

Design Modeling and Characterizations of SOI-based Parallel Cascaded MRR Array (PCMRRA) by Coupled Mode Theory

Hazura Haroon¹, Sahbudin Shaari², Menon, P.S², N. Arsad² and Hanim A.R¹

¹Centre of Electronics & Telecommunication Research Institute (CETRI),

Faculty of Electronic and Computer Engineering,

Universiti Teknikal Malaysia Melaka (UTeM), Malaysia.

²Institute of Microengineering and Nanoelectronics (IMEN),

Universiti Kebangsaan Malaysia (UKM), Malaysia

hazura@utem.edu.my

Abstract— We present a comprehensive analytical modeling of a Silicon-on- Insulator (SOI) -based Parallel Cascaded microring resonator Array (PCMRRA) by coupled mode theory (CMT) using the transfer matrix model. The transmission characteristics are analysed and the optimized design parameters are discussed. Analytical results show that higher order microring has a flatter passband and steeper roll-off. With a ring diameter of 12 μ m and a coupling gap of 100nm, the calculated FSR value is 14nm with the insertion loss of 0.43dB. For verification, we compare these results with the results obtained from the Finite Difference Time Domain (FDTD) commercially available software.

Index Terms— Microring resonator, Coupled-Mode Theory, Transfer Matrix Method, Cascaded Microring.

I. INTRODUCTION

High-index-contrast (HIC) of silicon-on-insulator (SOI) is an incredibly Silicon Photonics technology to miniaturize the size of planar lightwave circuits (PLCs) down to micrometer length scale[1]. Progress in the Silicon Photonics technology has increased the demonstrations of various photonic devices including microring resonator (MRR). MRR has been incorporated in many applications such as filtering, modulation, switching and multiplexing [2-5]. Unfortunately, designing a high performance of MRR filters is still a major challenge where the performance of MRR filter is highly dependent on the structural design and very fine etching processes. Therefore, accurate design parameters and tolerance analysis are crucial in the MRR filter device development. In this work, optimized design parameters are determined by the analytical modeling to study the effect of variation of each parameter to the overall performance.

To address the need for rapid design exploration, accurate but easily understandable device modeling are required. Several methods have been previously demonstrated to analyse the MRR filter transmittance, such as the finite difference time domain (FDTD) and conformal transformation method. Withal, both methods are time consuming and various approximations incorporated. We present the

modeling of MRR filter based on coupled-mode theory incorporated in the transfer matrix model. Our approach calculate an expression for the coupling coefficient, where then will be applied to determine other dependent parameters. Characterizations of the device performance including transmission loss, Free Spectral Range (FSR) and Quality Factor are theoretically analyzed. To evaluate the accuracy of the model, we compare the performance predictions of the model with results obtained from the Finite Difference Time Domain (FDTD) commercially available software by RSoft

II. THEORY

A. Device Design

The basic configuration of our MRR filter design comprises a circular waveguide microring resonator coupled to two bus waveguides as shown in Figure 1. Asymmetrical MRR design is considered, where two bus waveguides have the same width, W and the same coupling distance, g . A fully etched silicon waveguide is considered where the height of the waveguide, H is fixed at 550nm. The waveguide is approximately single mode, with the TE-mode light propagating around 1.55 μ m for 300nm waveguide width, W . The coupling distance between all waveguides, g is 100nm and the ring radius is 6 μ m. To simplify the analysis, we assume the bus waveguides and the ring waveguide have similar waveguide dimensions and refractive indexes. The resonances occur when the optical path length of a round-trip is a multiple of the effective wavelength. Thus the resonant wavelengths can be determined by [6]:

$$\lambda_c = \frac{n_{eff}}{m} L \quad (1)$$

where λ_c is the resonant wavelength, n_{eff} the effective index of the guided mode, L the circumference of the microring and m is an integer. The substrate cladding is composed of air ($n=1.0$ at 1.55 μ m) and lateral coupling is chosen between the bus waveguides and ring waveguides due to fabrication limitation.

