Performance Optimization of the Arbitrary Arrays with Randomly Distributed Elements for Wireless Sensor Networks

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***Abstract*—Beamforming using regular linear or planar arrays, in which their elements are uniformly spaced is widely studied for various applications. However, in the wireless sensor network applications, such regular arrays are not possible to build. Thus, they are usually built with randomly distributed planar elements. Generating the required beamforming from such randomly distributed arrays that can provide a significant improvement in the wireless sensor network performance is a real challenging issue. In this paper, the amplitude and phase of each random element within the arbitrary bounded area is optimized such that its corresponding array pattern acts as a beam-steerable with minimum sidelobe level and a certain beamwidth. Simulation results under various optimization constraints are given to show the effectiveness of the considered random array. The effect of changing the total number of array elements on the array performance, such as beamwidth, minimum sidelobe level, and the gain were also investigated.**

***Index Terms*—Arbitrary Array; Beamforming; Convex Optimization; Minimum Sidelobe Level; Randomly Spaced Elements.**

I. Introduction

In many applications, it is not possible to build a regular planar array of sensors due to some practical limitations. These applications include tumor localization in the biomedical investigation, environmental sensing including water quality monitoring, traffic control, seismic exploration, radio telescope that is composed of large number of randomly distributed sensors, forest fire detection, and flood control. Very recently, the wireless sensor networks (WSN) have been also effectively used in the monitoring areas for both military and civilian applications. In these applications, the sensors are deployed over a region where some phenomenon is to be monitored. In military, the sensors are used to detect the enemy intrusion, while in civilian scenarios, they are used to detect the geo-fencing of gas or oil pipelines [1]. To obtain the maximum performance in all of these above-mentioned applications, the sensor nodes in each wireless sensor network should be enabled to act as a smart array or collaborative beamforming. Then, a steerable beam can be easily formed to scan the horizon and find the desired direction, if the array elements within the bounded area share their information and transmit together in the synchronous manner. Unlike the uniformly distributed planar arrays, the sensors here are randomly distributed within an arbitrary bounded area. Each sensor node consists of a single element antenna. Combining these random nodes together results in a certain array configuration, which is more preferable than a single element antenna or even a simple linear array.

The arbitrary arrays consist of a random combination of sensor nodes whose main objective is to improve the performance of a wireless sensor network by enabling these random elements to work in a collaborative beamforming manner. The total electromagnetic fields of such arbitrary arrays is obtained by the coherent or incoherent combination of each sensor node. Accordingly, the radiation pattern in the far-field region of the arbitrary array with randomly distributed elements can be controlled by the location of each array elements, the phase excitation, and the amplitude excitation of each array elements.

In the regular planar arrays with uniform distances between array elements, the optimum beamforming with capability in enhancing the desired signals and at the same time suppressing the interference signals are obtained by properly optimizing the amplitude and/or the phase excitations of each array element by means of deterministic methods [2-3], locally adaptive algorithms [4-8] or by globally optimization algorithms [9-13]. In all of these algorithms, the distances between array elements are considered constant and the arrays have regular shapes such as rectangular and circular grids, thus, they are not included in the optimization process.

The contribution of this paper is as follows. First, an arbitrary planar array with a certain number of elements that are randomly distributed over a bounded area is considered. Then, the mathematical model of the radiation pattern for such array is derived. Next, various optimization criteria are applied to the array pattern to get maximum performance. These include the optimization of the amplitude and the phase excitations of each element in the randomly distributed planar array to obtain maximum radiation of the radio waves energy in the main beam direction, and minimize the wastes in the sidelobes region. The capability of placing deep nulls or generating asymmetric sidelobes for suppressing the jammer or interfering signals is also studied. In order to establish an efficient wireless communication link between any two wireless sensor networks, i.e., the one that near to the sensing information and the end user, a steerable beam that radiates or receives the concentrated radio waves energy in the desired direction is highly desirable. To achieve this goal, the designed arbitrary array should have a narrow beamwidth. This is directly proportional to the array size or aperture, i.e., the total number of the sensor elements that can be physically fit within the arbitrary and bounded planar area. A reduced number of sensors results in a lower array gain. This effect is extensively studied and an optimization process is introduced to find the maximum gain under narrower beamwidth. The convex optimization [14-16] is used to perform all the above-mentioned optimization constraints.

