Modeling, Analysis and Control of Chaotic Rucklidge System

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***Abstract*— This paper presents modeling, analysis and research of chaotic Rucklidge system based on programming interface that has been developed in LabView software environment. This study allows for generating and researching the main properties of chaotic Rucklidge system, focusing on time distribution of three chaotic coordinates and phase portraits. A range of the system parameter with which we can generate different period (controlled) attractors and a number of trajectories of chaotic Rucklidge system are also presented. The programming interface demonstrates the algorithm of masking and decrypt information carrier.**

***Index Terms*—Chaos; Rucklidge; Control; LabView.**

I. INTRODUCTION

Chaos theory has been applied in variety of areas, such as electronics schemes, secure communication systems, magnetism, plasmas, economy, and many others. [1-22]. Small deviation of parameters on the receiver makes it impossible to decrypt the received message. However, one of the significant problems of using deterministic chaos in communication systems is the selection of the same circuit parameters that generate chaotic signal. To address this issue, software has been created and one of them is the most modern system LabView (LabView'2015 (32-bit version for Windows)).

Information carrier can be masked for a certain chaotic laws using chaotic systems. A mechanism for signal generation with potential applications to communications and signal processing are provided of chaotic systems. Researchers have proposed different approaches or techniques, such as linear feedback control, OGY, inverse optimal control etc. Stabilizing of the unstable periodic orbits via parameter perturbation is a theoretical basis of most known methods for control chaos [23-25] to control chaos.

The paper is organized as follows. Section II presents differential equations that realize chaotic behavior, Lyapunov exponents, Kaplan-Yorke dimension and modelling results using LabView. In Section III, a range of system parameter with which we can generate different period (controlled) attractors and number of trajectories of chaotic Rucklidge system are presented. In Section IV, we present program interface that demonstrates masking and decrypt information carrier. Conclusions are provided in Section V.

II. MODELLING OF CHAOTIC RUCKLIDGE SYSTEM

Chaotic Rucklidge system can be used for security of the information carrier, and it can be described by three nonlinear differential equations:

$$\frac{dx}{dt} = -ax + by - yz$$

$$\frac{dy}{dt} = x \qquad (1)$$

$$\frac{dz}{dt} = -z + y^2$$

where: x, y, z = Dynamic variables
a, b = System parameters.

Chaotic oscillations are the if system parameters $a = 2$, $b = 6.7$, and dynamic variables $x = 1.2$, $y = 0.8$, $z = 1.4$ [26, 27].

Lyapunov exponent characterizes the rate of separation of infinitesimally close trajectories of a dynamical system. The rate of separation can be different for different orientations of initial separation vector. The Lyapunov exponents are determined as the logarithmic growth rates of the perturbations. When one of the Lyapunov exponents is positive, the system is chaotic.

The Lyapunov exponents for Rucklidge system:

$L_1 = 0.1877$, $L_2 = 0$, $L_3 = -3.1893$.

Figure 1 shows the Lyapunov exponents graphically.

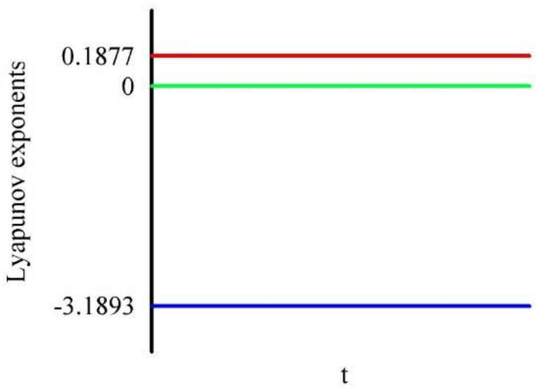

Figure 1: Graphical presentation of Lyapunov exponents

When there are positive, zero and negative Lyapunov exponents (+; 0; -), we can claim that Rucklidge system is chaotic.

For the system (1), the Kaplan–Yorke dimension equals to:

$$D_{KY} = 2 + \frac{L_1 + L_2}{|L_3|} = 2.0589 \quad (2)$$

For modeling and research of chaotic, Rucklidge system was used as one of the modern software environments LabView.

