Throughput Improvement of the Selective Repeat+Go-Back-N ARQ Scheme in Fading Channels with Diversity Combining

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***Abstract*—Various Automatic Repeat reQuest (ARQ) and Hybrid ARQ (HARQ) schemes have been employed to improve the performance of wireless communication systems operating in different environments. Various methods, some of them quite complex, have also been used to analyze the communication systems. In this report, we illustrate the simplicity of the transition diagram method when applied to the SR-GBN ARQ and HARQ schemes in fading environments and when diversity combining is employed. We derive the performance expressions for the schemes, which show that the SR-GBN HARQ scheme has better performance than the SR-GBN-ADP HARQ scheme.**

***Index Terms*—AWGN; Hybrid ARQ; Maximal Ratio Combining; Rayleigh Fading; Selective Combining; SR-GBN; Throughput Efficiency.**

I. INTRODUCTION

Automatic Repeat reQuest (ARQ) techniques have been widely used in combating errors dealing with digital information, when propagating through noisy and unstable channels. The techniques are based on the receivers requesting the retransmission of received information packets when the received information contained intolerable errors. The techniques include the Stop-and-Wait (SW) scheme, in which the transmitter only transmits new packets after it receives the status of the previous transmission from the receiver, the Selective Repeat (SR) scheme, in which the transmitter transmits new packets continuously and only retransmits packets received with error, and the Go-Back-N (GBN) scheme, in which packets are also transmitted continuously but when a packet fails, that packet and the N-1 packets are retransmitted. The SW scheme is simple but unpopular because the channel remains unutilized when the transmitter is waiting for the responses from the receiver. The SR scheme is better than the previously mentioned because the channel is fully utilized. However, it is impractical because an infinite buffer is needed at the receiver when the channel is noisy. The GBN scheme improves on the SR scheme by limiting the required buffer size at the receiver to N packets. However, its performance decreases when the channel is noisy due to the requirement to transmit N packets each time a packet fails. The combination of the SR and GBN schemes for the SR-GBN scheme reduces the limitations of the GBN scheme by applying a number of SR retransmissions before moving to the lost of GBN retransmissions. The analysis of the basic SW, SR, GBN and mixed mode SR-GBN and other ARQ schemes can be found in various reports [1-3], while the application of the basic ARQ schemes to model molecular communication is given in [4].

In the efforts to improve the performance of the ARQ-based systems, Forward Error Correction (FEC) is also attempted at the receiver to avoid or reduce the retransmissions. This leads to Hybrid ARQ (HARQ) techniques, in which FEC is incorporated in the basic ARQ schemes mentioned above. With HARQ, failed packets are only retransmitted if attempts to correct the errors in them fails. The operation and performance of HARQ techniques can be found in [1-2, 5-6]. Analysis to use HARQ to improve performance through code combining systems is found in [7], while the evolution of methods and techniques aimed at avoiding the use of retransmissions is reported in [8]. When data transmissions are carried out in fading channels, the performance of the systems becomes irregular, and the combined diversity is applied as one of the techniques to improve the performance [9]. In particular, the current demand for multiple services at any place and at any time has led to the requirement for faster and more robust communication systems and techniques [10]. Comparative analysis of various combining techniques is given in [11-14], where it is shown that the Maximal Ratio Combining (MRC) technique has the best performance. Analysis on the application of various combining techniques in MIMO systems aimed at mitigating Rayleigh fading is reported in [15-16]. In these reports, the MRC technique is also reported to have superior performance. The analysis in [17] gives a review on the application of antenna diversity in OFDM systems, while a throughput-based antenna selection method for a system employing the SR-GBN scheme is given in [18].

In this paper, we report on the performance of the SR-GBN ARQ scheme as it is utilized in a simple mixed format, when used in fading channels, which then used with FEC, and finally, with diversity combining. The analysis employs the model used in [3], in contrast to the Markov based and signal flow based methods [19-20] and incorporates fading, FEC and diversity combining progressively to it.

