Multistep Linear Predictor for Slow Fading Channel

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***Abstract*—Most of the channel estimation methods for OFDM transmission symbols have been developed under the assumption of a slowly fading channel, where the channel impulse response (CIR) is assumed to be stationary within an OFDM block. Such assumption may not always be true. In this paper, two cases are considered, i.e. when the channels are stationary within a block and time-varying within a block. The equalisation is done in time-domain with reduced computational complexity and with reduced time averaging. The RChol algorithm is used to solve the least squares equation. The simulation results show that the proposed algorithm has better performance compared to other previously proposed methods with reduced time averaging.**

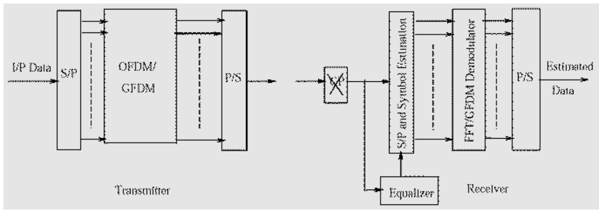
***Index Terms*—Linear Predictor; OFDM; Fading Channel.**

I. INTRODUCTION

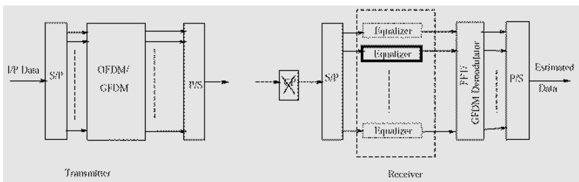
Blind channel estimation has a great deal of attention for recent studies of orthogonal frequency-division multiplexing (OFDM) systems. Most of existing blind and semi-blind methods are based on second-order statistics (SoS) of a long vector, whose size is equal to or larger than the number of sub-carriers to estimate the correlation matrix. Such techniques have extremely high computational complexity because of huge matrices and also not suitable for time-varying channel [1], [2]. By exploiting the frequency correlation among adjacent sub-carrier, a subspace-based estimation method that requires a significantly smaller number of time samples is proposed in [5], and It is assumed that the channel is stationary over the time-averaging intervals [6], [7]. However, in practice, a wideband radio channel may have significant changes, even within one OFDM data block. Fast time-varying multipath (fading) channel cannot be considered as time-invariant during a block period, and the channel variations lead to the loss of sub-carrier orthogonality. ICI corrupts the demodulated sub-carrier symbol. Even for a small value of the Doppler spread which degrades the BER performance. Such a small variation of the sub-channel matrices over the coherence bandwidth is analysed in [5]. In contrast, the estimation of frequency selective MIMO-OFDM channel based on second-order statistics of a short vector using linear prediction (LP) is given in [3], [4]. The LP-based methods need a vector size slightly larger than the channel length.

In this paper, the time-domain equaliser is used to estimate the channel coefficient for the slow-fading channel. The channel remains constant over a block in such case the channel has to be estimated at any OFDM symbol at *k*th sub-carrier within a block. However, for the fast-fading channel which introduces ICI, the channel has to be estimated for each OFDM symbol. If the channel is constant over an OFDM block, i.e., for slow fading channel, averaging the succeeding frame or symbol may not be effective and results in poor BER. The RChol algorithm introduced in [8] significantly reduces the computational complexity for the fast-fading time-varying channel which exploits the special structure of the time-varying channel matrix and performance of the same is analysed for different single-carrier transmission techniques in [9]. In this paper, we optimise the RChol algorithm for slow fading channel to solve the least squares equation and to find the tap-weights of the filter by averaging OFDM symbol at any of the kth sub-carrier of the succeeding OFDM blocks.

Figure 1 shows these two case, where the channel and the transmitted symbols are estimated before taking FFT at the receiver. Figure 1(a) is for the fast-fading channel where the channel has to be estimated at each transmitted symbol and Figure 1(b) is for a slow-fading channel where the channel has to be estimated for any of the kth sub-carrier within a block.



(a) Wideband Fast Fading



(b) Wideband Slow Fading

Figure 1: OFDM Transmitter Receiver Block

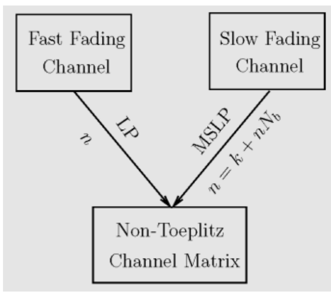


Figure 2: Decision Flow Chart

The rest of the paper is organised as follows. In section II system model is addressed. Section III briefs the LP and the MMSE equaliser for the time-varying Channel. In section IV, the performance analysis is presented. Section V concludes the contribution of the proposed work.