II. SIGNAL MODEL

The arbitrary array with randomly distributed elements located within a bounded planar area is considered to perform the desired beam configuration. This beam has the task to scan and search the horizon to identify the desired direction and accordingly establish the communication link between any considered wireless sensor networks. Unlike the regular uniform planar arrays, in which the elements are spaced apart by a constant value of $\mathrm{d} = 0.45\lambda$ in both x and y coordinates, the random planar arrays are composed of randomly distributed elements. A regular uniform planar array (UPA) and a random planar array (RPA) under the same number of sensor elements are illustrated in Figure 1 and 2 respectively.

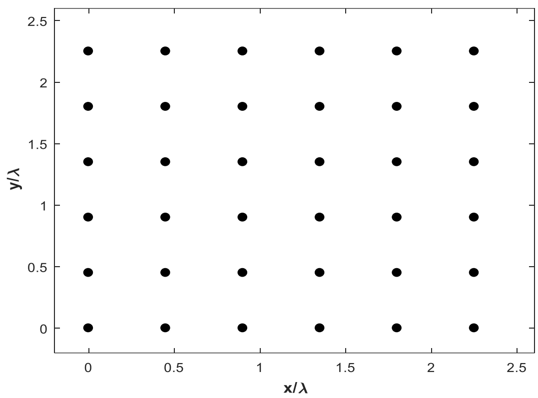
The considered random array composed of N isotropic elements, and assume K narrowband signals arriving from different directions in terms of elevation and azimuth planes $\theta_1, \phi_1,\ \theta_2, \phi_2,\ \ldots,\ \theta_K, \phi_K$. Considering that the incoming signals, i.e., the desired signal and the interfering signals, are located in the far-field and they can be assumed as isotropic sources, the total signal arriving at the nth sensor element in discrete time t is calculated as follows:

$$x_n(t) = \sum_{i=1}^{K} S_i(t) e^{-jk_i\tau_n} + n(t) \quad (1)$$

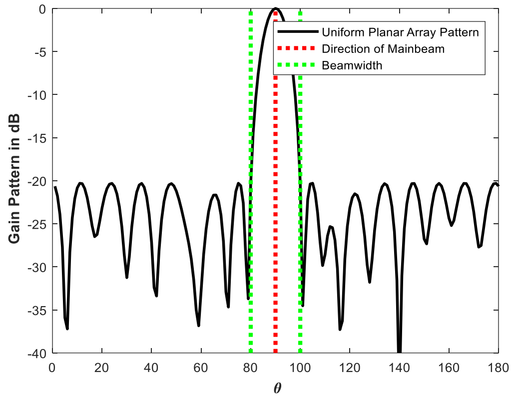
where: $\mathrm{S_i}(\mathrm{t})$ = Envelop of the ith signal source including both the phase and quadrature components
$k_i = \frac{2\pi}{\lambda_i}$ = Wave-number vector
$n(t)$ = White Gaussian noise signal
τ = Time delay of the arrived signal at any array elements with respect to the reference element and it can be written as follows:

$$\tau = \frac{d_x \sin(\theta_i)\cos(\phi_i) + d_y \sin(\theta_i)\sin(\phi_i)}{v_o} \quad (2)$$