II. SYSTEM MODEL AND ANALYSIS

In this analysis, we adopted the following notations: T = round trip delay + τ, τ = packet duration, L packet length in bits, k = information bits in the packet, m = parity bits in the packet such that $L = k + m$, and $R = k/L$ = coding rate. Hence, if the transmitter starts to transmit a packet at t_0, it finishes transmitting the last bit at $t_0 + \tau$, the data then reaches the receiver and the errors are processed before the receiver transmits the *ACK / NACK* (*ACKnowledgement / Negative ACKnowledgement*). Here, the transmitter receives and finishes the processing in time to start transmitting the next packet at $t_0 + T$. Assuming the length of the *ACK* or *NACK* is very small compared to the data packet length, and the a negligible time is taken to process the *ACK* or *NACK*, we can define the process using this formula:

$$S = \frac{T-\tau}{\tau} = \frac{T}{\tau} - 1 = N - 1 \tag{1}$$

where: N = integers, and
S = integers, if
T = Multiple of τ

Moreover, we assume that the feedback channel (for *ACK* and *NACK*) is error free, and time can be represented in terms of frames, with each frame occupying N slots each of duration τ. We then start with the analysis of the SR-GBN scheme, which can be represented by a transition diagram based on a previous analysis [3] as given in Figure 1, with N = 4 and v = 2. We note that q is the probability of packet transmission failure, p (= 1 - q) is the probability of packet transmission success, v is the number of SR retransmissions before the system enters the GBN mode, and the parameter z (corresponding to τ) is only used for the slots, in which the packet of interest is transmitted, or any other packet is transmitted to aid the retransmission of the packet of interest.

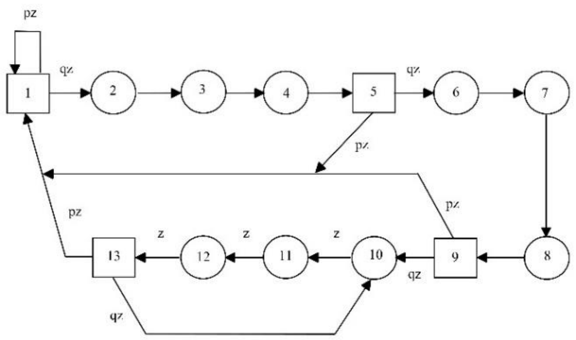

Figure 1: The SR-GBN ARQ scheme with N = 4, v = 2

In Figure 1, the system remains in state 1 as long as the transmitted packets received error free. If an error occurs, the system transits to state 2, at the same time retransmitting the failed packet. The response from the receiver for this packet is received in state 5: If it is an *ACK*, the system returns to state 1, otherwise it retransmits the packet and goes to state 6. The next decision on the packet is made in state 9: If a *NACK* is received, the system enters the GBN mode in states 10 to 13 and will return to state 1. The transfer functions for general v can then be obtained by inspecting Figure 1 to give:

$$\Phi_{12}(z) = \frac{qz}{1-pz}$$
$$\Phi_{21}(z) = \sum_{n=0}^{v-1} (qz)^n pz + \frac{(qz)^v pz^N}{1-qz^N} \tag{2}$$

From which we obtain the delays:

$$L_{12} = \frac{d}{dz}\left(\Phi_{12}(z)\right)\Big|_{z=1} = \frac{1}{q}$$
$$L_{21} = \frac{d}{dz}\left(\Phi_{21}(z)\right)\Big|_{z=1} = \frac{1+(N-1)q^v}{p} \tag{3}$$

