it with the Kalman filter, called the Linear Quadratic Gaussian (LQG). This research investigated the LQG applied in the Two-Wheeled Balancing Robot. According to the obtained data from MPU6050 (Accelero-Gyro sensor), Kalman Filter was firstly designed by adjusting the matrix R and Q. In the same way, LQR was also designed by manually tuning the matrix Q(1,1), Q(2,2) and R. The results of Kalman Filter showed that while Q_{acc}, Q_{gyro}, and R are 0.001, 0.003, and 1, respectively, the noise of the sensor can be successfully decreased. At the same time, while Q(1,1), Q(2,2), R of LQR are set to 1650, 25, and 3, respectively, the Two-Wheeled Robot can be stabilized in the set-point with the lowest J-function (1365.86). The verification experiment indicates that the controller can maintain the system stability even when the external disturbance is present.**

***Index Terms*— Balancing Robot; Kalman Filter; LQG; LQR.**

I. INTRODUCTION

The application of a Two-Wheeled Balancing Robot can be found in several applications such as Segway, rocket propeller, walking robot, and boat stability [1-4]. This plant is analyzed based on the structure of inverted pendulum, which is naturally unstable. There have been many control methods proposed to solve the problems appearing in the Two- Wheeled Balancing Robot control. The work of [5] and [6] proposed PID and Fuzzy control method to stabilize the balancing robot, which is firstly modeled as an inverted pendulum. In another work, Junfeng et al. proposed a sliding mode control method to stabilize the position [7]. They used computer simulations to verify their method. In the work in [5-6], although the proposed control methods could stabilize the plant, it still focuses on the response qualities without considering the consumed power by the plant. Therefore, several other researches proposed the optimal control, LQR, to balance both the quality of response and the consumed power [8,9]. Realizing it in a real plant, the LQR has become complex due to the quality of feedback sensor that usually uses the accelero-gyro sensor, which contains a lot of noise; thus, the Kalman Filter is then employed and combined to LQR, called Linear Quadratic Gaussian (LQG).

This research proposed an optimal control method (LQG) applied in the Two-Wheeled Balancing Robot. The plant was firstly modeled by measuring all physical parts. The model, in state-space form, was then evaluated in terms of its controllability and observability. In this research, the LQG method consists of the Kalman Filter and LQR, which can be designed separately. The Kalman Filter was firstly designed by tuning matrix Q and R of the Accelero-Gyro sensor; thus, the enhanced feedback sensor can be obtained. The LQR was subsequently designed by tuning matrix Q(1,1), Q(2,2), and R. The performance index (J-function) was finally employed to evaluate the effectiveness of the chosen parameter.

II. PLANT MODELING

The modeling process of Two-Wheeled Balancing Robot was obtained by firstly representing it as an inverted pendulum (depicted in Figure 1 [10,11]). All forces influencing the plant were then investigated as shown in Figure 2.

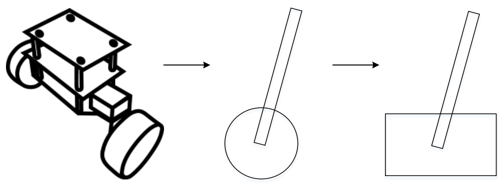

Figure 1: Balancing robot and inverted pendulum

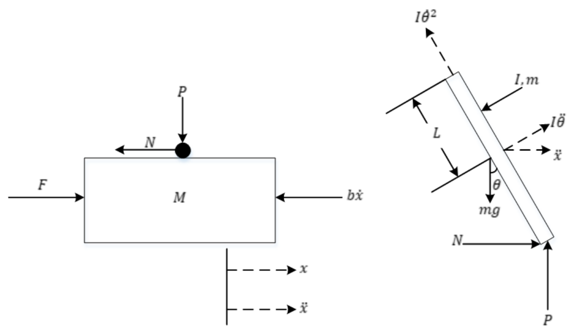

Figure 2: Forces in inverted pendulum [10]

where: x = Cart position (m)
$\dot{x}$ = Cart velocity (m/s)
$\ddot{x}$ = Cart acceleration (m/s^2)
F = Force (N)
N = Horizontal Force (N)
P = Vertical Force (N)
m = Pendulum Mass (kg)
M = Cart mass (kg)
b = Friction (N/m/s)
l = Length of Pendulum’s equilibrium point (m)
I = Moment of Inertia (kgm^2)
θ = Degree (°)
$\dot{\theta}$ = angular velocity (°/s)
$\ddot{\theta}$ = angular acceleration (°/s^2)