where: v_o = Speed of light in free space
θ = Elevation angle
ϕ = Azimuth angle

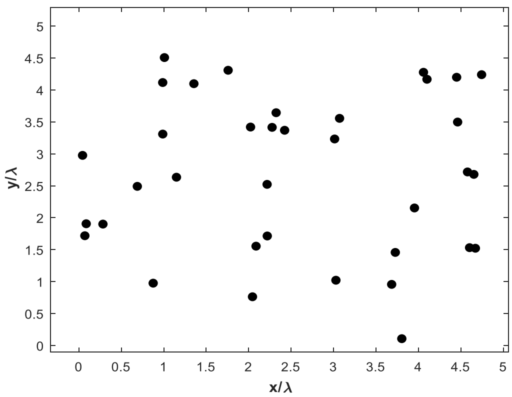


(a) Element distribution

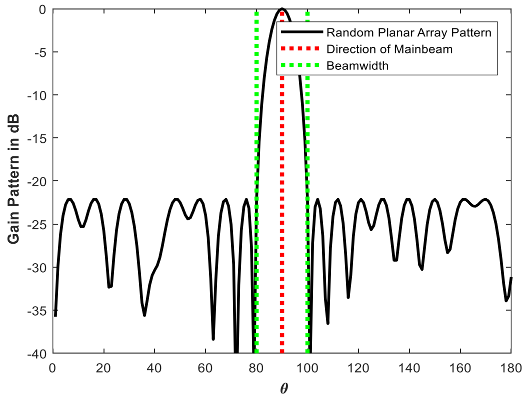


(b) Array pattern

Figure 1: Regular uniform planar array pattern and its element distribution



(a) Element distribution



(b) Array pattern

Figure 2: Random planar array pattern and its element distribution

Note that for the considered random planar array, the elements spacing d_x and $d_y$in (2) are both random variables. In order to find the gain pattern of the random array, the steering vector needs to be determined first.

$$a(\theta_i, \phi_i) = [1, e^{-jk_i\tau_1}, e^{-jk_i\tau_2}, \ldots, e^{-jk_i\tau_{N-1}}]^T \quad (3)$$

Then, the total steering matrix containing all of the incoming sources can be expressed as follows:

$$A(\theta, \phi) = [a(\theta_1, \phi_1), a(\theta_2, \phi_2), \ldots, a(\theta_K, \phi_K)] \quad (4)$$

Finally, the gain pattern of the random planar array can be written as:

$$y(\theta,\phi) = A(\theta,\phi)\, w \tag{5}$$

where: w = complex function which represents the array weights vector, i.e., the amplitude and the phase excitations

From (5), it is clear that the gain pattern of the array is a linear function of *w*. Thus, the shape of the gain pattern can be changed by simply imposing some constraints on it and optimize accordingly the array weights. Note that the optimization variables are the amplitude and the phase excitations of the array weights *w* as will be illustrated in the following section.

III. RANDOM PLANAR ARRAY

In this section, the radiation fields of each random elements in the considered wireless sensor network are coherently combined to form the desired beam toward the look direction. Clearly, the configured beam may be directly affected by the number of the sensor nodes within the bounded area of the WSN where, in general, a reduced number of sensor elements results in a lower peak gain. Thus, an optimization process with some required constraints is needed to optimize the array performance such as minimum sidelobe level, narrower beamwidth, and maximum gain or directivity. It is assumed that the total number of the sensor nodes within the bounded area of the WSN is constant. The constraints on the array pattern could be in the form of minimizing the width of the mainbeam, i.e., first null-to-null beamwidth, under pre-specified peak sidelobe level, which in turn minimizes the consumed energy during the beam scanning in the searching or receiving modes. An asymmetric sidelobes constraint becomes indispensable when it is required to reduce the ground clutter effect. This is mainly due to the fact that the sensors in many applications are mounted on the ground and their plane waves may be very near to the ground. As a result, the clutter echoes may greatly limit the performance of the designed array during the horizon scan. Therefore, a specific constraint on the sidelobe level to get lower sidelobes on the ground side is also presented in this section.