And subsequently, the packet based throughput efficiency:

$$\eta = \frac{L_{12}}{L_{12}+L_{21}} = \frac{p}{1+Sq^{v+1}}. \tag{4}$$

The transition diagram and transfer function for the SR scheme can be obtained from the analysis of the SR-GBN scheme by extending v to infinite value and neglecting packets other than the packet of interest to obtain:

$$\eta_{SR} = p. \tag{5}$$

Likewise, the corresponding transition diagram and transfer function for the SW scheme can be obtained from the SR scheme by noting that all N slots in a SW frame are attributed to the single packet transmitted in the frame to obtain:

$$\eta_{SW} = \frac{p}{N}. \tag{6}$$

We note that since in the conventional ARQ schemes only ED (Error Detection) is employed, the probability of a packet being received without errors is given by:

$$P_B = (1-P_e)^L \tag{7}$$

where: P_e = Probability of a received bit in the packet being in error

When FEC is employed, the probability of a packet being received with correctible errors is given by:

$$P_B = \sum_{i=0}^{t} \binom{L}{i} P_e^i (1-P_e)^{L-i} \tag{8}$$

where: t = Error correcting capability of the FEC code

With these definitions, the true throughput efficiency of the SR-GBN scheme is expressed as:

$$\eta_{SR-GBN} = \frac{P_B R}{1+SQ_B^{v+1}} \tag{9}$$

where: Q_B = 1 - P_B
R = Coding rate

If BPSK transmission is employed, then the probability of bit error in AWGN (Additive White Gaussian Noise) channels is given by:

$$P_e = \frac{1}{2} erfc\left\{\sqrt{\frac{E_b}{N_0}}\right\} \tag{10}$$

where E_b/N_0 = Signal to Noise Ratio (SNR)
E_b = Energy per bit
$N_0/2$ = Double sided Power Spectral Density (PSD) of the AWGN

When there is a Rayleigh fading in the channel, the pdf of the envelope amplitude x of the received signal is of the form [13].

$$f(x) = \begin{cases} \frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}} & 0 \le x < \infty \\ 0 & otherwise \end{cases} \tag{11}$$

where: σ = RMS value of the received signal before envelope detection
σ^2 = Time-averaged power of the received signal before envelope detection

The conditional probability of bit error with fading then becomes:

$$P(e \mid h) = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{N_0} h^2}\right) \tag{12}$$

where: h = Complex multipath Rayleigh scaling factor

Using $\gamma = \sqrt{\frac{E_b}{N_0} h^2}$ gives:

$$P_e = \int_0^\infty \frac{1}{2} erfc\left(\sqrt{\gamma}\right) \frac{\gamma}{a^2} e^{-\frac{\gamma}{2a^2}} d\gamma \tag{13}$$

where: $a^2 = \frac{E_b}{N_0}$.

Subsequently, using Equation (14) in Equation (13) gives Equation (15).

$$erfc(x) = 2Q\left(\sqrt{2}x\right)$$
$$Q(x) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} e^{-\frac{x^2}{2\sin^2\theta}} d\theta, x > 0 \tag{14}$$

$$P_e = \frac{1}{2}\left(1 - \sqrt{\frac{a^2}{1+a^2}}\right) \tag{15}$$

The effects of fading can be applied to the SR-GBN scheme by inserting Equation (15) in Equation (7) and using it in Equation (9).

When diversity combining is employed to mitigate Rayleigh fading, the probability of bit error for the Maximum Ratio Combiner is given in [13-14] as:

$$P_e = P^M \sum_{k=0}^{M-1} \binom{M-1+k}{k} \left(1-P\right)^k \tag{16}$$

where: P = Equation (17)
M = Number of antennas at the receiver

$$P = \frac{1}{2} - \frac{1}{2}\left(1 + \frac{1}{E_b / N_0}\right)^{-1/2} \tag{17}$$

The corresponding probability of bit error for the Selective Combiner scheme is given in [13-14] as:

$$P_e = \frac{1}{2} \sum_{k=0}^{M} \left(-1\right)^k \binom{M}{k} \left(1 + \frac{k}{E_b / N_0}\right)^{-1/2} \tag{18}$$

These results can be used in Equation (7), (8) and (9) to give the performance when diversity combining is applied. The effects of fading when HARQ is employed can be investigated by inserting Equation (15) into Equation (8) and (9). Likewise, the effects of diversity combining can be investigated by inserting Equation (16) and (18) into Equation (8) and (9).

