Implementation of Particle Filtering in TDOA Positioning

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***Abstract*—This paper describes the implementation of particle filtering (PF) estimation method in TDOA positioning to mitigate the effect of NLOS signal propagation on the TDOA measurements. The measurement errors were modelled according to the circular disk scatter model (CDSM) ranging from 0 to 600 m. In this paper, we consider static PF which uses one TDOA measurement to estimate one fixed MT position. The network layout is set up with five base stations (BS) that resolve to a total of ten measured TDOAs in every PF iteration. The performance of the static PF was compared to two basic estimation methods namely robust intersection estimation (RIE) and linear least square (LLS). The simulation results show the stability of static PF over a wide range of measurement errors and giving an almost constant estimation error at various CDSM radiuses. Static PF outperforms RIE and LLS with the estimation error of less than 40 m and 60 m for 67% and 90% of the time respectively.**

***Index Terms*—TDOA Positioning; Particle Filtering; NLOS environment; Circular Scatterers.**

I. Introduction

The growth of wireless mobile communication systems is very rapid. The widely offered information access in outdoor and indoor environments resulted in higher demand on the development of location-based services (LBSs). The motion activity of the mobile terminal (MT) that is provided by LBSs is very important in vehicle navigation, fraud detection, tracking, healthcare applications, monitoring and transportation systems [1]. The performance of these applications in ensuring the continuity of mobile multimedia applications relies on the positioning accuracy of the MTs. This requires alternative positioning techniques to the existing GPS-assisted positioning.

Terrestrial signals from mobile communication networks offer a promising potential to be explored in enhancing the accuracy of localization and positioning (L&P) techniques. Despite the extensive number of research being carried out throughout the last decade, mitigating the multipath and non-line-of-sight (NLOS) propagation errors remains the main research issue especially in the time-based positioning. In time difference of arrival (TDOA) positioning techniques, multipath propagation causes bias in the time of arrival (TOA) measurements [2] which leads to uncertainties in TDOA measurements. The multipath effect is intensified in an NLOS condition where the direct path between BSs and MT is absent due to the presence of scatterers in the network surrounding.

In general, the network-based L&P techniques can be categorized into three main types which are proximity-based, fingerprinting and measurement-based technique. Proximity-based positioning techniques manipulate the unique geographical information (cell ID) of the closest (serving) BS as the reference point in estimating the target's position. Despite CID technique being the simplest and fastest positioning technique, it offers very low accuracy which may lie within the distance of cell radius [3]. Fingerprinting positioning as proposed in [4], [5] promise high estimation accuracy. However, the implementation of fingerprinting techniques involves high process overhead during the offline data set up. Therefore, an accurate yet low cost technique is highly anticipated.

The main interest of this work is to pursue TDOA positioning which is a measurement-based positioning technique. This technique manipulates the time measurements done by MTs as the input to the position estimation method carried out by the location server. The methods for position estimation for indoor and outdoor TDOA positioning have been surveyed in [6], [7]. Among the methods used are the geometrical-based, numerical-based and probabilistic approach. Among the three methods, a probabilistic approach based on particle filtering (PF) is found to be a versatile and robust estimation method. PF is a Bayesian filtering method that is widely used in robotic positioning and tracking applications. It also attracts researchers enhancing positioning accuracy in LTE indoor environments [8], LTE outdoor environments [9] and urban UAS navigation [10].

In this work, PF is implemented in TDOA positioning to handle multiple simultaneous TDOA measurements and mitigate the effect of NLOS errors. The objective of this work is to show the performance of PF estimation method in treating uncertainties of TDOA measurements due to multipath errors in NLOS environment. The static PF was implemented to estimate the MT's position in NLOS environments where the TDOA measurements are affected by the circularly located scatterers. Only one TDOA measurement for each pair of BSs was used where the motion model of the target MT is not considered. The performance of the static PF is compared to two basic estimation methods namely robust intersection estimation (RIE) and linear least square (LLS).