Referring to the array gain pattern that was expressed in (5), it can be seen that this is a quasi-convex problem and can be solved using a bisection [16]. The optimization problem is formulated as the minimization of the gain pattern outside the mainbeam region under different constraints as follows:

$$\text{minimize} \quad max|y(\theta,\phi)| \tag{6}$$

for θ,ϕ outside the mainbeam region subject to:

$$y(\beta_x,\beta_y) = 1 \tag{7}$$

where: $\beta_x = sin(\theta_0)\,cos(\phi_0)$ and $\beta_y = sin(\theta_0)\,sin(\phi_0)$ are progressive phase shifts in x and y directions that are necessary to direct the mainbeam to the *(θ_0,ϕ_0)*.

In this case, the optimization is formulated such that, first it has unit sensitivity (distortionless) at the desired direction, β_x,β_y, second it obeys a constraint on a minimum sidelobe level outside the mainbeam region.

As mentioned earlier, it is also desirable to minimize the width of the mainbeam (i.e., minimize the consumed energy during the horizon scan of the main beam of the array) under the condition of pre-specified minimum possible sidelobe level (minSLL). Thus, the optimization problem may be written as:

$$\text{minimize} \quad max|y(\theta,\phi)| \tag{8}$$

for θ,ϕ outside the main beam subject to:

$$y(\beta_x,\beta_y) = 1 \tag{9}$$

$$\begin{gathered} |y(\theta_i,\phi_i)| \le 10^{minSLL/20} \\ \theta_i \in (\Omega_{BW},180) \\ \phi_i \in (\Psi_{BW},360) \end{gathered} \tag{10}$$

where: Ω_{BW} = Required half-beamwidths in the elevation plane
Ψ_{BW} = Required half-beamwidths in the azimuth plane

The constraint in (9) aims at preserving the normalized peak gain in the look direction, while the constraint in (10) is for obtaining the pre-specified sidelobe level.

Effective and efficient design of the random planar array should be also able to deal with ground clutter problem, which is unavoidable in most of the WSN applications. As mentioned, to suppress these clutter echoes, the array pattern should be designed with asymmetric sidelobes or placing nulls at strong interfering directions, as illustrated in the following optimization:

$$\text{minimize} \quad max|y(\theta,\phi)| \tag{11}$$

for θ,ϕ outside the main beam subject to:

$$y(\beta_x,\beta_y) = 1 \tag{12}$$

$$|y(\theta_i,\phi_i)| \le SLL_i^{left} \quad \theta_i \in (\Omega_{BW},180) \tag{13}$$

$$|y(\theta_i,\phi_i)| \le SLL_i^{right} \quad \theta_i \in (0,-\Omega_{BW}) \tag{14}$$

$$\left|y(\theta_j,\phi_j)\right| \le Null_j \quad j = 1,2,\ldots,J \tag{15}$$

where: SLL_i^{left} = Required sidelobe levels on the left sides of the mainbeam in the designed planar array pattern
SLL_i^{right} = Required sidelobe levels on the right sides of the mainbeam in the designed planar array pattern. (Here, the asymmetric sidelobes is considered only in the elevation plane, nevertheless the azimuth plane is a straightforward)
$Null_j$ = Required number of the nulls toward interfering signals
J = total number of interfering signals

The constraints in (13) and (14) are for obtaining the required asymmetric sidelobe levels, while the constraints in (15) is for placing the required nulls at the interfering directions.

IV. SIMULATION RESULTS

Extensive numerical examples are provided in this section to optimize the array performance, in terms of maximum sidelobe reduction, minimum beamwidth, and maximum gain or directivity. For the random planar array, the array size is characterized in terms of wavelengths and the total number of sensor elements N. This is in contrast to the regular uniform planar arrays, which commonly list the number of elements along each axis, for example $N = 6$ and $M = 6$ for a total of 36 sensor elements. In the random planar array, it used $N = 36$ to indicate that there are 36 sensor elements within the bounded arbitrary planar area.

