The Effect of the Choice of the Mapping on the Performance of Map Equalizer

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***Abstract*—In this paper, we propose to study the effect of the choice of the mapping on the performance of iterative receivers for frequency selective channels. We use at the transmitter a BICM (Bit Interleaved Coded Modulation) based scheme obtained by the concatenation of a binary code and a modulator separated by an interleaver. Because of the interleaving, the optimal receiver is too complex. We consider here an iterative receiver composed of a Maximum *A Posteriori* (MAP) equalizer and a MAP decoder. During the iterations, the equalizer and the decoder exchange extrinsic information and use them as *a priori* in order to improve their performance. An optimization of the mapping is required to obtain a gain by using the iterative receivers. In order to make the mapping choice easier, we propose to perform an analysis based on the Gaussian approximation. This analysis relies on only one simulation of the equalizer.**

***Index Terms*—Mapping, optimization, BICM, turbo-equalizer, MAP criterion.**

I. INTRODUCTION

The optimal receiver for a frequency selective coded channel performs joint equalization and decoding which makes its complexity generally prohibitive. A solution achieving a good complexity/ performance trade-off is to use an iterative receiver composed of a soft-input soft-output (SISO) equalizer and a SISO [9]. The basic idea behind iterative processing is to exchange extrinsic information among the equalizer and the decoder in order to achieve successively refined performance. The optimal SISO algorithm, in the sense of minimum bit error rate (BER), to be used for equalization and decoding is the symbol MAP algorithm [2]. Hence, in this paper, we consider an iterative receiver composed of a MAP equalizer and a MAP decoder. We propose a BICM (BICM: Bit Interleaved Coded Modulation) based scheme obtained by the concatenation of a binary code and a modulator separated by an interleaver [1]. We propose to study, by simulation and analysis, the effect of the choice of the mapping of the constellation (Gray, Set Partitionnig ...) on the performance of the iterative receiver for frequency selective channels.

II. SYSTEM MODEL

We consider a coded data transmission system over a frequency selective channel depicted in Figure 1. The output information bit sequence is first encoded with a convolutional encoder. The output of the encoder is interleaved, mapped to the symbol alphabet *A*. We assume that transmissions are organized into burst of *T* symbols. The channel is supposed to be invariant during one burst. The received baseband signal sampled at the symbol rate at time *k* is

$$z_k = \sum_{l=0}^{L-1} h_l x_{k-l} + n_k \tag{1}$$

where L is the channel memory. In this expression, n_k are modeled as independent samples of real white Gaussian noise with normal probability density function (pdf) $N(0,\sigma^2)$ where $N(\alpha,\sigma^2)$ denotes a Gaussian distribution with mean α and variance σ^2. The term h_l is the l^{th} tap gain of the channel, which is assumed to be real valued. Let $\underline{x} = (x_{1-L},\ldots,x_{T-1})^T$ be the (L+T- 1)-long vector of coded symbols and $\underline{n} = (n_0,\ldots,n_{T-1})^T$ be the T –long noise vector. The output of the channel is the T –long vector $\underline{z} = (z_0,\ldots,z_{T-1})^T$ defined as

$$\underline{z} = H\underline{x} + \underline{n} \tag{2}$$

where *H* is a $T \times (T+L-1)$ Toeplitz matrix with its first row equal to $(h_0, h_1, h_2, \ldots h_{L-1}, 0, ..0)$ and its first column $(h_0, 0, ..0)^T$.

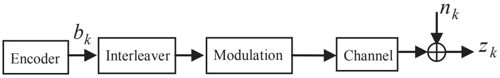

Figure 1: Transmitter structure

III. ITERATIVE RECEIVER

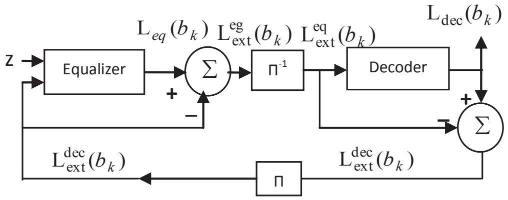

Figure 2: Transmitter structure

As shown in Figure 2, the receiver consists of two soft-input soft-output (SISO) processors, the equalizer and the decoder. We consider only the MAP approach for both equalization and decoding using the BCJR algorithm [2] [4]. The MAP equalizer computes the *a posteriori* probabilities (APPs) on the coded bits, $LLR_{pos}(b_k) = \ln \frac{p(b_k = 1/\underline{z})}{p(b_k = 0/\underline{z})}$ and outputs the log-likelihood ratios (LLRs) [9]:

$$LLR_{ext-eq}(b_k) = LLR_{pos-eq}(b_k) - LLR_{priori-eq}(b_k)$$

$$= \ln \frac{p(b_k = 1/\underline{r})}{p(b_k = 0/\underline{r})} - \ln \frac{p(b_k = 1)}{p(b_k = 0)} \qquad (3)$$