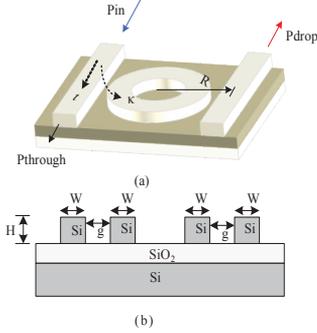


Figure 1(a): Layout and (b) cross section of the proposed MRR.

The number of microring resonators used can be increased to improve selectivity, i.e. the box-like shape of the passband, and/or to exploit the Vernier principle. Therefore, we also study the effect of incorporating multiple microrings in parallel configuration or so called a parallel-cascaded MRR array (PCMRRA) as illustrated in Figure 2. By selecting proper values of design parameters, the box-like spectral response is expected in PCMRRA configurations.

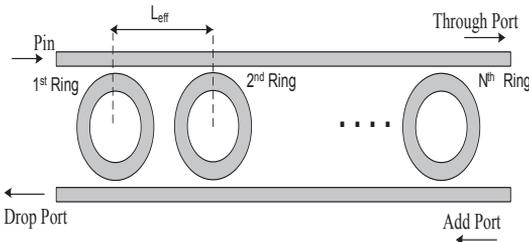


Figure 2: Schematic layout of parallel cascaded MRR filter

B. Coupled Mode Theory and Transfer Matrix Method

One of the most fundamental approaches in the photonic device modeling and design is the coupled mode theory (CMT) and transfer matrix techniques, due to its conceptual simplicity and the physical insights. Using these standard approaches, we derive the expressions to calculate coupling coefficient and transfer functions of the MRR filter. From the transmission response, some well-known MRR filter figures of merit can be defined including FSR and Q-factor. Here, we will briefly review the basic features of the method and the analytical equations; to provide a more understandable model.

We consider the MRR filter, comprising of a single ring coupled to a double bus waveguide as in Figure 1. For better understanding of the mentioned method, Figure 3 is considered where the coupling between waveguide for microring filter can be calculated using the standard coupling method for directional coupler and delay unit as shown in Figure 3.

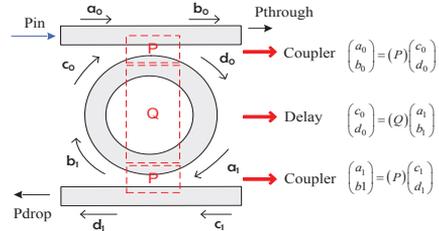


Figure 3: Visualization of the couplers and delay unit from microring resonator

The transmission of the input and output coupler, P can be calculated by [7]:

$$P = \frac{1}{\kappa} \begin{pmatrix} -t & 1 \\ -1 & t^* \end{pmatrix} \quad (2)$$

where κ is the coupling coefficient, and κ^2 determine the ratio of power coupled between the bus and ring waveguides. Assuming the case of lossless coupling, we have $\kappa^2 + t^2 = 1$, where t is the transmission coefficient.

As the field propagates around the ring, it accumulates a phase shift and attenuated. The optical phase delay and the waveguide loss, Q can be described as:

$$Q = \frac{1}{\kappa} \begin{pmatrix} 0 & e^{-i\beta\pi R} \\ e^{i\beta\pi R} & 0 \end{pmatrix} \quad (3)$$

where R is the ring radius and β is the propagation constant which is equal to:

$$\tilde{\beta} = \frac{2\pi}{\lambda_o} n_{eff} - j \frac{\alpha}{2} \quad (4)$$

Here, α is the loss per unit length in the microring, λ_o is the free space wavelength and n_{eff} is the effective refractive index. The transfer matrix between two bus waveguides is:

$$\begin{pmatrix} a_0 \\ b_0 \end{pmatrix} = (PQP) \begin{pmatrix} c_1 \\ d_1 \end{pmatrix} \equiv (M) \begin{pmatrix} c_1 \\ d_1 \end{pmatrix} \equiv \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \begin{pmatrix} c_1 \\ d_1 \end{pmatrix} \quad (5)$$