II. SYSTEM MODEL

Let s_k be the symbol transmitted over kth sub-carrier where $k \in \{0,1,...NC-1\}$ and a block of symbol be represented as $s := [s_0, s_1, ...s_{Nc-1}]^T$. The inverse Fourier transform (IDFT) of s gives the time-domain OFDM signal and *m*th OFDM block can be denoted as $\boldsymbol{x} = [x(m,0), x(m,1), ...x(m,Nc-1)]^T$ where m ∈ Z and:

$$\boldsymbol{x} = \boldsymbol{F}^H \boldsymbol{s} \tag{1}$$

where $\boldsymbol{F}$ is Fourier transform matrix be:

$$F = \frac{1}{\sqrt{N_c}} e^{-2pi\frac{pq}{N_c}}$$

for $p \in \{0...Nc-1\}$ and $q \in \{0...Nc-1\}$.

After adding cyclic prefix (CP), where the length of CP is needed not be longer than or equal to the maximum excess delay of the channel, each OFDM symbol is then transmitted by the corresponding transmit antenna.

The multipath slow-fading channel for SIMO for *m*th block is represented by $\boldsymbol{h}(m)$ such that:

$$\boldsymbol{h}(m) = \begin{bmatrix} \boldsymbol{h}_1(m,0) & \boldsymbol{h}_1(m,1) & ... & \boldsymbol{h}_1(m,L-1) \\ \boldsymbol{h}_2(m,0) & \boldsymbol{h}_2(m,1) & ... & \boldsymbol{h}_2(m,L-1) \\ \vdots & \vdots & \vdots & \vdots \\ \boldsymbol{h}_m(m,0) & \boldsymbol{h}_m(m,1) & ... & \boldsymbol{h}_M(m,L-1) \end{bmatrix} \tag{2}$$

$$\boldsymbol{h}(m) = [\boldsymbol{h}_m^0 \quad \boldsymbol{h}_m^1 \quad \cdots \quad \boldsymbol{h}_m^{L-1}] \tag{3}$$

and the received symbol vector at receiver as:

$$\boldsymbol{y}(m) = \sum_{l=0}^{L-1} \boldsymbol{h}_m^0 x(m-l) + \boldsymbol{v}(m) \tag{4}$$

A. Multistep LP for Slow Fading Channel

In case of the slow-fading channel, the channel remains constant over an OFDM block, and the channel matrix has the Toeplitz structure. The channel matrix for slow fading OFDM symbol at *m*th block is represented by $\tilde{\boldsymbol{H}}_s(m) \in C^{NcM\times(N+L-1)}$ as:

$$\begin{bmatrix} \boldsymbol{h}_m^0 & \boldsymbol{h}_m^1 & \cdots & \boldsymbol{h}_m^{L-1} & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{h}_m^0 & \cdots & \boldsymbol{h}_m^{L-2} & \boldsymbol{h}_m^{L-1} & \cdots & \boldsymbol{0} \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{h}_m^0 & \cdots & \cdots & \boldsymbol{h}_m^{L-1} \end{bmatrix} \tag{5}$$

and at *(m-1)*th block is represented by $\tilde{\boldsymbol{H}}_s(m) \in C^{NcM\times(N+L-1)}$ as:

$$\begin{bmatrix} \boldsymbol{h}_{m-1}^0 & \boldsymbol{h}_{m-1}^1 & \cdots & \boldsymbol{h}_{m-1}^{L-1} & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{h}_{m-1}^0 & \cdots & \boldsymbol{h}_{m-1}^{L-2} & \boldsymbol{h}_{m-1}^{L-1} & \cdots & \boldsymbol{0} \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{h}_{m-1}^0 & \cdots & \cdots & \boldsymbol{h}_{m-1}^{L-1} \end{bmatrix} \tag{6}$$

and received signal vector as:

$$\boldsymbol{y}_{N_c}(m) = \tilde{\boldsymbol{H}}_s(m)\boldsymbol{x}(m) \tag{7}$$

As the channel remains constant over a block, for time domain equaliser, we do not need to find the channel coefficients at each sub-carrier, and channel estimation at any of the *k*th sub-carrier for each block is sufficient.