In the first example, the regular periodic planar array shown in Figure 1 consists of 36 equal-spaced sensors with 0.45λ inter-element spacing for comparison purpose. Here, the gain pattern of such array is optimized according to (6) and (7). The minimum sidelobe level is -20.30 dB for a given half beamwidth equal to 10^{o}. This result was obtained by optimizing both the amplitude and phase of each array sensor, as shown in Table I. On the other hand, the results of the randomly distributed planar array under the same number of sensor elements N=36 is shown in Figure 2. For fair comparison, the optimization constraints in (6) and (7) were also used to form the required array pattern. The minimum sidelobe level in the resulting randomly distributed planar array pattern is found to be 22.13dB (An improvement in the sidelobe reduction about 1.83 dB with respect to that of the regular planar array). The optimized amplitude and phase of each random element is also shown in Table 1. By comparing the results of the uniformly and randomly distributed planar arrays, it is clear that the random array provides lower sidelobe level under the same conditions and constraints. This proves the effectiveness of the designed arbitrary array and at the same time makes the random planar array superior to its counterpart in forming the desired beam for the WSN.

In the second example, we investigated the possibility of obtaining optimum beam width in the randomly distributed planar array under the given value of sidelobe level. In this example, we considered two different cases. In the first case, the given sidelobe level was -20dB, while in the second case the given sidelobe level was -30 dB. In both cases, it was required to find the minimum beamwidth according to (8), (9), and (10). Figure 3 shows the array patterns for these two cases. The optimum half beamwidths for the first and the second cases were $10^{o}$and 14^{o} respectively. The location, amplitude and phase of each sensor element are shown in Table I. It can be seen that the designed array pattern satisfies the required constraints.

In the next example, the constraints in (11), (12), (13), (14), and (15) are applied to obtain asymmetric radiation pattern. The results are shown in Figure 4. Here, the SLL_i^{left}, $SLL_i^{right}$are chosen to be -20dB and -40dB respectively. It is also assumed a single null at 20^{o}. The location, amplitude, and phase of each array sensor are shown in Table I. As can be seen, the designed arbitrary array is well optimized and satisfying all the required constraints.

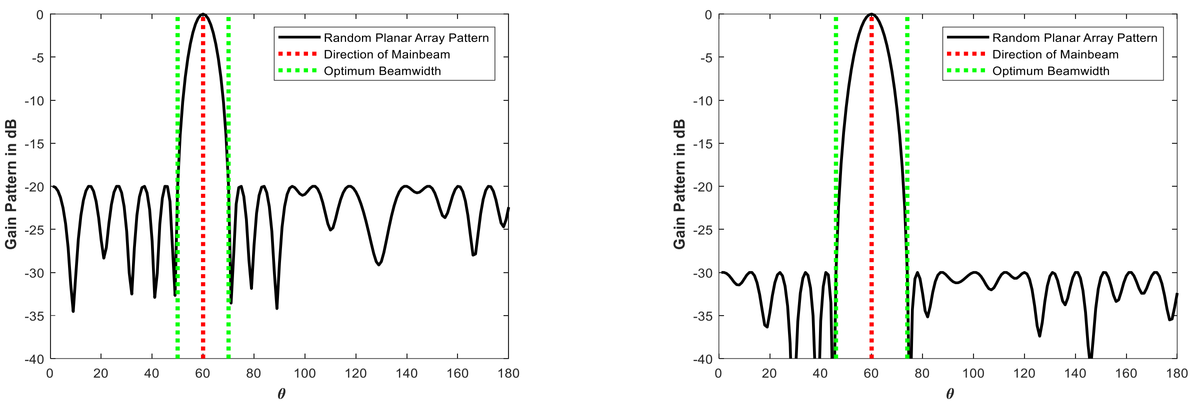


Figure 3: Random planar array patterns with optimum beamwidths for minimum sidelobe level=-20dB (left) and -30dB (right)