The SR-GBN scheme can also be implemented using the Alternate Data/Parity packet scheme to give the SR-GBN-ADP scheme. The throughput of the SR-GBN scheme can therefore be compared with that of the SR-GBN-ADP scheme.

III. Results and Discussions

The performance of the SR-GBN scheme is given in Figure 2 to 8. In these results, the packet length was fixed at L = 1023 bits, round trip delay at N = 10, number of data bits in the frame at k = 973 bits, error correcting capability at t = 5 bits, and SR retransmissions in the GBN scheme at v = 1 unless otherwise stated. Also, the throughput efficiency is based on packets, rather than information bits. The comparison in the throughput performance of the SW, SR, GBN and SR-GBN ARQ schemes in AWGN channels is given in Figure 2. It is seen as expected that the SW scheme has the lowest performance, while the ideal SR scheme has the highest. Also, the SR-GBN scheme has higher performance than the GBN scheme, and this increases with the number of SR retransmissions v. It can be concluded from these results that increased number of SR retransmissions in the SR-GBN mode makes the GBN scheme approach the configuration of the SR scheme in operation and performance.

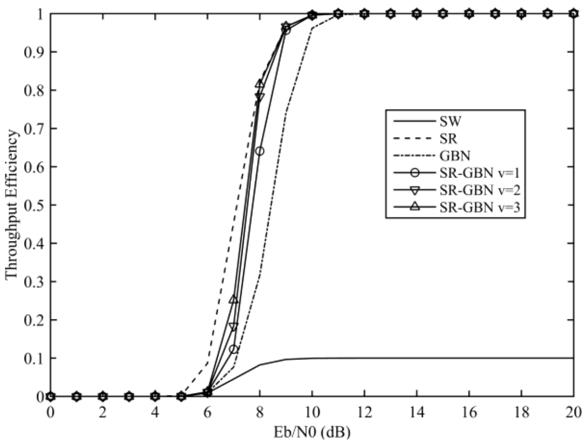

Figure 2: Throughput efficiency vs Eb/N0 for the SW, SR, GBN and SR-GBN ARQ schemes

The effects of fading on the performance of the SW, SR, GBN and SR-GBN schemes are given in Figure 3. Comparing the results with those in Figure 2, it is seen that fading lowers the throughput of all the schemes although their relative performance is maintained. Since the errors in a digital communication system occur when deciding on which bit was transmitted, it can be concluded that the effects of fading are to increase the uncertainties in the decision making function of the communication system.

As a measure to mitigate fading, the effects of applying diversity in the form of MRC are given in Figure 4. The results show that when MRC is applied to the SR-GBN scheme, the performance increases with the number of antennas used, even exceeding that of the single antenna ideal SR scheme. Since increasing the number of receive antennas increases the complexity and cost of the receiver, the given results suggest that the use of two antennas is sufficient. The comparison of the MRC and SC diversity techniques is given in Figure 5, where it is seen that the MRC technique yields better results compared to the SC technique, a result that has also been obtained in [13, 14].

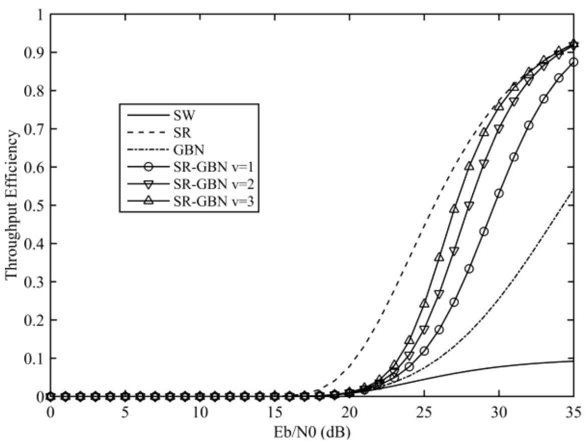

Figure 3: The effects of fading on the throughput of the SW, SR, GBN and SR-GBN ARQ schemes