The remainder of this paper is organized as follows: Section II provides the background study of TDOA positioning technique. Section III briefly described the three position estimation methods which are considered in this work. Then, the simulation set up is described, and the results are discussed in Section IV. Finally, the findings are concluded in Section V.

II. TDOA Positioning Technique

Current research of network-based L&P techniques have considered several measurement parameters which are TOA [11], TDOA[12], received signal strength indicator (RSSI)[13], the angle of arrival (AOA)[14] and a hybrid of those parameters methods [15]. In this work, we focus on TDOA measurements in NLOS propagation environments.

In TDOA positioning techniques, the position of the MT is estimated based on the TDOA measurements from multiple BSs. The positions of constant time differences of TDOA represented by hyperbola curves, which are, focused at the location of the corresponding BS pairs as shown in Figure 1. As seen in the figure, the distance between the MT and BS_1, BS_2, and BS_3 are represented by d_1, d_2, and d_3, respectively. The distances correspond to the time taken by the signal from each of the BSs to reach the MT is referred to as TOA. The value of TDOAs is calculated by differencing the TOAs in a pair-wise manner. For example, the TDOA of BS_1-BS_2 pair is determined as $|TOA_2 - TOA_1|$ which corresponds to the distance difference of $|d_2 - d_1|$ as shown in the figure. Each point on the hyperbola curve, (x_h, y_h) represents a possible position of MT. Therefore, at least three BS pairs producing three hyperbola curves are needed to solve one unique intersection point. In this case, the MT position estimation has a single solution that resolves accurately at the MT true position.

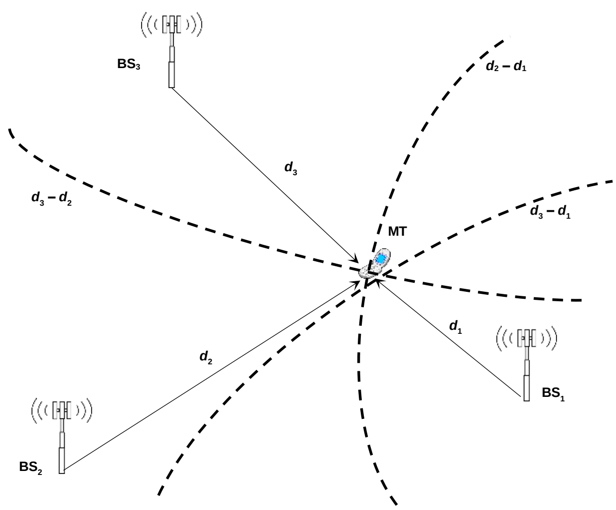

Figure 1: Trilateration method in TDOA positioning

The intersection of the hyperbola curves is obtained using an analytical method. For the three BSs network in Figure 1, the intersection can be found by solving three equations, namely Equation (1) through (3).

$$d_2 - d_1 = \sqrt{(x_2 - x)^2 + (y_2 - y)^2} - \sqrt{(x_1 - x)^2 + (y_1 - y)^2} \quad (1)$$

$$d_3 - d_1 = \sqrt{(x_3 - x)^2 + (y_3 - y)^2} - \sqrt{(x_1 - x)^2 + (y_1 - y)^2} \quad (2)$$

$$d_3 - d_2 = \sqrt{(x_3 - x)^2 + (y_3 - y)^2} - \sqrt{(x_2 - x)^2 + (y_2 - y)^2} \quad (3)$$

The term $|d_j - d_i|$ is the measured TDOA of BS_j-BS_i pair and [x_j, y_j] is the position of BS_j. If the position of all BSs is known and the TDOA measurements are error-free, the estimated MT position is given by [x, y].

In a real implementation, however, the mobile network systems are more complex. The propagation channels are exposed to noises; errors and the number of detected BSs may be more than three. Multipath and NLOS errors are the main threats that add uncertainties to time measurements in TDOA positioning. The erroneous measured TDOA leads to arbitrary positions of hyperbola curves intersections, and the estimation process becomes even more complex if the number of detected BSs is more than three. As the hyperbola is not a linear function, using the analytical method will produce an unacceptable error, and the estimated results may deviate far from the true value.