Considering only 1 input, c_1 will be zero, hence the final transfer functions for the through port [T] and drop port [D] signals are [8]:

$$|T| = \frac{b_o}{a_o} = \frac{m_{22}}{m_{12}} = \frac{\sqrt{1-\kappa_{in}} - \sqrt{1-\kappa_{out}} e^{-i\tilde{\beta}2\pi R}}{1 - \sqrt{(1-\kappa_{in})(1-\kappa_{out})} e^{-i\tilde{\beta}2\pi R}} \quad (6)$$

$$|D| = \frac{d_1}{a_o} = \frac{1}{m_{12}} = \frac{-\sqrt{\kappa_{in}\kappa_{out}} e^{-i\tilde{\beta}\pi R}}{1 - \sqrt{(1-\kappa_{in})(1-\kappa_{out})} e^{-i\tilde{\beta}2\pi R}} \quad (7)$$

In the parallel coupled configuration, multiple microrings are periodically separated from each other by a spacing L_{eff} , as depicted in Figure 2. The overall transfer matrix can be simply expressed by multiplying the individual resonator transmission matrices by the spacing phase matrices as below:

$$\begin{pmatrix} input \\ drop \end{pmatrix} = (M_{R1} M_{L_{eff1}} M_{R2} M_{L_{eff2}} \dots M_{RN}) \begin{pmatrix} throughput \\ add \end{pmatrix} \equiv (M) \begin{pmatrix} throughput \\ add \end{pmatrix} \quad (8)$$

where N is the number of MRR and $[M_R]$ is the transmission of individual resonator as in Eq. (5), $M_{L_{eff}}$ can be computed

by:

$$M_{L_{eff}} = \begin{pmatrix} e^{i\tilde{\beta}L_{eff}} & 0 \\ 0 & e^{i\tilde{\beta}\pi R} \end{pmatrix} \quad (9)$$

The net coupling coefficient can be obtained by integrating over the entire region, which interaction between waveguides fields occurs and considering the waveguide mode phase mismatch. For symmetrical lateral coupling waveguide structures, κ is analytically computed according to the theory developed in [9]:

$$\kappa = \frac{2\eta^2\gamma \exp(-\gamma \cdot g)}{\tilde{\beta}(W + \frac{2}{\gamma})(\eta^2 + \gamma^2)} \quad (10)$$

where g is the distance between the bus waveguide and the microring waveguide, while η and γ are as follows:

$$\eta = \sqrt{n_{core}^2 k_o^2 - \tilde{\beta}^2} \quad (11)$$

$$\gamma = \sqrt{\tilde{\beta}^2 - n_{clad}^2 k_o^2} \quad (12)$$

It is noted that $k_o = (2\pi/\lambda_o)$, n_{core} is the refractive index of the core waveguide and n_{clad} is the refractive index of cladding.

Once the coupling coefficient is known, the performance of the coupled microring can be evaluated for the FSR and Q-factor. FSR is the frequency separation between two successive resonances and is given by [9]:

$$FSR \approx \frac{\lambda_o^2}{n_g(\lambda)L_{eff}} \quad (13)$$

where n_g is the group refractive index. Meanwhile, the Q-factor of the m^{th} resonance is estimated by computing the ratio of the center wavelength to the -3dB bandwidth.

III. RESULTS AND DISCUSSION

In this section, the transmission characteristics of single and parallel cascaded microring resonator will be discussed. In the calculations, it is assume that the waveguide width, W , the waveguide height, H and the free space wavelength considered are 300nm, 550nm and 1550nm, respectively. The ring radius of $6\mu\text{m}$ is taken into account throughout this study.

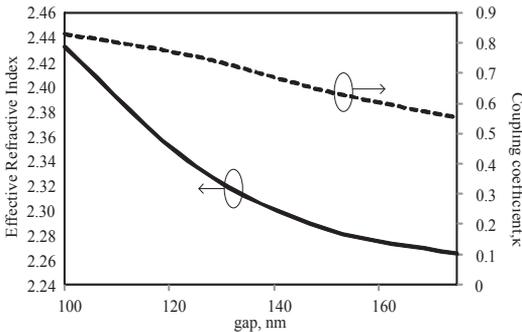


Figure 4: Theoretical coupling coefficient and effective refractive index values using CMT