Which are the *a posteriori* LLRs $LLR_{pos-eq}(b_k)$ minus the a *priori* LLRs $LLR_{priori-eq}(b_k)$. These a priori LLRs are provided by the decoder. At the first receiver iteration, $LLR_{priori-eq}(b_k) = 0$ since no a priori information are available. The LLRs $LLR_{ext}(b_k)$ are then deinterleaved and provided to the decoder as input information, in order to refine its calculations. The MAP decoder computes the APPs

$$LLR_{pos-de}(b_k) = \ln\left(\frac{p(b_k = 1/\underline{r})}{p(b_k = 0/\underline{r})}\right),$$

$\underline{r} = (L_{ext}(b_{1-L}),...,L_{ext}(b_{T-1}))^T$, and outputs the LLRs

$$LLR_{ext-de}(b_k) = LLR_{pos-de}(b_k) - LLR_{priori-de}(b_k)$$

$$= \ln \frac{p(b_k = 1/\underline{r})}{p(b_k = 0/\underline{r})} - \ln \frac{p(b_k = 1)}{p(b_k = 0)} \qquad (4)$$

These LLRs are then interleaved and provided to the equalizer as a priori, $LLR_{priori-de}(b_k)$ at the next iteration [6]. After some iterations, hard decisions are taken on the information bits by the decoder.

IV. PERFORMANCE ANALYSIS

A. Case of QPSK modulation

The considered modulation is a QPSK modulation with a complex modulation output. It has four states with alphabet elements:

$$A_1 = \left\{\frac{1+j}{\sqrt{2}}; \frac{-1+j}{\sqrt{2}}; \frac{-1-j}{\sqrt{2}}; \frac{1-j}{\sqrt{2}}\right\} \qquad (5)$$

Figure 3 and 4 are examples of the constellation using Gray coding.

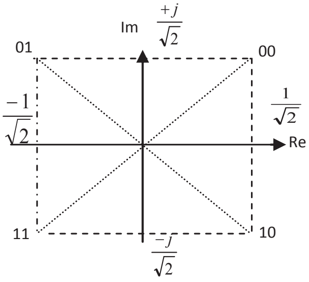

Figure 3: Constellation of QPSK modulation using Gray Mapping

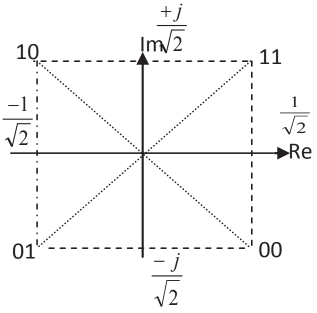

Figure 4: Constellation of QPSK modulation using no Gray Mapping

In our simulations, the channel is supposed to be invariant during the burst and perfectly known by the receiver. The impulse response of the channel is (0.5 ; 0.71 ; 0.5).

In the simulations, the data is encoded by a non-recursive nonsystematic convolutional code of polynomial (5.7) in octal format and with rate ½ and constraint length *k= 3*. The coded bits are then interleaved by a random interleaver before being modulated QPSK symbols according to a Gray mapping or not. The performances of the receiver are estimate by tracing the curves of Bit Error Rate (BER) with respect to SNR at each iteration.

Figures 5 and 6 show the Bit Error Rate (BER) curves with respect to the SNR at the output of the turbo-equalizer for a

QPSK modulation using, respectively Gray and non Gray mapping. We can see that the Gray mapping gives the best performances that the not Gray mapping at the first iteration. Hawever , the performances of the turbo-equalizer with non Gray mapping becomes better in the following iterations.