III. BRIEF OVERVIEW OF MULTISTEP LINEAR PREDICTOR

The channel matrix for slow fading can be represented by Equation (9) for $n \in Z$, and the size of the channel matrix is reduced from N_C to $N < NC$, Figure 2 shows the condition for the slow fading and fast-fading channel. For the slow-fading channel, the RChol algorithm works as multistep linear predictor (MSLP) solver and linear predictor for a fast-fading channel where the channel matrix is represented by $\boldsymbol{H}(n)$. To distinguish channel matrix $\tilde{\boldsymbol{H}}_s$ from $\boldsymbol{H}$, we are replacing time-index m by n such that $m=n$ and:

$$n = \begin{cases} n, & \text{Fast Fading} \\ k + nN_c, & \text{Slow Fading} \end{cases} \tag{8}$$

Hence, the multistep slow fading channel can be represented by the time-varying channel matrix by $\boldsymbol{H}(n-1) \in C^{NM\times(N+L-1)}$ as:

$$\begin{bmatrix} \boldsymbol{h}_{n-1}^0 & \boldsymbol{h}_{n-1}^1 & \cdots & \boldsymbol{h}_{n-1}^{L-1} & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{h}_{n-2}^0 & \cdots & \boldsymbol{h}_{n-2}^{L-2} & \boldsymbol{h}_{n-2}^{L-1} & \cdots & \boldsymbol{0} \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{h}_{n-N}^0 & \cdots & \cdots & \boldsymbol{h}_{n-N}^{L-1} \end{bmatrix} \tag{9}$$

After removing CP in each link at the receiver and stacking N successive OFDM symbol at a *k*th sub-carrier over N received block the received vector be:

$$\boldsymbol{y}_N^m(n-1) = [y^T(n-1), \boldsymbol{y}^T(n-2) \ldots \boldsymbol{y}^T(n-N)]$$

$$\boldsymbol{y}_N^m(n-1) := \boldsymbol{H}(n-1)\boldsymbol{x}_N(n-1) + \boldsymbol{v}_N(n-1) \tag{10}$$

and the covariance matrix equal to:

$$\boldsymbol{R}_N^m(n-1) = E[\boldsymbol{y}_N^m(n-1)\boldsymbol{y}_N^{mH}(n-1)]$$

$$\boldsymbol{R}_N^m(n-1) = \begin{bmatrix} \boldsymbol{R}_{11}^{m,1} & \boldsymbol{R}_{12}^{m,1} & ... & \boldsymbol{R}_{1N}^{m,1} \\ \boldsymbol{R}_{21}^{m,1} & \boldsymbol{R}_{22}^{m,1} & ... & \boldsymbol{R}_{2N}^{m,1} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{R}_{N1}^{m,1} & \boldsymbol{R}_{N2}^{m,1} & ... & \boldsymbol{R}_{NN}^{m,1} \end{bmatrix} \tag{11}$$

and the prediction error for *m*th block and *k*th sub-carrier can be computed by Equation (12).

$$\boldsymbol{e}^m(k) = \boldsymbol{y}^m(k) - \boldsymbol{w}_N^{mT}\boldsymbol{y}_N^m(n-1) \tag{12}$$

Let $\tilde{\mathbf{P}}(n) = E[\boldsymbol{y}_N^m(n-1)\boldsymbol{y}^{mH}(k)]$ then the tap weight of the predictor can be represented as:

$$\boldsymbol{w}_N^m = \boldsymbol{R}_N^{m\dagger}\tilde{\boldsymbol{P}} \tag{13}$$

To estimate the Cholesky factors of correlation matrix explicitly, the RChol algorithm is used [8], [9] as:

$$\boldsymbol{R}_N(n-1) := \mathbf{C}(n-1)\boldsymbol{D}(n-1)\boldsymbol{C}^H(n-1) \quad (14)$$

Then

$$\mathbf{C}(n-1)\boldsymbol{D}(n-1)\boldsymbol{C}^H(n-1)\boldsymbol{w}_N^m = \widetilde{\boldsymbol{P}}(n) \quad (15)$$

can be solved by using forward substitute method.

From Equation (12) of prediction error, the prediction error matrix $\boldsymbol{P}_e$ be:

$$\boldsymbol{P}_e(n) = R_{yy}(0) - \boldsymbol{w}_N^m\widetilde{\boldsymbol{P}}(n) \quad (16)$$

Given prediction error variance which has rank one, $\boldsymbol{h}_m^0$ can be estimated by Eigen-value tracking method as discussed in [11] as:

$$\boldsymbol{h}_m^0 = \alpha_n\theta_1\boldsymbol{\mu}_1 \quad (17)$$

where θ_2^2 is the highest eigenvalue of prediction error variance $\boldsymbol{P}_e$ and $\boldsymbol{\mu}_1$ is the corresponding eigenvector and α_n such that $|\alpha_n| = 1$ then the OFDM symbol at kth sub-carrier for mth block can be estimated as:

$$x^m(k) = \boldsymbol{h}_m^{0\dagger}\boldsymbol{e}^m(k) \quad (18)$$

After taking FFT of the estimated OFDM symbols, we get the estimated data symbols as:

$$\hat{\boldsymbol{s}} = \boldsymbol{F}\hat{\boldsymbol{x}} \quad (19)$$

Though the simulation is shown for OFDM system, the same technique can be applied to the Next Generation (5G) like Generalized Frequency Domain Multiplexing (GFDM) [12].