Equations (1) and (2) are the result of the horizontal and the normal forces (N). By substituting (2) to (1) and summing the perpendicular force, the dynamic equation, (4), can be obtained.

$$M\ddot{x}+b\dot{x}+N=F \tag{1}$$

$$N=m\ddot{x}+ml\ddot{\theta}\cos(\theta)-ml\ddot{\theta}^2\sin(\theta) \tag{2}$$

$$(M+m)\ddot{x}+ml\ddot{\theta}\cos(\theta)-ml\ddot{\theta}^2\sin(\theta)=F \tag{3}$$

$$P\sin(\theta)+N\cos(\theta)-mg\sin(\theta)=ml\ddot{\theta}+ml\ddot{\theta}\cos(\theta \tag{4}$$

For eliminating *P* and *N*, all moments around the center of mass were summed, therefore (5) can be derived. By substituting (4) to (5), the dynamic equation is to be (6).

$$-P\sin(\theta)-N\cos(\theta)=l\ddot{\theta} \tag{5}$$

$$(l+ml^2)\ddot{\theta}+mgl\sin(\theta)=-ml\ddot{x}\cos(\theta) \tag{6}$$

Equation (3) and (6) represent the dynamic part of the inverted pendulum. Because the equations are nonlinear, the linearization of them is thus needed. The pendulum has distance π radian from the stable condition and $\theta = \pi$. Assuming that $\theta = \pi + \phi$ (where ϕ represents small changing of the pendulum), the $\cos(\theta) = -1$, $\sin(\theta) = -\phi$ and $\left(\frac{d(\theta)}{dt}\right)^2=0$

After linearization (3) and (6), the new equations are to be (7) and (8) with u as an input signal (in this research it is voltage signal sent from the microcontroller). Equation (7) and (8) are subsequently converted by Laplace Transformation to be (9) and (10).

$$(M+m)\ddot{x}+b\dot{x}-ml\ddot{\phi}=u \tag{7}$$

$$(l+ml^2)\ddot{\phi}-mgl\phi=ml\ddot{x} \tag{8}$$

$$(M+m)x(s)s^2+bx(s)s-ml\phi(s)s^2=u \tag{9}$$

$$(l+ml^2)\phi(s)s^2-mgl\phi(s)=mlx(s)s^2 \tag{10}$$

Based on (10), *x(s)* is stated as (11), which is then substituted to (9). The overall transfer function can be derived as (13).

$$x(s)=\left[\frac{(l+ml^2)}{ml}-\frac{g}{s^2}\right]\phi(s) \tag{11}$$

$$\begin{aligned}&(M+m)\left[\frac{(l+ml^2)}{ml}-\frac{g}{s^2}\right]\phi(s)s^2+\\&b\left[\frac{(l+ml^2)}{ml}-\frac{g}{s^2}\right]\phi(s)s-ml\phi(s)s^2=u(s)\end{aligned} \tag{12}$$

$$\frac{\phi(s)}{u(s)}=\frac{mls}{qs^3+b(l+ml^2)s^2-mgl(M+m)s-bmgl} \tag{13}$$

where: $q=(M+m)(l+ml^2)-(ml)^2$

With the same method, the transfer function of cart position *x(s)* as output is derived as (14).

$$\frac{x(s)}{u(s)}=\frac{mls}{qs^3+b(l+ml^2)s^2-mgl(M+m)s-bmgl} \tag{14}$$

Equation (13) and (14) are then represent the state-space equation as (15) and (16).