III. Position Estimation Methods

There are numerous methods of position estimation being studied. In this work, we compare the performance of static PF to two basic methods which are intersection estimation [16] and linear least square (LLS) [17].

A. Robust Intersection Estimation (RIE)

Intersection estimation method is the simplest method to estimate the MT position by finding the intersection of the hyperbola curves. The intersection estimation method must be robust to deal with erroneous TDOA measurements where the intersections are not unique. In this work, we simulate TDOA positioning by using a robust intersection estimation (RIE) method which is modified from the basic intersection calculation in [16]. RIE has the capability of finding all possible intersection points and truncate the duplications. RIE computes the intersection point of each pair of hyperbola curves functions. The estimated MT position is then calculated by averaging all the intersection points.

B. Linear Least Square (LLS)

The LLS is a simple method that can be used to estimate the MT position in TDOA positioning. The non-linear equations of hyperbola curves are simplified as in (4).

$$A_m x + B_m y + C_m = 0 \quad (4)$$

where:

$$A_m = \left[\frac{2x_m}{TDOA_{m,1}}\right] - \left[\frac{2x_2}{TDOA_{2,1}}\right] \quad (5)$$

$$B_m = \left[\frac{2y_m}{TDOA_{m,1}}\right] - \left[\frac{2y_2}{TDOA_{2,1}}\right] \quad (6)$$

$$C_m = TDOA_{m,1} - TDOA_{2,1} - \left[\frac{x_m^2 + y_m^2}{TDOA_{m,1}}\right] - \left[\frac{x_2^2 + y_2^2}{TDOA_{2,1}}\right] \quad (7)$$

for $m = 3,4,\ldots n$. Then, then the position of MT (x,y) is estimated by using matrix operation as in (8).

$$\begin{bmatrix} x \\ y \end{bmatrix} = -\begin{bmatrix} A_3 & B_3 \\ \vdots & \vdots \\ A_n & B_n \end{bmatrix}^T \begin{bmatrix} C_3 \\ \vdots \\ C_n \end{bmatrix} \quad (8)$$

C. Particle Filtering (PF)

PF is an iterative estimation method that utilizes the Bayes rule which computes the posterior distribution of the state vector based on the previous and current state observations [18]. The implementation of PF involves three steps. The first step is the initialization stage where *N* particles denoted as $p(S_0)$ were randomly sampled around the initial distribution of the system. The second step is the sampling stage where *N* samples $\tilde{x}_k^i$ were drawn from $p\left(\tilde{x}_k|\tilde{x}_{k-1}^i\right)$. Lastly, the weight (likelihood) of each of the samples was computed based on Gaussian distribution model in Equation (9), where *x* is the estimated sample and μ is the observation value at the current time. The weight of each particle was normalized for the resampling process. In the resampling stage, *m* lowest weighted particles were eliminated. To maintain the number of *N* samples, *m* highest weighted particles were duplicated to replace the eliminated ones.

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)} \tag{9}$$

The resampling step is very crucial in determining the estimation accuracy.

IV. Simulation Results and Performance Analysis

A. Simulation and TDOA Measurements Setup

We ran a series of simulations to show the performance of static PF in enhancing the accuracy of TDOA positioning over NLOS signal propagation. The system layout was set up in an 800x600m^2 area with five BSs located at fixed Cartesian coordinates as summarized in Table 1. For simplicity, we assume that BS_1 as the serving BS and the initial position of the MT was set to [0, 0]. We consider an NLOS environment where the scatterers are modelled according to the circular disk scatter model (CDSM) [17]. The set up generates NLOS errors within the range of 0 to 600 m. The system layout is shown in Figure2 while Figure3 depicts the TDOA measurements distribution of 1000 samples generated at the CDSM radius of 300m. The measured TDOA range is between 22ns to 2µs which corresponds to the distance difference constants of 5 to 600 m. By assuming that the reference signal transmitted by all five BSs was detected by the MT, ten TDOA measurements correspond to ten possible BS pairs were computed.