IV. Performance Analysis

To evaluate the performance of the RChol algorithm, we consider a SIMO-OFDM system with n_t=1 transmit antenna and n_r={4} receive antennas and QPSK modulation. The number of sub-carriers N_C is set to be 64. For the BER performance, we consider the time-variant channel with prediction order $N = 8$ for the multipath $L = 6$. All the results are obtained by averaging 200 independent Monte Carlo run averaged over 200 OFDM block (12800 OFDM symbols).
The time-variant channel is approximated by complex-exponential basis expansion model (CE-BEM) as introduced in [13] for single carrier and [14, 15, 16] for multicarrier like OFDM system as:

$$\boldsymbol{h}(n;l) = \sum_{q=0}^{Q-1}\boldsymbol{h}_q(l\}e^{jw_q n} \quad (20)$$

where $e^{jw_q n}$ is the basis function and $\{\boldsymbol{h}_q(l)\}_{q=0}^{Q-1}\}$ is the M_r-column time-invariant BEM coefficient vector.

From Equation (7) and Equation (10), we can conclude that the size of the correlation matrix is reduced from N_C to N. The practical way to compute the estimation of the correlation matrix is by the time-averaging as given in Equation (21)

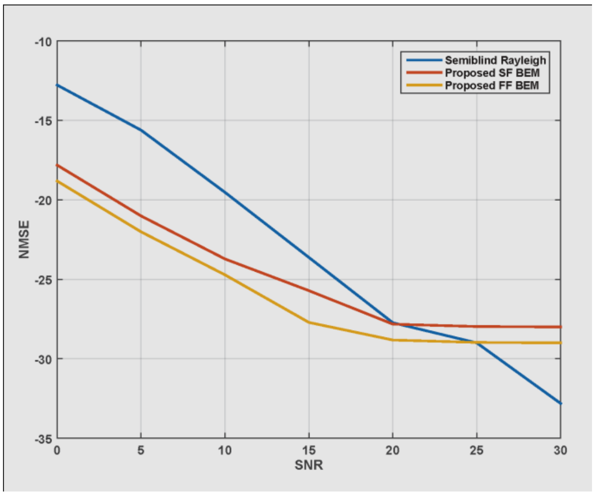


Figure 3: NMSE vs SNR

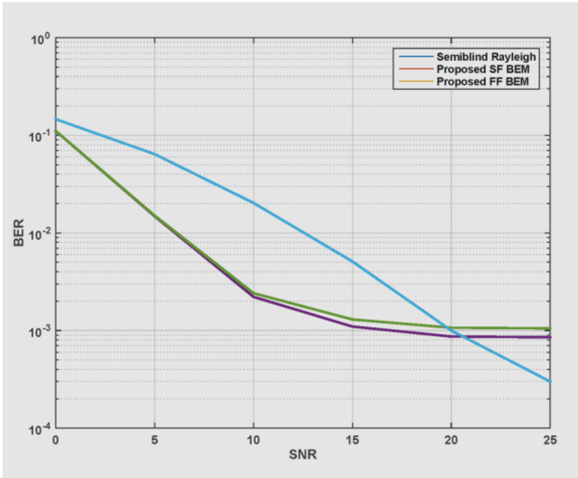


Figure 4: BER vs SNR

$$\boldsymbol{R}_N^m(n-1) = \frac{1}{T_{avg}}\sum_{n=1}^{T_{avg}}\boldsymbol{y}_N^m(n-1)\boldsymbol{y}_N^{mH}(n-1) \quad (21)$$

Figure 3 shows the normalised mean squared error for channel estimation which compares the performance of the slow-fading channel and fast fading channel with the semi-blind linear prediction based slow-fading channel estimation proposed in [3]. This figure shows that the proposed method for slow fading has almost same performance as that of fast fading channel and at low SNR the NMSE is significantly improved over the reference method in [3].

Figure 4 shows BER versus SNR plot which shows that the response of the RChol algorithm to the fast fading as well as slow fading channel approximated by basis expansion model is nearly the same and significantly outperform the proposed method in [3] at low SNR but at high SNR performance degrades compare to the proposed method in [3].

V. Conclusion

This paper shows that the time-domain equaliser significantly reduces the size of the correlation matrix. From a received symbol vector, the RChol algorithm directly computes the Cholesky factors of a correlation matrix and is optimised for a slow-fading as well as for a fast-fading channel. The simulation results show that the proposed algorithm has better performance compared to other previously proposed methods with reduced time averaging.

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