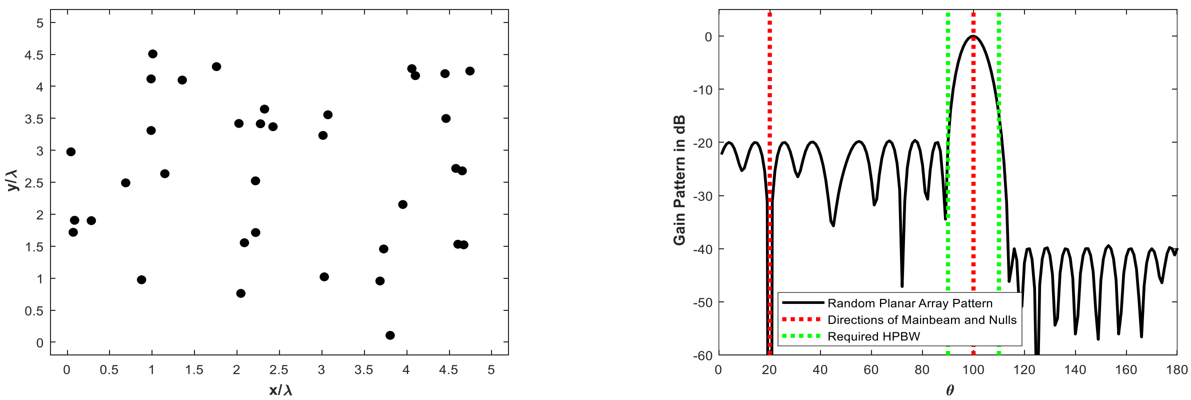


Figure 4: Random planar array pattern with asymmetric sidelobes and a single null control