Table 1
System Parameters

Parameter	Value
No. of BSs	5
Position of BSs	[440 -110; 200 550; -370 600; -700 -70; -200 -820]
Initial position of MT	[0, 0]
Propagation model	CDSM
Propagation condition	NLOS

The effect of NLOS environment over TDOA measurements can be clearly seen in Figure 2. The hyperbola curves plot produced correspond to the measured TDOA from five BSs intersect at several points, and the position of the intersections are arbitrary. There are also possibilities where two curves intersect at more than one point. This condition is far different when compared to the error free case as described in Figure 1. Therefore, treating this condition by using a basic intersection method will not satisfy the L&P requirements.

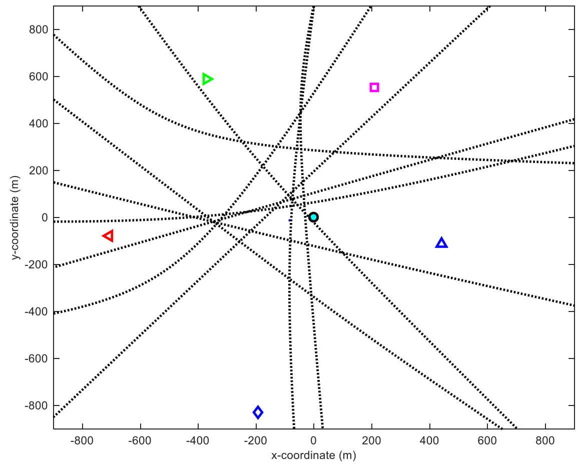

Figure 2: System layout and hyperbola curve plots corresponding to the measured TDOAs in NLOS environment.

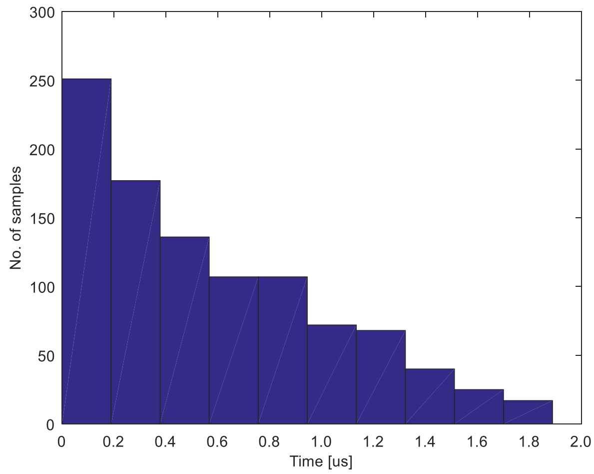

Figure 3: Distribution of TDOA measurements (Rd = 300m).

B. Implementation of Particle Filtering

We applied a static PF estimation method where only one measurement from five BSs was considered. With five BSs deployed in the network, ten measured TDOAs are computed. The measurements were treated as the current state observations, which became the reference points in the filtering iterations.

In the first state of PF implementation, the transitional prior $p\left(\tilde{x}_k|\tilde{x}_{k-1}^i\right)$ is computed. The term $\tilde{x}_k$ represents the state value at time *k* that consists of the position vector, $[x_i, y_i]$ for particle *i*. We generate 1000 random particles around the initial MT position with the position standard deviation, σ of 100 m. The distribution of the position of the particle is depicted in Figure 4. Then, we calculated ten datasets of the estimated TDOA for each of the particles, $TDOA_p^i$ which corresponds to ten possible BS pairs. The estimated values were used to compute the particles' weight in the next implementation state.

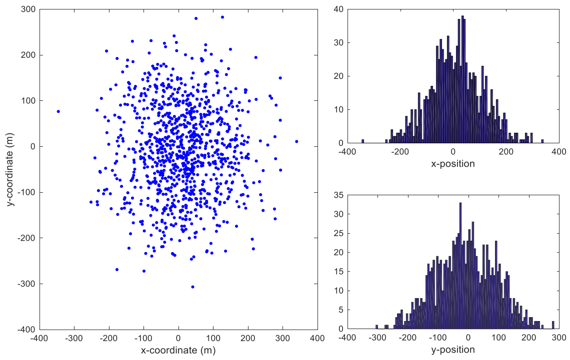

Figure 4: Distribution of estimated positions sampled around initial MT's position (σ = 100m).