Figure 4 graphed the calculations results of effective refractive values and coupling coefficients. The calculations

only considered the TE-mode behavior of the microring filter. It is noticeable that, the effective refractive index decreases as the gap separation between waveguide increases. Since the coupling coefficient is dependable on the effective index value, the same trend can be observable for the coupling coefficient as we varied the gap separation. The effective refractive index influences the propagation constant value as in eq(4), therefore the lower the effective refractive indexes values, the lower the propagation constants, thus lessen the power coupled between waveguides, which can estimated by the coupling coefficient values. As comparison, the simulated effective index value obtained for a gap of 100nm is 2.5446 as compared to 2.4321 obtained from the theoretical modeling. Hence, the deviation between the theoretical modeling and the simulation validation is around 4%, which can considered small.

To analyze the effect of gap separation on Q-factor and loss, we varied the gap separation from 100nm to 175nm. The results as shown in Figure 5 is for a single mode first order microring filter. It can be seen that as the gap increases the Q-factor and insertion loss raised. Yet again, the difference between theoretical and simulation is small. Moreover, we can concluded that the separation gap plays an important role in optimizing the performance of the microring resonator. Therefore, an accurate selection of gap separation value is crucial to confirms a well-functioning device.

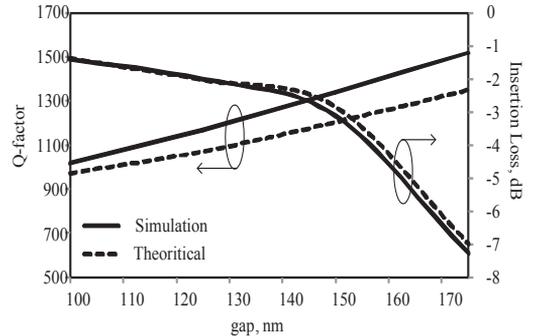


Figure 5: Q-factor and insertion loss values against gap separation

The influence of the variations of ring radius on Q-factor and FSR for first order microring filter are depicted in Figure 6. Here we fixed $g=100\text{nm}$. We can conclude that both are highly dependent on the ring radius. There are a slight different of FSR values between the theoretical model and simulation for ring radii below $9\mu\text{m}$. For instance, for the ring radii of $16\mu\text{m}$, the $FSR_{theoretical}$ is 16nm, compared to 14nm for simulation. The different is possibly due to the discretized nature of the numerically simulated structure. Table 1 summarized the 1st order microring resonator's performance for the ring radii of $8\mu\text{m}$ and the separation gap of 125nm.

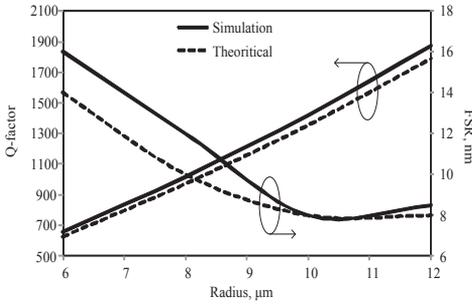


Figure 6: Q-factor and FSR value against ring radius

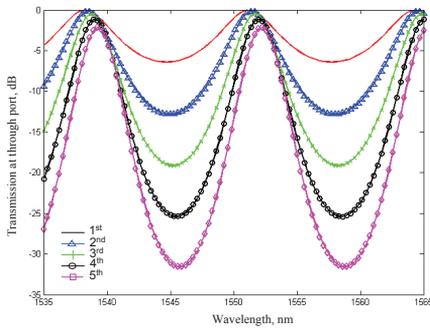


Figure 7: Transmission spectrum observed at the drop port

To investigate the performance of higher order parallel cascaded microring resonator, we theoretically designed up to 5th order microrings where each ring is coupled to the adjacent resonator. Figure 7 and 8 illustrate the transmission spectrum of the microring resonator observed at the drop port and through port, respectively. From Figure 7, it should be noted that flatter passband and steeper roll-off (box-like response) occurs as the number of microring order grows. This box-like response suits the applications of optical delay lines and Butterworth-like filter. In addition, it can be clearly seen that the flattening of passband is realized without giving a great impact to the output power. However, at the through port, higher order microring yields larger sidelobes. This is strongly influenced by loss in microring.