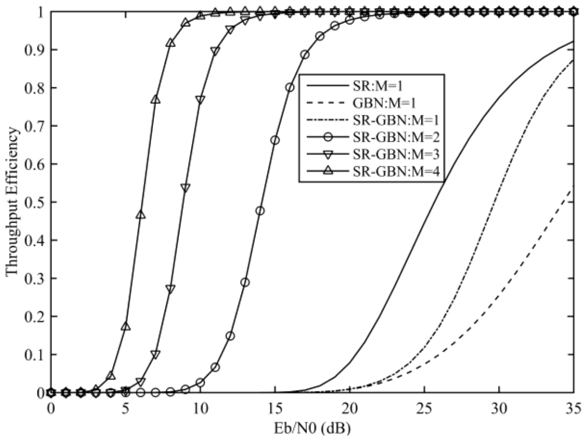

Figure 4: The effects of MRC on the throughput of the SR, GBN and SR-GBN ARQ schemes

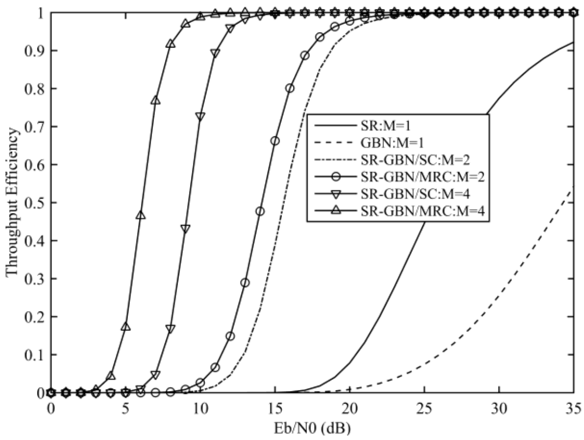

Figure 5: Comparison of the effects of MRC and SC on the performance of the SR-GBN ARQ scheme

Figure 6 shows the performance when HARQ is employed for the SW, SR and SR-GBN schemes in channels with and without fading. Comparing Figure 2 and 6 shows that when there is no fading, the use of HARQ greatly improves the performance of the communication systems for all types of ARQ. Likewise, a comparison of Figure 3 and 6 shows that when there is a fading, the use of HARQ improves the performance of the communication systems. Note that Figure 6 alone shows that fading affects the performance of the communication system for all types of ARQ schemes, even when FEC is employed.

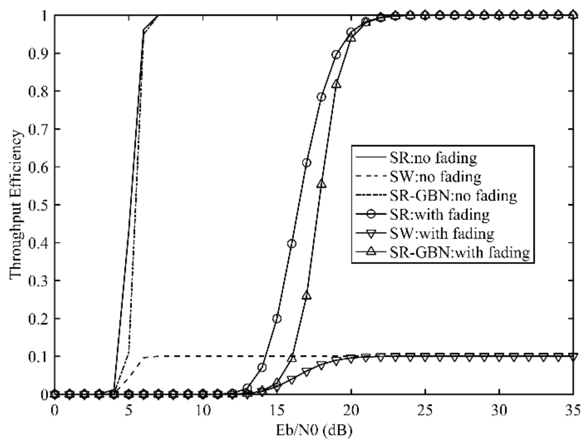

Figure 6: The effects of HARQ on the performance of the SW, SR and SR-GBN schemes with and without fading

Figure 7 shows the performance, when MRC is employed in communication systems utilizing HARQ. First, based on the comparison of Figure 6 and 7, the use of just 2 antennas improves the performance of the communication systems significantly. Secondly, Figure 7 shows that use of more received antennas further improves the performance of the communication system for all types of ARQ schemes. Therefore, when both HARQ and MRC are applied to communication systems operating in fading environments, there is great improvement in performance.