LabView can be used as a graphical programming platform. It helps engineers to implement all stages of development of small and large projects for example, from the creation of prototype to the final testing. In this development environment, the best integration of hardware and software components with the latest computer technologies is combined. LabView contains many tools for solving current and upcoming challenges with enormous potential for innovation, future success and effectiveness. LabView includes powerful multi-function tools that allow conducting the development of any applications and any types of measurements. With these tools, scientists and engineers can work in the widest range of applications and spend much less time for development.

Figure 2 shows a block-scheme that implements chaotic Rucklidge system. Three nonlinear differential equations (1) were recorded to the functional main part of the block-scheme that is called the formula node.

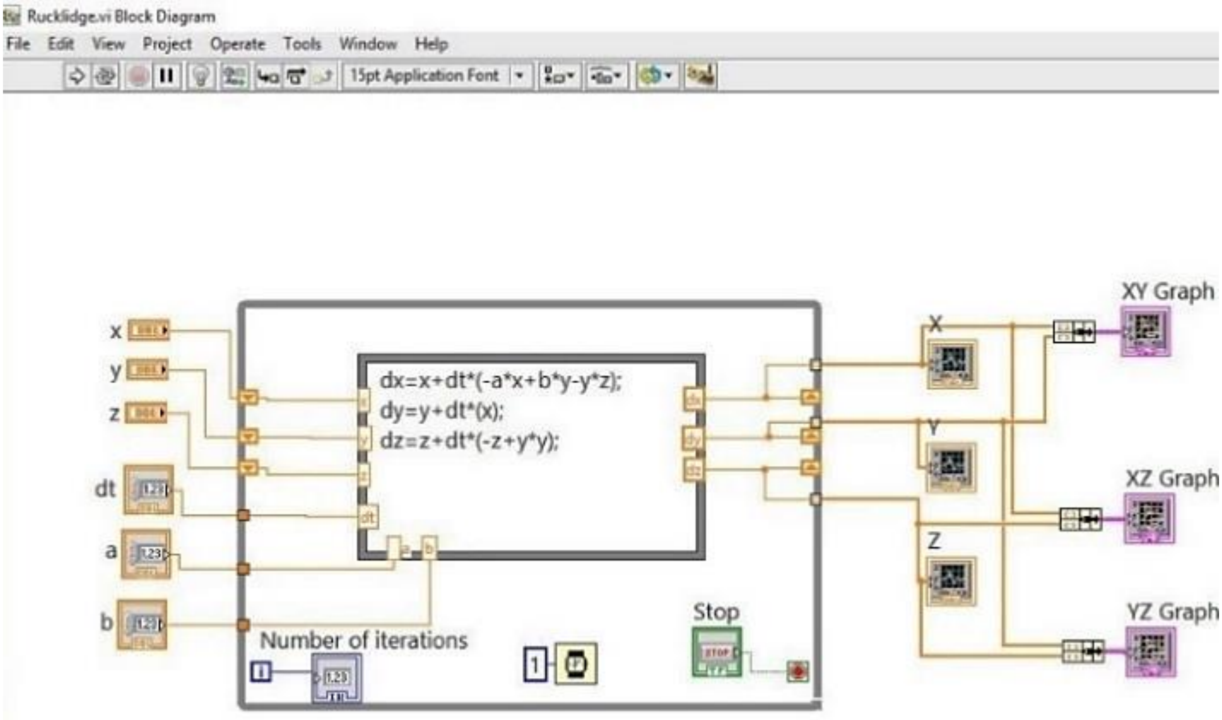

Figure 2: A block-scheme that implements chaotic Rucklidge system

The values of system parameters (*a*, *b*) and dynamic variables (*x*, *y*, *z*) were brought to the formula node. The solution of equations in three dimensions is an opportunity to demonstrate from the block-scheme output. The change of initial conditions and system parameters allows researcher to analyze and research the behavior of the oscillations. This block-scheme can serve as a one of the functional blocks of secure communication systems based chaos.

III. CONTROL OF CHAOTIC RUCKLIDGE SYSTEM

To control the chaotic Rucklidge system, we exchanged one of the system parameters. If we exchange system parameter ($1 \leq a \leq 3$), we will get different oscillations as chaotic and controlled.

Table 1 shows the values of parameter *a*, with which we can control the chaotic attractors of the Rucklidge system. Figure 3 shows an example of the control of the Rucklidge system for system parameters $a = 2.5$, $b = 6.7$, dynamic variables $x = 1.2$, $y = 0.8$, $z = 1.4$, number of iterations $N = 5000$.