In the second state, the likelihood for each of the particles was computed as in Equation (10) and (11). As seen in Equation (11), the likelihood was determined based on the Gaussian distribution model with the measured TDOA, $TDOA_m$ as the mean value and the accuracy of the measurements, σ_{TDOAm} as the standard deviation.

$$p(z_k|\tilde{x}_k^i) = \mathcal{N}(TDOA_p^i; TDOA_m, \sigma_{TDOAm}) \tag{10}$$

$$p(z_k|\tilde{x}_k^i) = \frac{1}{\sigma_{TDOAm}\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{TDOA_p^i - TDOA_m}{\sigma_{TDOAm}}\right)} \tag{11}$$

The value of the likelihood of the particles determines the particles' weight which is a normalized value as given in Equation (12) and (13). The total weight of the particles is accumulated likelihood contributed by all BSs. During the resampling process, particles that possess higher weight survive in the next iteration while the particles with the lowest weight were eliminated. We used an importance-resampling method where the highest weighted particles were duplicated to replace the eliminated ones with the equivalent numbers. The accuracy of the estimated values increases as the number of iterations increases until the simulation converges. The converged state value $\tilde{x}_k$ is considered as the estimated MT position.

$$w_k^i = \frac{w_k^i}{W} \tag{12}$$

where:

$$W = \sum_{i=1}^{Np} w_k^i \tag{13}$$

C. The Performance of static PF

The performance of the static PF was compared to RIE and LLS based on the estimation accuracy parameter of the root mean square error (RMSE). The NLOS measurement errors were generated at six different CDSM radius ranges of 100, 200, 300, 400, 500 and 600 m. For each of the error ranges, graphs in Figure 5 compare the accuracy of estimated MT position produced by the three estimation methods. As seen from the graphs, static PF constantly gives high estimation accuracy for all error ranges. The estimation error of static PF is almost constant which falls around 50 m from the MT true position for all error ranges. Conversely, the estimation error produced by RIE increases linearly with the increment of measurement error. Even though RIE gives the highest accuracy when the CDSM radius is set to a low range, its performance declines linearly with the increasing of CDSM radius. The graph of LLS shows that it offers the least positioning accuracy when compared to other methods. On average, the estimation error of LLS is 450 m from the true MT position. Even though LLS shows the stability of the algorithm, its accuracy is lower than that of static PF by a factor of 8.

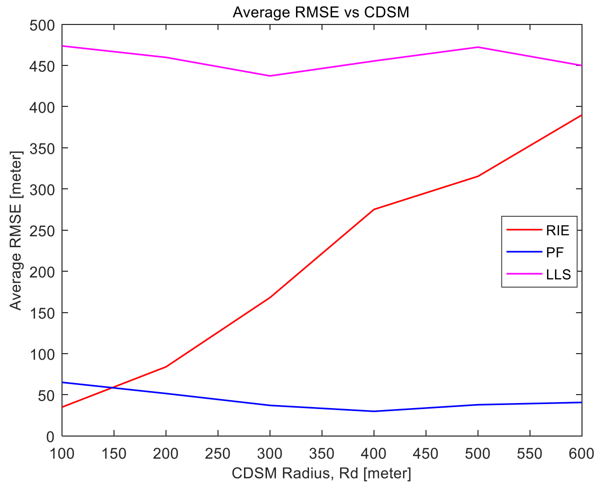

Figure 5: Comparison of estimated position accuracy between static PF, RIE, and LLS.

Figure 6 shows the cumulative distribution function (CDF) of estimated position errors for static PF, RIE, and LLS when the radius of scatterers is set to 300 m. From the graphs, it is obvious that static PF outperforms RIE and LLS with the estimation error is less than 50m and 60m for 67% and 90% of the time, respectively. The estimation error of RIE is higher than static PF by the factor of 3 and 6 for 67% and 90% of the time, which resolve to 200 m and 500 m, respectively. LLS gives the lowest performance with the estimation error of less than 550 m and 750 m for 67% and 90% of the time respectively.