$$\begin{bmatrix}\dot{x}\\ \ddot{x}\\ \dot{\phi}\\ \ddot{\phi}\end{bmatrix}=\begin{bmatrix}0 & 1 & 0 & 0\\ 0 & \frac{-(I+ml^2)b}{I(M+m)+Mml^2} & \frac{m^2gl^2}{I(M+m)+Mml^2} & 0\\ 0 & 0 & 0 & 1\\ 0 & \frac{-mlb}{I(M+m)+Mml^2} & \frac{mgl(M+m)}{I(M+m)+Mml^2} & 0\end{bmatrix}\begin{bmatrix}x\\ \dot{x}\\ \phi\\ \dot{\phi}\end{bmatrix}+\begin{bmatrix}0\\ \frac{I+ml^2}{I(M+m)+Mml^2}\\ 0\\ \frac{ml^2}{I(M+m)+Mml^2}\end{bmatrix}u \tag{15}$$

$$y=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}x\\ \dot{x}\\ \phi\\ \dot{\phi}\end{bmatrix}+\begin{bmatrix}0\\ 0\end{bmatrix}u \tag{16}$$

In this paper x and $\dot{x}$ are not used, therefore the equation can be simplified to be (17) and (18). Figure 3 and Table 1 are the plant and its parameter's information, respectively. The system Equations are finally derived to be (19) and (20).

$$\begin{bmatrix}\dot{\phi}\\ \ddot{\phi}\end{bmatrix}=\begin{bmatrix}0 & 1\\ \frac{mgl(M+m)}{I(M+m)+Mml^2} & 0\end{bmatrix}\begin{bmatrix}\phi\\ \dot{\phi}\end{bmatrix}+\begin{bmatrix}0\\ \frac{ml^2}{I(M+m)+Mml^2}\end{bmatrix}u \tag{17}$$

$$y=\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}\phi\\ \dot{\phi}\end{bmatrix}+\begin{bmatrix}0\end{bmatrix}u \tag{18}$$

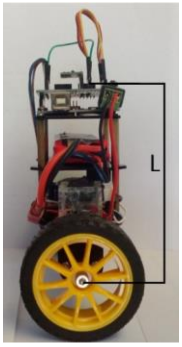

Figure 3: The balancing robot

Table 1
Parameter of the Plant Model

Parameter	Value
M	0.285 (kg)
m	0.285 (kg)
l	0.0565 (m)
g	9.8 kgm/s^2
L	0.113 (m)
I=mL^2	0.003639165

$$\begin{bmatrix}\dot{\phi}\\ \ddot{\phi}\end{bmatrix}=\begin{bmatrix}0 & 1\\ 38.5447 & 0\end{bmatrix}\begin{bmatrix}\phi\\ \dot{\phi}\end{bmatrix}+\begin{bmatrix}0\\ 6.9002\end{bmatrix}u \qquad (19)$$

$$y=\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}\phi\\ \dot{\phi}\end{bmatrix}+\begin{bmatrix}0\end{bmatrix}u \qquad (20)$$

III. CONTROLLER DESIGN

In the controller design, the system model was firstly evaluated to analyze its controllability and observability. In this research, the parameters of Kalman Filter and LQR are designed separately. The detail of the controller design is explained in the following sub-section.

A. Control Ability (CA) and Observability (OA)

According to (19) and (20), the CA and OA were calculated by (21) and (22). Both matrices have a rank equal to 2 (full rank), they are hence controllable and observable.

$$CA=\begin{bmatrix}B & AB\end{bmatrix}=\begin{bmatrix}0 & 6.9002\\ 6.9002 & 0\end{bmatrix} \qquad (21)$$

$$CA=\begin{bmatrix}C\\ CA\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \qquad (22)$$

B. Kalman Filter

This research uses the Accelero-Gyro sensor (MPU6050) as feedback of the robot position. Equation (23) is the general equation for the predicted state. According to the work of [12], the equation is going to be (24). Equation (25) is then employed to calculate the predicted covariance matrix.