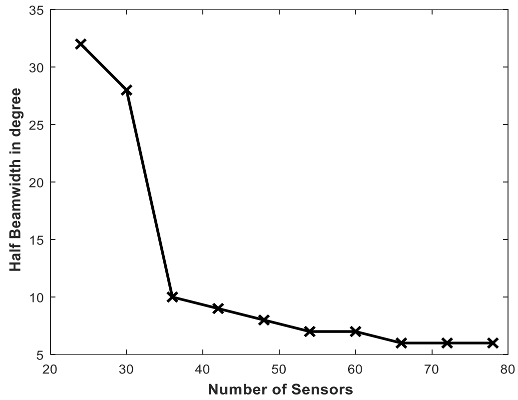


Figure 5: Variations of beamwidth versus number of sensor elements

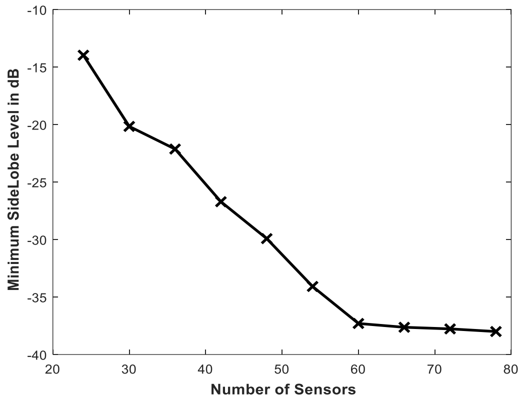


Figure 6: Variations of minimum sidelobe level versus number of sensor elements

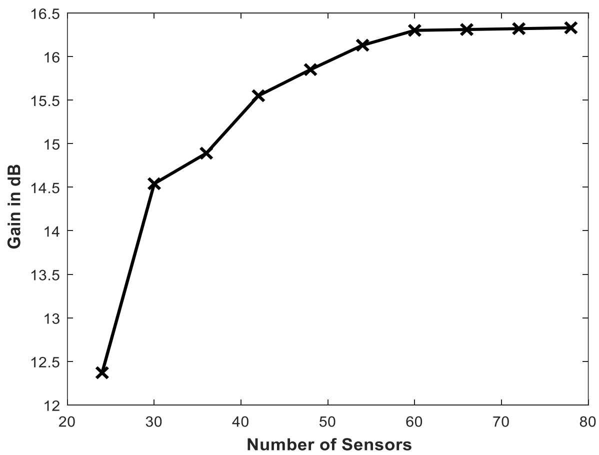


Figure 7: Variations of peak gain versus number of sensor elements

Finally, the effect of the total number of sensor elements on the beamwidth, minimium sidelobe level, and gain of the considered randomly distributed planar array according to the constraints that were given by (6) and (7) were evaluated. Figure 5 shows the variation of the half beamwidth as a function of the total number of sensors. Here in this case, the minimum sidelobe level was chosen to be -20dB and the mainbeam direction was pointed toward $\theta = 90^0$ and $\phi = 0^o$.

As can be seen that the beamwidth becomes narrower for the increased number of random sensor elements. Figure 6 shows the minimum obtainable sidelobe level as a function of the total number of sensors. From this figure, it is clear that a maximum reduction in the sidelobe level can be obtained for larger array elements. Note that the half beamwidth for this case was chosen to be 10^0 in the elevation plane and the number of random elements was increased within a fixed bounded planar area. Figure 7 shows the variations of the array gain with respect to the total number of sensor elements. Clearly, as expected, the array gain increases with increased total number of array elements.

V. CONCLUSIONS

The performance of the Wireless Sensor Networks can be significantly improved by applying the technique of collaborative beamforming or smart arrays. In this paper, the required beamforming shape was successfully and efficiently formed from the use of an arbitrary array with randomly distributed elements. The amplitude and the phase of each random element were optimized by means of the convex optimization to generate the required array pattern taking into consideration some constraints such as minimum sidelobe level, narrower beamwidth, maximum gain, and asymmetric sidelobes with null controls. The simulation results showed that the beam patterns of the designed random array were accurately formed and fulfilled all the required constraints. It is also found that the formed beam of the considered random planar array was significantly affected by the total number of the random elements within the arbitrary bounded planar area. Thus, further investigation may be carried out to optimize the array performance under a smaller number of random elements using compressed sensing method. This case is of a great importance in practical consideration where some array sensors may fail in its operation and the resulting array pattern should be maintained with less deterioration.

Table 1
Element’s Amplitude, Phase, and Location of the Regular and Randomly Distributed Planar Array Patterns Given in Figures 1-4