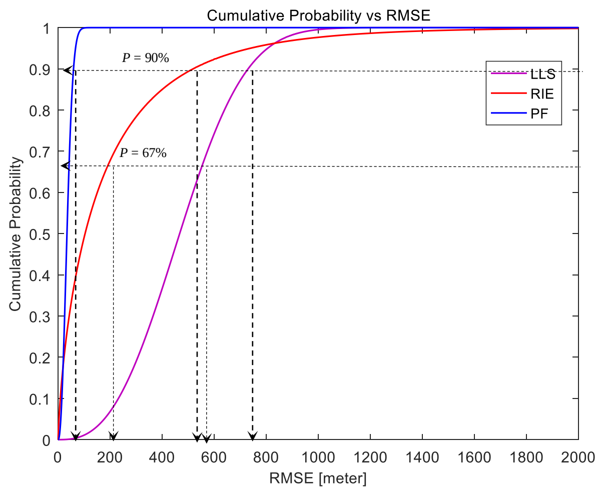

Figure 6: CDF of estimated position errors for static PF, RIE, and LLS (Rd = 300m).

Figure 7 shows the CDF of estimated position errors for static PF at various CDSM radius ranges. From the graphs, it is obvious that the performance of static PF is consistent through all the error ranges. Overall, the minimum estimation error achieved by static PF is less than 40 m and 60 m for 67% and 90% of the time respectively which occur at CDSM radius of 400 m. While the maximum estimation error achieved by static PF is less than 70 m and 115 m for 67% and 90% of the time, respectively which occur at CDSM radius of 100 m.

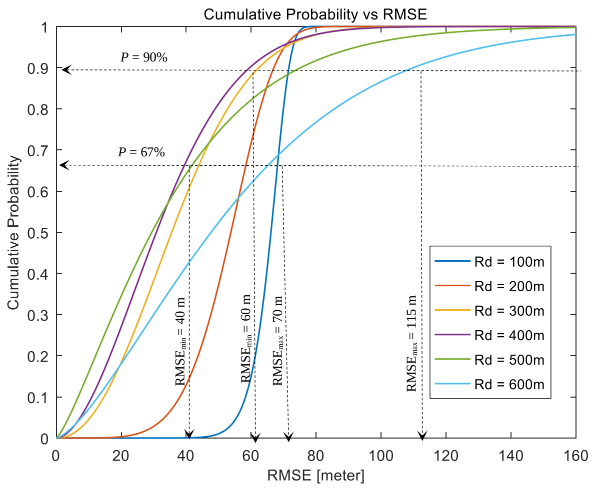

Figure 7: CDF of Position Errors in NLOS Environment by using static PF.

V. Conclusion

In this paper, we have implemented static PF to estimate the MT position in TDOA positioning. The intention is to mitigate the effect of NLOS signal propagation on the TDOA measurements. The measurement errors were generated according to CDSM ranging from 0 to 600 m. The performance of the static PF was compared to two basic estimation methods, which are RIE and LLS. Based on the results, we can conclude that static PF can handle multiple simultaneous TDOA measurements and a good candidate to mitigate the effect of NLOS errors. Static PF is very stable over a wide range of measurement errors for giving an almost constant estimation error at various CDSM radius. It performs the best when the CDSM radius is set to 400 m with the estimation of less than 40 m and 60 m for 67% and 90% of the time respectively. Even though this achievement seems to fulfill the requirements of FCC, the investigation must be further expanded to consider many other factors. In our future works, we will simulate and enhance the PF estimation method in TDOA positioning over in more realistic and accurate channel models.

Acknowledgement

Authors would like to thank Universiti Teknikal Malaysia Melaka (UTeM) and Ministry of Higher Education (MOHE), Malaysia for sponsoring this work under project RACE/F3/TK3/FKEKK/F00299.

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