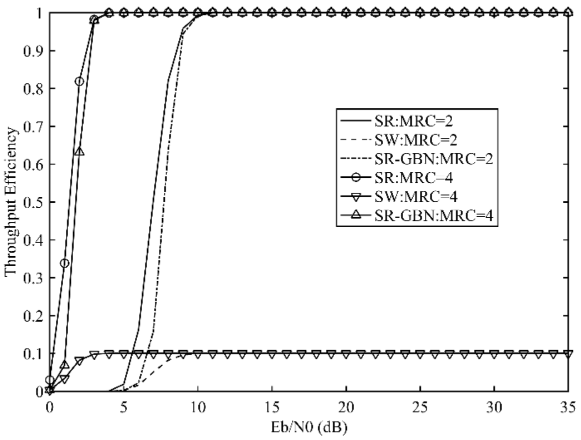

Figure 7: The effects of HARQ and MRC on the performance of the SW, SR and SR-GBN ARQ schemes

Figure 8 shows the performance of the communication system when MRC is employed in systems utilizing HARQ with the SR scheme used as a reference. First, it is seen that the SR-GBN scheme has higher performance than the SR-GBN-ADP scheme. Secondly, it is seen from Figure 6, 7 and 8 that the use of just 2 antennas improves the performance of the communication systems significantly. Thirdly, when both HARQ and MRC are applied to communication systems operating in fading environments, there is a great improvement in performance. Noting that the SR-GBN-ADP scheme is essentially a Type II HARQ scheme, while the SR-GBN HARQ scheme is a Type I HARQ scheme, the decision processing at the receiver during retransmissions is more complex for the SR-GBN-ADP scheme.

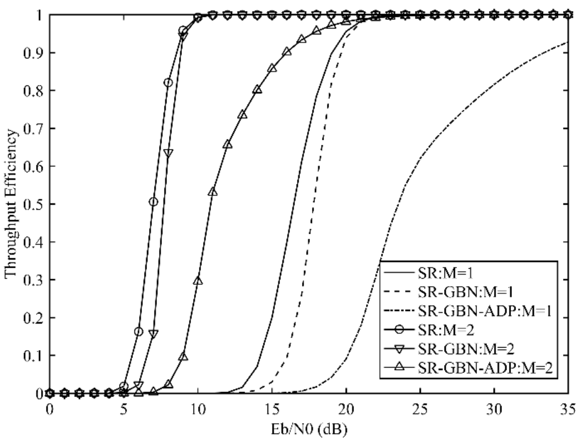

Figure 8: Comparison of the SR-GBN and SR-GBN-ADP HARQ schemes

We note that in this analysis, the employed SR-GBN HARQ schemes used only one SR retransmission. As seen in Figure 2 and 3, the use of more SR retransmissions lead yields higher performance. The use of many SR retransmissions in the SR-GBN scheme, however, makes the receiver more complex, and the use of higher order diversity makes the systems even more complex. A trade-off between the increased SR retransmissions and higher order diversity is, therefore necessary in order to avoid the attendant increase in complexity.

We also note that the alternate data/parity scheme used with the SR-GBN ARQ in this analysis can be regarded as a truncated data/parity scheme, which simplifies the analysis. The simplification, however, is obtained at the expense of lower performance. Other popular Type II HARQ schemes include the Chase Combining (CC) and the Incremental Redundancy (IR), which are best analyzed using simulation techniques, and in which the IR scheme outperforms the CC scheme [5]. In order to investigate these HARQ schemes analytically, however, the use of a different modeling approach over the model used in this report is required. The adopted model, however, must represent practical implementation. In general, practical design approaches are used to yield systems that are less complex and easier to analyze, provided that they yield acceptable performance.

IV. CONCLUSION

The analysis of the SR-GBN ARQ scheme has been done to investigate the effects of fading, HARQ and diversity combining. The results have shown that HARQ greatly improves the performance of the systems. It has been further shown that fading greatly degrades the performance of the systems. The incorporation of diversity combining has shown that there is improved performance, when there is only AWGN in the channel, and when fading is also present. Finally, when both HARQ and MRC are applied to systems operating in fading conditions, the performance improves significantly. Realization of such systems, however, favor less complex implementations which also yields simpler models that are easier to analyze, provided that the achieved performance is acceptable.

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