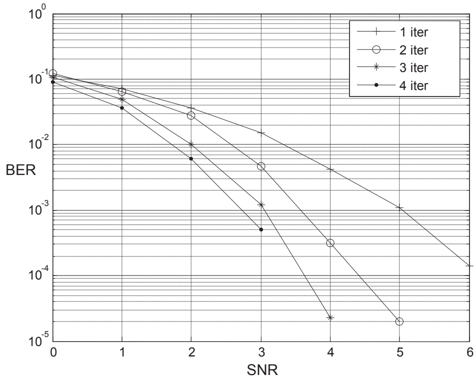

Figure 5: BER versus SNR: performances for 4 iterations of the turbo-equalizer for QPSK modulation (Gray mapping).

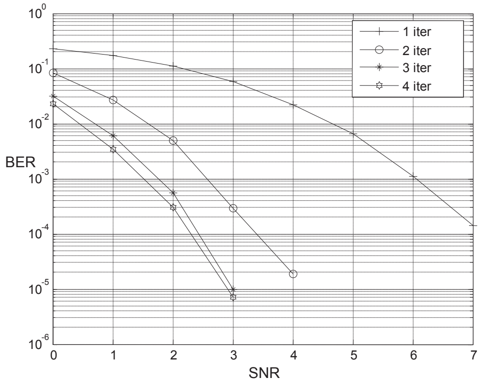

Figure 6: BER versus SNR: performances for 4 iterations of the turbo-equalizer for QPSK modulation (no Gray mapping).

B. *Case of a 8-PSK modulation*

Figures 7, 8 and 9 show three examples of constellation with Gray mapping respectively, partitioning of the constellation and mapping M8 proposed in [8].

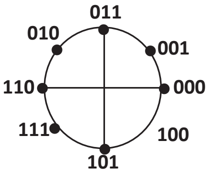

Figure 7: Gray Mapping

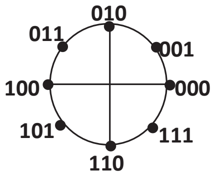

Figure 8: SP Mapping

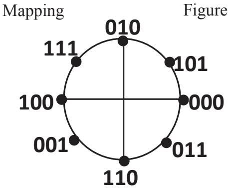

Figure 9: M8 Mapping

Figures 10, 11 and 12 illustrate performances for 4 iterations of the turbo-equalizer in the case of 8-PSK modulation using respectively Gray mapping, mapping with partitionnement of the constellation (SP) and M8 mapping proposed in [8]. We note that in the first iteration, the Gray mapping offer the best performance than the SP and M8 mapping. Conversely, the performances of turbo-equalizer with SP and M8 mapping are better than those obtained with the Gray mapping for subsequent iterations. Performances improve over iterations.

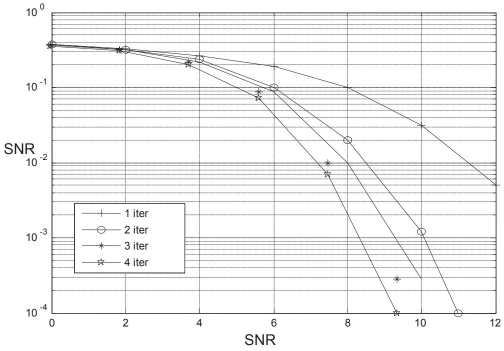

Figure 10: BER versus SNR: performances for 4 iterations of the turbo-equalizer for 8-PSK modulation (Gray mapping)

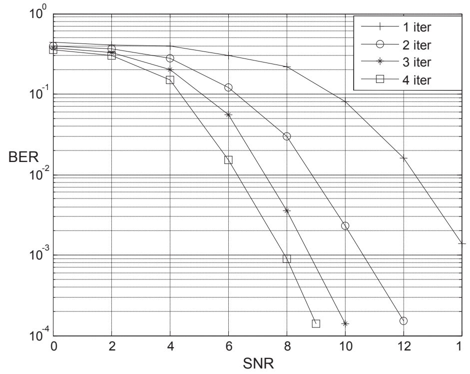

Figure 11: BER versus SNR: performances for 4 iterations of the turbo-equalizer for 8-PSK modulation (SP mapping)