$$X_{kp}=A.X_{k-1}+B.U_k+W_k \qquad (23)$$

where: X_{kp} = Predicted State
X_{k-1} = Previous
U_k = Control Variable Matrix
W_k = Predicted State Noise Matrix

$$\begin{bmatrix}Xa_{kp}\\ Xg_{kp}\end{bmatrix}=\begin{bmatrix}1 & dt\\ 0 & 1\end{bmatrix}\begin{bmatrix}Xa_{k-1}\\ Xg_{k-1}\end{bmatrix} \qquad (24)$$

where: X_{akp} = Predicted state of Accelero
X_{gkp} = Predicted state of Gyro
dt = Time sampling

$$P_{kp}=AP_{k-1}A^T+Q_k dt \qquad (25)$$

where: P = Process Covariance Matrix
Q = Process Noise Covariance Matrix

In this work, matrix Q is defined as (26). The values of Q_{acc} and Q_{Gyro} are manually tuned so that the Kalman Filter has a good response.

$$Q_k=\begin{bmatrix}Q_{acc} & 0\\ 0 & Q_{Gyro}\end{bmatrix} \qquad (26)$$

The next process is to calculate Kalman gain based on (27). In this research, the value of H is set to be 1. On the other hand, the value of R is adjusted so that the Kalman Filter has a good response. The output of the Kalman Filter is finally derived based on (28). Process Covariance is subsequently updated by using (29) for the next calculation.

$$K=\frac{P_{kp}.H^T}{H.P_{kp}.H^T+R} \qquad (27)$$

where: K = Kalman Gain
R = Sensor Noise Covariance Matrix

$$X_k=X_{kp}+K\left[Y_k-H.X_{KP}\right] \qquad (28)$$

$$P_k=\left[I-K.H\right]P_{kp} \qquad (29)$$

C. Linear Quadratic Gaussian (LQG)

The LQG method consists of Kalman Filter and Linear Quadratic Regulator (LQR), which can be designed separately. According to (19) and system that has zero set-point, the vector of matrix U is defined as (30).

$$u=-Kx \qquad (30)$$

To obtain the K matrix, (31) is employed whereas Algebraic Riccati Equation (ARE) in (32) is applied to find matrix P.

$$K=R^{-1}B^T P \qquad (31)$$

$$A^T P+PA-PBR^{-1}B^T P+Q=0 \qquad (32)$$

In this research, the value of Q and R are adjusted so that the lowest of the Performance Index, (33), can be derived.

$$J=\int_0^\infty \left(X^T QX+U^T QU\right)dt \qquad (33)$$

IV. EXPERIMENT SETUP

Figure 4 and 5 show both the block diagram and the structure of the hardware. The 8 bit microcontroller is employed to implement the Kalman filter, LQR algorithm and calculation of the J-function. The output of the controller is subsequently sent to the L298 driver, which is directly attached to the DC motor 25GA370. The feedback of the plant is measured by using MPU6050, a gyro-accelerometer sensor.