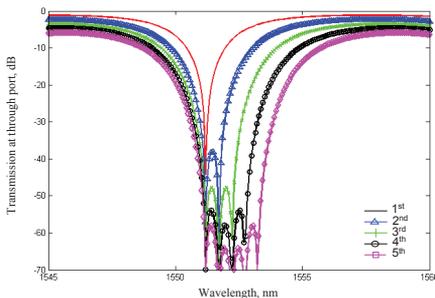


Figure 8: Transmission spectrum observed at the through port

Table 1
Performance summary

Parameter	Theoretical	Simulation
Resonant λ (nm)	1546	1546
n_{eff}	2.3358	2.5449
FSR (nm)	11	12
Q-factor	-3.05	-3.14
IL (dB)	1071	1167

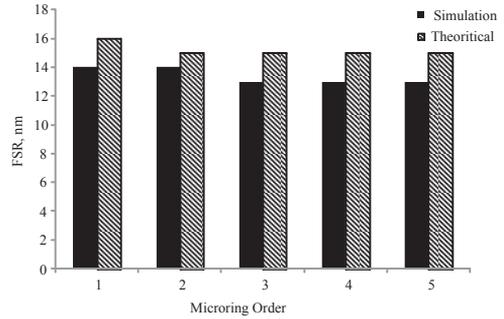


Figure 9: FSR for higher microring order

IV. CONCLUSION

We have presented initial results for analytical modeling of parallel cascaded microring resonator array up to 5th order by using the coupled mode theory. The influence of the microring resonator parameters on the coupling coefficient, FSR and Q-factor has been discussed. The efficiency of the model has been validate with FDTD simulation software and the developed model proves to allows fast and accurate solution for device design problem.

ACKNOWLEDGMENT

This research is supported by funding from UTeM (PJP/2013/FKEKK (44C)/S01261). Universiti Kebangsaan Malaysia is also acknowledged for their contribution via grant no GUP-2012-012.

REFERENCES

- [1] B. Mulyanti, P.S. Menon, S. Shaari, T. Hariyadi, L. Hasanah, H. Haroon. "Design and optimization of coupled Microring Resonators (MRRs) in silicon-on-insulator", *Sains Malaysiana* 43 (2), 2014, 247-252.
- [2] H. Haroon, S. Shaari, P.S. Menon, H. Abdul Razak and M. Bidin, "Application of statistical method to investigate the effects of design parameters on the performance of microring resonator channel dropping filter", *Int. J. Numer. Model.*, 26: 670-679. doi: 10.1002/jnm.1899,2013.
- [3] Y. Chao, W. Fung, and L.J. Guo, "Polymer microring resonators for biochemical sensing applications", *Optics Express*, 18, 2010, pp. 393-400.
- [4] S. Shaari, A. R. Hanim, B. Mardiana, H. Hazura and P. S. Menon, "Modeling and analysis of lateral doping region translation variation on optical modulator performance", 4th Asian Physics Symposium *AIP Conference Proceedings*, 1325, September, 2010, pp. 297-300.

- [5] A. Yariv, "Universal relations for coupling of optical power between microresonators and dielectric waveguides", *Electronic Letters*, 36, 2000, pp. 321-322.
- [6] C-Y Chao, and L.Jay Guo, "Design and Optimization of Microring Resonators in Biochemical Sensing Applications", *IEEE J.of Lightwave Technol.*, 24, 2006, pp. 1395-1402.
- [7] Zen Pheng, "Coupled Multiple Micro-Resonators Design And Active Semiconductor Micro-Resonator Fabrication", Thesis Dissertation University Of Southern California, 2007.
- [8] Y. B.E. Little, S.T. Chu, H.A Haus, J.Foresi, and J-P. Laine, "Microring Resonator Channel Dropping Filter", *J.of Lightwave Technol.* ,15 1997, pp. 998-1050.
- [9] P. Rabiei, W.H. Steier, Z.Cheng, and L.R Dalton, 'Polymer micro-ring filters and modulators', *J. of Lightwave Technol.*, 20, 2002, pp. 1968-1975.