The result of the modelling is a 3-period controlled attractor in 2D graphs, presenting the time distributions of chaotic coordinates X, Y, and Z.

Table 1
The values of coefficient for control of chaotic oscillations

Range of control coefficient (parameter *a*)	NUMBER OF TRAJECTORIES
$a = 1.0$	1
$a = 1.1$	4
$a = 1.2$	5
$a = 1.3$	9
$a = 1.4$	9
$a = 1.5$	8
$a = 1.6$	6
$a = 1.7$	5
$a = 1.8$	4
$a = 1.9$	3
$a = 2.0$	4
$a = 2.1$	5
$a = 2.2$	7
$a = 2.3$	5
$a = 2.4$	6
$a = 2.5$	3
$a = 2.6$	6
$a = 2.7$	4
$a = 2.8$	5
$a = 2.9$	1
$a = 3.0$	2

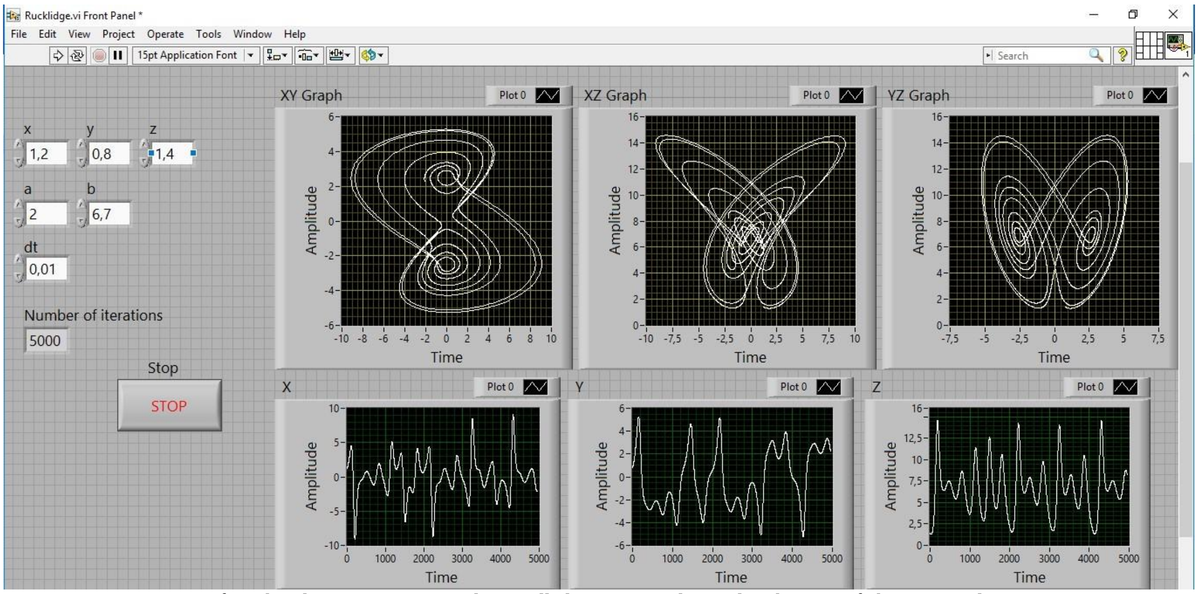

Figure 3: Interface that demonstrate 3-period controlled attractor and time distributions of chaotic coordinates X, Y, Z