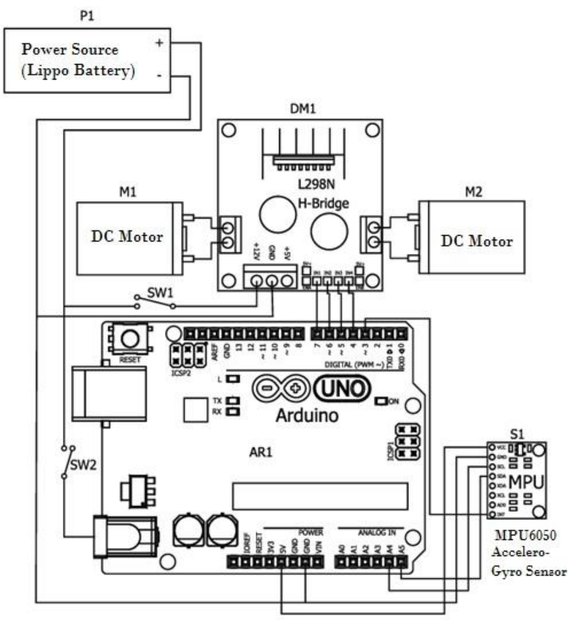

Figure 4: The hardware setup

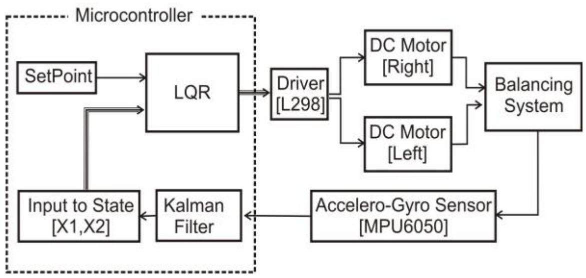

Figure 5: The block diagram

V. Results And Discussion

A. Kalman Filter

The combination of Q and R matrix are randomly chosen as follow: Q_{acc} = {0.4; 0.01; 0.001}, Q_{gyro} = {0.2; 0.03; 0.003} and R = {1;50; 100}. The best result is depicted in Figure 6.

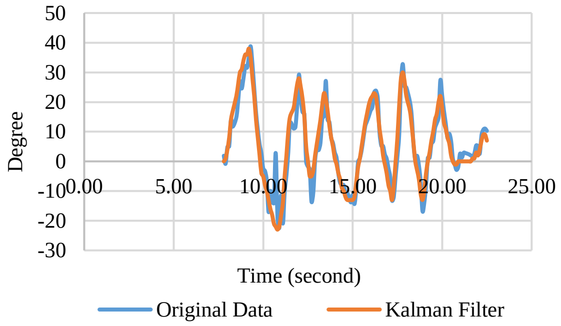

Figure 6: Resulted signal with R =1, Q_{acc} = 0.001 and Q_{gyro} = 0.003

B. Linear Quadratic Gaussian

After obtaining the Kalman Filter parameter, the next process is to tune matrix Q and R manually and calculate the value of Performance Index (J-function). Table 2 is the tuning result for matrix Q. The lowest value of J-function with $Q(1,1)$ and $Q(2,2)$ are 1650 and 25, respectively. Figure 7 is the plant response of the chosen parameters.

Table 2
The Result of Tuning for Matrix Q

$Q(1,1)$	$Q(2,2)$	K_1	K_2	*J-Function*
1650	25	46.5885	6.2051	1656.6
1700	30	47.1937	6.6090	1665.05
1750	35	47.7903	6.9894	1740.2

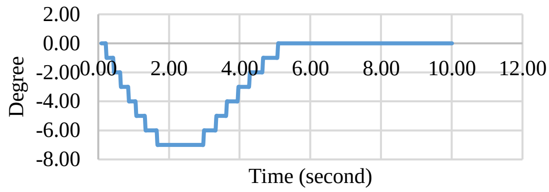

Figure 7: The response with $Q(1,1)$ = 1650 and $Q(2,2)$ = 25

Table 3 is the result of J function, while the value of R is varied. With $Q(1,1)$ = 1650, $Q(2,2)$ = 25 and R = 3, the lowest of J function can be derived and the steady-state is 8.297 second. Figure 8 shows the response.

Table 3
The Result of Tuning for Matrix R

R	K_1	K_2	*J-Function*
1	46.5885	6.2051	1656.60
2	34.8470	4.7540	1818.19
3	29.6942	4.1158	1365.86

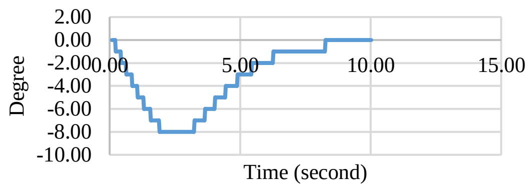

Figure 8: The response with $Q(1,1)$ = 1650, $Q(2,2)$ = 25 and R = 3

The response of the controller is finally verified by giving external disturbances, as depicted in Figure 9. The disturbance started from 25.074-th second, while the steady state is in 26.34-th second (1.266 seconds).