	UPA (Figure 1)			RPA (Figure 2)			RPA (Figure 3)			RPA (Figure 4)		
n	Amp.	Phase	d_x, d_y	Amp.	Phase	d_x, d_y	Amp	Phase	d_x, d_y	Amp	Phase	d_x, d_y
1	0.0000	12.05	0 ,0	0.07	135.92	4.75, 4.23	0.15	-94.71	4.75, 4.23	0.10	127.38	4.75, 4.23
2	0.0043	-167.12	0.45,0	0.28	-105.43	1.15, 2.62	0.21	-170.30	1.15, 2.62	0.19	-58.80	1.15, 2.62
3	0.0163	9.23	0.9,0	0.34	-73.14	3.03 ,1.01	0.65	174.77	3.03 ,1.01	0.28	-50.23	3.03 ,1.01
4	0.0051	139.86	1.35, 0	0.25	94.79	2.42, 3.36	0.65	-147.46	2.42, 3.36	0.68	-128.58	2.42, 3.36
5	0.0011	-92.39	1.80,0	0.27	-91.62	4.45,4.19	0.34	96.06	4.45,4.19	0.14	-9.74	4.45,4.19
6	0.0000	81.42	2.25,0	0.04	-149.11	3.81,0.09	0.06	75.07	3.81,0.09	0.07	-96.80	3.81,0.09
7	0.0043	-167.04	0,0.45	0.25	-83.80	2.28,3.40	0.95	14.73	2.28,3.40	1.00	58.05	2.28,3.40
8	0.2475	-166.83	0.45,0.45	0.34	94.53	0.09,1.89	0.30	-22.57	0.09,1.89	0.21	144.57	0.09,1.89
9	0.7175	-162.23	0.90, 0.45	1.00	93.57	4.10,4.15	1.00	-81.61	4.10,4.15	0.78	163.68	4.10,4.15
10	0.3436	-137.30	1.35, 0.45	0.07	42.44	2.22,2.51	0.24	-119.34	2.22,2.51	0.08	151.58	2.22,2.51
11	0.0645	-97.36	1.80, 0.45	0.09	-69.63	3.07,3.54	0.51	148.06	3.07,3.54	0.21	-81.28	3.07,3.54
12	0.0011	-93.59	2.25, 0.45	0.25	52.05	3.95,2.14	0.2	-141.48	3.95,2.14	0.25	-156.84	3.95,2.14
13	0.0166	12.89	0, 0.90	0.10	8.78	4.60,1.52	0.96	-147.71	4.60,1.52	0.27	-69.26	4.60,1.52
14	0.6955	-167.40	0.45, 0.90	0.04	106.62	3.69,0.94	0.09	128.16	3.69,0.94	0.14	27.54	3.69,0.94
15	1.0000	-157.25	0.90, 0.90	0.17	-19.39	0.88,0.96	0.17	-61.95	0.88,0.96	0.02	89.18	0.88,0.96
16	0.7985	-150.43	1.35, 0.90	0.32	79.54	2.02,3.41	0.23	-121.93	2.02,3.41	0.42	-143.18	2.02,3.41
17	0.1740	-88.48	1.80, 0.90	0.18	-128.58	4.67,1.51	0.90	46.44	4.67,1.51	0.16	136.48	4.67,1.51
18	0.0042	86.17	2.25, 0.90	0.24	-39.78	4.58,2.70	0.80	90.99	4.58,2.70	0.31	-123.52	4.58,2.70
19	0.0043	-166.99	0,1.35	0.38	108.39	2.05,0.75	0.25	-8.95	2.05,0.75	0.18	146.25	2.05,0.75
20	0.2471	-166.81	0.45, 1.35	0.37	87.31	4.46,3.48	0.21	-57.59	4.46,3.48	0.40	172.93	4.46,3.48
21	0.7169	-162.24	0.90,1.35	0.29	-98.39	0.28,1.89	0.25	130.59	0.28,1.89	0.32	-6.56	0.28,1.89
22	0.3430	-137.29	1.35, 1.35	0.02	-144.68	1.76,4.30	0.18	-174.37	1.76,4.30	0.07	58.62	1.76,4.30
23	0.0643	-97.35	1.80,1.35	0.94	-91.94	4.06,4.26	0.87	102.47	4.06,4.26	0.67	-7.32	4.06,4.26
24	0.0011	-93.48	2.25,1.35	0.29	-71.66	0.04,2.96	0.31	159.65	0.04,2.96	0.13	-46.57	0.04,2.96
25	0.0000	5.08	0,1.80	0.48	-77.66	0.69,2.48	0.30	158.58	0.69,2.48	0.25	-11.15	0.69,2.48
26	0.0041	-166.72	0.45,1.80	0.07	63.77	1.01,4.49	0.03	-62.84	1.01,4.49	0.04	161.53	1.01,4.49
27	0.0167	9.34	0.90,1.80	0.20	-85.63	0.99,4.10	0.12	156.47	0.99,4.10	0.09	-63.10	0.99,4.10
28	0.0051	136.51	1.350,1.80	0.20	94.45	3.01, 3.22	0.28	-26.78	3.01, 3.22	0.32	129.72	3.01, 3.22
29	0.0010	-91.12	1.80,1.80	0.24	-97.34	1.36,4.08	0.32	-174.48	1.36,4.08	0.16	18.57	1.36,4.08
30	0.0000	59.09	2.25,1.80	0.15	173.88	0.99,3.30	0.10	-53.68	0.99,3.30	0.10	-81.76	0.99,3.30

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