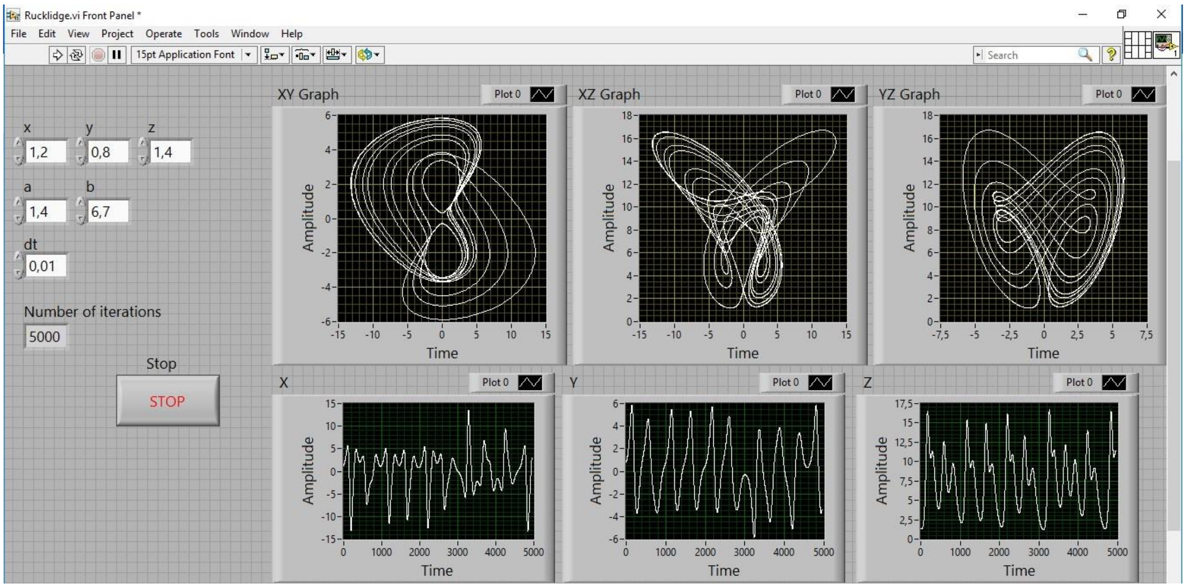

Figure 4: Interface that demonstrate 9-period controlled attractor and time distributions of chaotic coordinates X, Y, Z

Figure 4 shows an example of control of chaotic Rucklidge system for system parameters $a = 1.4$, $b = 6.7$, dynamic variables $x = 1.2$, $y = 0.8$, $z = 1.4$, number of iterations $N = 5000$.

The result of the modelling is a 9-period controlled attractor in 2D graphs, presenting the time distributions of chaotic coordinates X, Y, and Z.

IV. Chaotic Masking and Decryption of The Information Carrier

The coherent receivers usually are dynamical systems that resemble the chaos producing transmitters. They achieve synchronization with the transmitter, enabling the synchronization to extract the information signal from the received chaotic signal. In order to achieve synchronization, the parameters of the transmitter have to be known. They can be considered as the encryption key of the message; thus, coherent receptions allows for some privacy of the information transmission.

Figure 5 demonstrates the presence of the chaotic signal between the transmitter and receiver. In this case, the use of chaos in secure communication systems has been proposed. The design of these systems depends on the self-synchronization property of the Rucklidge attractor. As shown in Figure 5, the transmitter and the receiver systems are identical.

Figure 6 presents the program interface, which demonstrates the masking of the carrier of information based on a Rucklidge system (1).

The masking of the carrier of information based on chaos is provided by blending information with the chaotic signal. A sinusoidal signal (useful signal) was used as information (input) with amplitude of *1 V* and system parameters $a = 2$, $b = 6.7$, dynamic variables $x = 1.2$, $y = 0.8$, $z = 1.4$. System parameters and dynamic variables are the keys for the masking information.

Algorithm for the decryption has opposite effect.