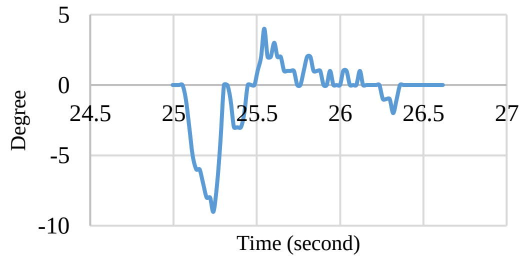

Figure 11: Robot response against external disturbance

VI. Conclusion

This research deals with the optimal control method (LQG) applied in the two-wheel balancing robot. The method consists of the Kalman Filter and LQR, which can be designed separately. The Kalman Filter was firstly designed by tuning matrix *Q* and *R* of the Accelero-Gyro sensor so that the feedback sensor without noise could be obtained. The LQR was then designed by tuning matrix *Q(1,1)*, *Q(2,2)* and *R*. For verification the effectiveness of the chosen parameter, the performance index (*J*-function) was subsequently employed. The results of Kalman Filter showed that with Q_{acc}, Q_{gyro} and *R* are 0.001, 0.003, and 1, respectively, the noise of the response could be minimized. Similarly, while *Q(1,1)*, *Q(2,2)*, and *R* are set to 1650, 25 and 3, respectively, the two-wheeled robot could be stabilized in the set-point. The proposed control could also maintain the robot position with the presence of the external disturbance.

Acknowledgment

We would like to thank UPPM Politeknik Negeri Bandung for the research Grant "Penelitian Mandiri 2019"

References

[1] R Babazadeh, AG Khiabani, H Azmi, “Optimal control of Segway personal transporter,” *2016 4th International Conference on Control, Instrumentation, and Automation (ICCIA),* Iran, 2016.

[2] Adi K, Hardianto, Edi SK, Indra RK “Modeling and control of ballast system to improve stability of catamaran boat,” *2015 International Conference on Advance Mechatronics, Intelligent Manufacture, and Industrial Automation (ICAMIMIA),* Indonesia, 2015.

[3] WY Dzan, SY Chang, KC Hsu “Designing and Building of a Catamaran and Its Stability Analysis,” *2013Second International Conference on Robot, Vision and Signal Processing,* Kitakyushu, Japan, 2013.

[4] M Migdalovici, Luige V, Daniela B, Gabriela V, Mihai R “Stability Analysis of the Walking Robots,” *Procedia Computer Science,* V

[5] Rasoul S, Mehdi TM "An experimental study on the PID-Fuzzy controller on a designed two-wheeled self-balancing autonomous robot," *2016 4th International Conference on Control, Instrumentation, and Automation (ICCIA),* Surabaya Indonesia, 2016.

[6] Derry P, Eko HB, Fernando A “Movement control of two wheels balancing robot using cascaded PID controller,” *2015 International Electronics Symposium (IES),* Surabaya Indonesia, 2015.

[7] Junfeng W, Yuxin L, Zhe W “A robust control method of two-wheeled self-balancing robot,” *2011 6th International Forum on Strategic Technology*, Harbin, 2011.

[8] Mustafa E “Embedded LQR controller design for self-balancing robot,” *2018 7th Mediterranean Conference on Embedded Computing (MECO)*, Budva Montenegro, 2018.

[9] Saqib I, Adeel M, MT Razzaq, J Iqbal “Advance sliding mode control techniques for Inverted Pendulum: Modelling and simulation,” *an International Journal Engineering Science and Technology*, Vol 21, Issue 4, pp. 753-759, 2018.

[10] Hellman H, Sunnerman H “Two-Wheeled Self-Balancing Robot: Design and control based on the concept of an inverted pendulum,” Bachelor’s Thesis in Mechatronic, KTH Royal Institute of Technology in Stockholm, Sweden, 2015

[11] Bill M, Dawn T “Inverted Pendulum: System Modeling,” *Control Tutorial for Matlab and Simulink,* Department of Mechanical Engineering at Tufts University and The University of Michigan, USA, 2017

[12] Dendi M, Feriyonika, Eril M “Segway design by using PID controller and Kalman Filter,” Undergraduate thesis, Politeknik Negeri Bandung, Indonesia, 2016.