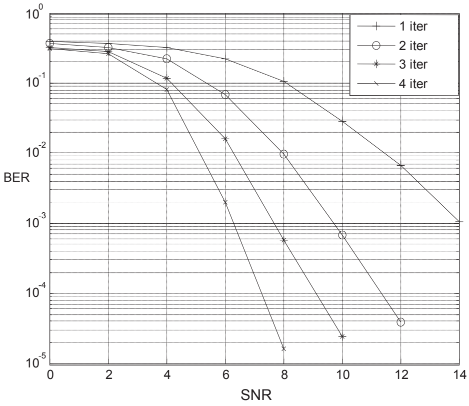

Figure 12: BER versus SNR: performances for 4 iterations of the turbo-equalizer for 8-PSK modulation (M8 mapping)

C. *Analysis of the turbo-equalizer*

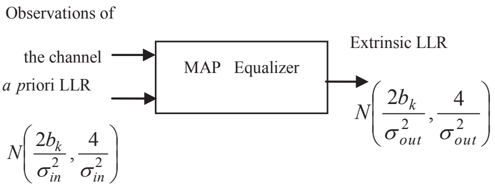

Figure 13: Theoretical analysis of the equalizer

Figures 13 show the variance σ_{out}^2 at the output of the equalizer according to the variance of entry σ_{in}^2 for different mappings respectively for SNR = 2dB and SNR = 7dB. From these curves, we notice that when σ_{in}^2 decrease (the *a priori* information to the input of the equalizer become reliable), σ_{out}^2 also decrease. This shows that there is an improvement of the performances of the turbo-equalizer during the iterations. We also notice that in the case of QPSK modulation, the no Gray mapping to have earnings in term of variance with regard to the Gray mapping for tow values of SNR. This allows confirming the results obtained by simulation. For 8-PSK modulation, the M8 mappings provide a better output (σ_{out}^2 lower) than the SP mapping for $\sigma_{in}^2 \leq 0.15$ when SNR = 2dB and $\sigma_{in}^2 \leq 0.17$ when SNR = 7dB. The Gray mapping leads generally to less reliable LLR in the output of the equalizer (the variance is bigger) that SP and M8 mappings. In the analysis of the iterative receiver (figure 13), we supposed that the extrinsic LLR in the output of the equalizer and the decoder is Gaussians and symmetric [3] (the variance is double of the value absolved from the average). We suggest, verify these hypothesis.

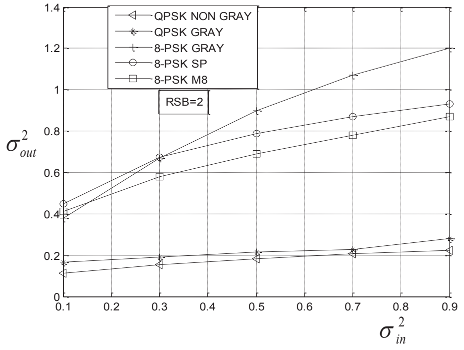

Figure 14: Theoretical analysis of the equalizer for different mapping (SNR=2dB)

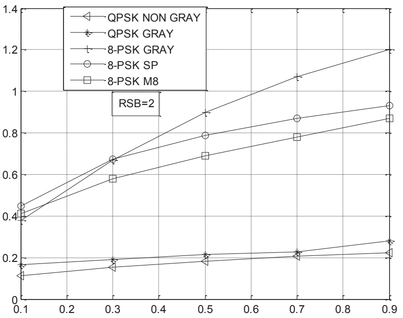

Figure 15: Theoretical analysis of the equalizer for different mapping (SNR=7dB)

In the analysis of the turbo-equalizer, it was assumed that the extrinsic LLR output of the equalizer and decoder are Gaussian and symmetric (the variance is double the absolute value of the average). In the following, we propose to verify this hypothesis by simulations.

D. Gaussian and Symmetry hypothesis verification

To verify the choice of the model we take for LLR in the analysis, we will determine by simulation at each iteration, the kurtosis of the densities of extrinsic LLR output of the equalizer and decoder. For a Gaussian random variable, the kurtosis is 3. We will also determine the ratio of the variance on the average densities of extrinsic LLR and see if that ratio is 2 in order to validate the hypothesis of symmetry [5]-[7]. The following tables give the kurtosis and the ratio of the variance on the average extrinsic LLR distributions versus the number of iterations of the turbo-equalizer for QPSK and 8-PSK, respectively, for SNR = 2dB and SNR = 7dB.