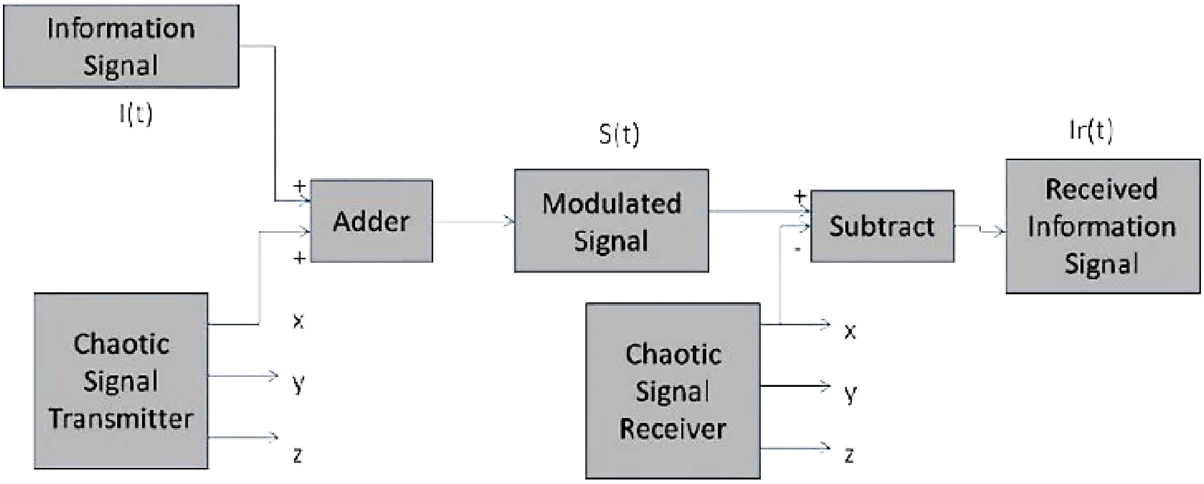

Figure 5: Transmitter and receiver systems

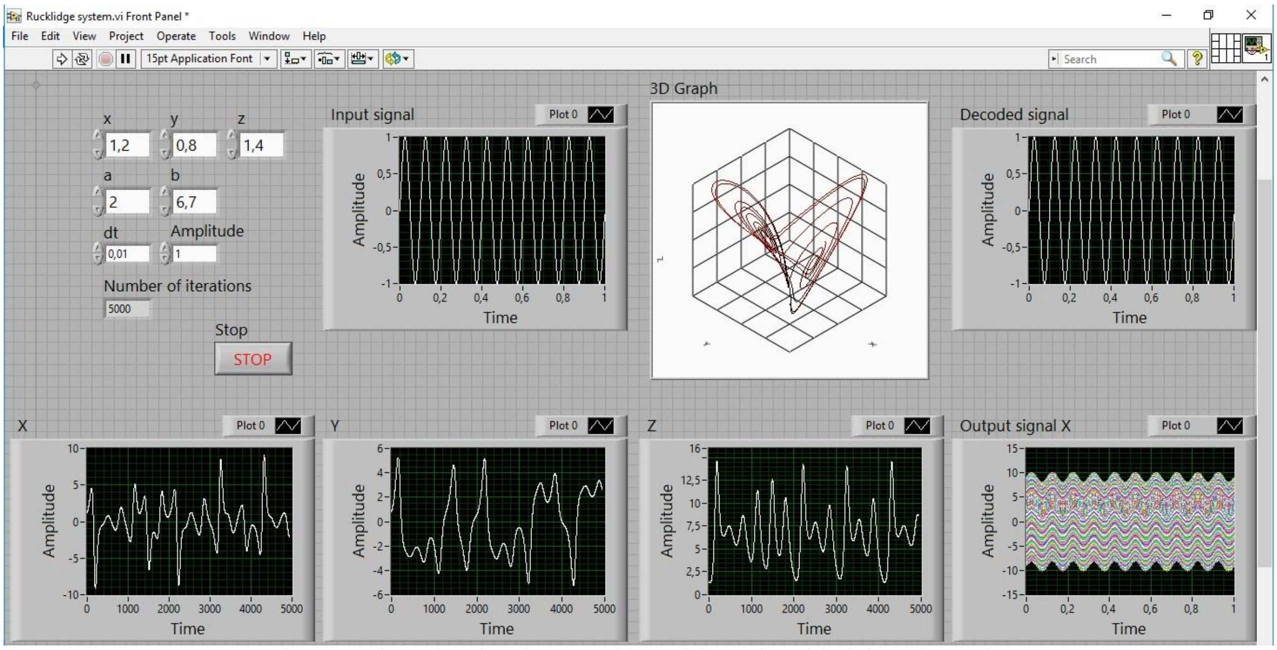

Figure 6: Software interface show's masking and decryption of the information carrier

V. CONCLUSION

This study conducts experiments of the control of chaotic Rucklidge system. The range of the system parameter with which generated different period (controlled) attractors and number of trajectories of chaotic Rucklidge system is shown in Table 1. This range of the system parameter can be used as keys for masking and decryption of information carrier in modern systems transmitting and receiving information.

ACKNOWLEDGMENT

This work was carried out at the Department of Radio Technics and Information Security of the Yuriy Fedkovych Chernivtsi National University.

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