From these tables, we notice that the kurtosis and the ratio of the variance on the average extrinsic LLR output of the decoder and equalizer approach respectively 3 and 2 as the number of iterations increases. This is true for both modulations considered. The values given in the tables therefore do justify model selection we made for the densities of LLR in the analysis.

Table 1
The values of the kurtosis and the ratio of the variance on the average of extrinsic LLR distribution in the output of the equalizer case of a BPSK modulation.

Equalizer	Iter 1	Iter 2	Iter 3	Iter 4
Kurtosis(SNR=2dB)	3.74	3. 38	3.2	3.11
variance/average (SNR=2dB)	2.4	2.2	2.15	2.1
Kurtosis(RSB=7dB)	3.73	3.039	3.004	3.0026
variance/ average (SNR=7dB)	2.01	2.015	2.7	2.1

Table 2
The values of the kurtosis and the ratio of the variance on the average of extrinsic LLR distribution in the output of the decoder case of a BPSK modulation.

Decoder	Iter 1	Iter 2	Iter 3	Iter 4
Kurtosis(SNR=2dB)	4.3	3.03	2 .97	2.99
variance/average (SNR=2dB)	2.6	2.18	2.07	2.01
Kurtosis(SNR=7dB)	3.18	3.17	3.2	3.1
variance/average (SNR=7dB)	2.5	1.91	1 .98	1.99

Table 3
The values of the kurtosis and the ratio of the variance on the average of extrinsic LLR distribution in the output of the equalizer case of a QPSK modulation (non Gray).

Equalizer	Iter 1	Iter 2	Iter 3	Iter 4
Kurtosis(SNR=2dB)	3.73	3.03	2.97	2.99
variance/moyenne (S=2dB)R	2.6	2.18	2.02	2.00
Kurtosis(SNR=7dB)	3.49	3.05	3.07	3.07
variance/moyenne (SNR=7dB)	2.3	2.07	2.09	2.09

Table 4
The values of the kurtosis and the ratio of the variance on the average of extrinsic LLR distribution in the output of the decoder case of a QPSK modulation (non Gray)

Decoder	Iter 1	Iter 2	Iter 3	Iter 4
Kurtosis(SNR=2dB)	3.3	2.89	3 .13	3.015
variance/average (SNR=2dB)	2.4	2.21	1.97	2.02
Kurtosis(SNR=7dB)	3.23	3.2	3.18	3.1
variance/average (SNR=7dB)	2.5	1.95	1 .98	1.99

Table 5
The values of the kurtosis and the ratio of the variance on the average of extrinsic LLR distribution in the output of the equalizer case of a 8-PSK modulation (SP mapping).

Equalizer	Iter 1	Iter 2	Iter 3	Iter 4
Kurtosis(SNR=2dB)	4.23	3.63	3.23	3.11
variance/average (SNR=2dB)	2.65	2.48	2.28	2.10
Kurtosis(SNR=7dB)	3.8	2.91	2.97	3.09
variance/average (SNR=7dB)	2.34	2.19	2.09	2.29

Table 6
The values of the kurtosis and the ratio of the variance on the average of extrinsic LLR distribution in the output of the decoder case of a 8-PSK modulation (SP mapping).

Decoder	Iter1	Iter 2	Iter 3	Iter 4
Kurtosis(SNR=2dB)	4.5	3.75	3 .33	3.19
variance/average (SNR=2dB)	2.75	2.31	2.12	1.97
Kurtosis(SNR=7dB)	2.88	3.24	3.2	3.13
variance/average (SNR=7dB)	2.22	2.2	2 .18	2.09

V. CONCLUSION

In this paper, we consider an iterative receiver composed of o Maximum *a posterior* (MAP) equalizer and a MAP decoder. We proposed to study the effect of the choice of mapping on the iterative receiver performance for selective frequency channel. We use at the transmitter a BICM (Bit Interleaved Coded Modulation). A good choice of mapping is required to obtain a gain with this technique. Simulation results show that the non Gray mapping achieves a gain in term of the SNR in the case of QPSK modulation more than the Gray mapping during the iterations. Nevertheless, performances of turbo-equalizer with SP and M8 mapping are better than those obtained with the Gray mapping for subsequent iterations. We propose to perform an analysis based on the Gaussian approximation. This analysis relies on only one simulation of the equalizer which significantly reduces the computational complexity. The simulations of the iterative receiver confirm the results obtained by the analysis.

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