

Optimum Allocation of Reactive Power Sources for Voltage Stability Improvement and Loss Minimization in Power Distribution Systems

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Abstract—The real power losses in distribution systems are generally quite appreciable, constituting a major portion of the overall system losses. Low power factors and poor voltage profiles are the main reasons for higher losses and unsatisfactory conditions of power supply to the consumers. Installation of reactive power sources at suitable locations in distribution systems is usually suggested for the dual purpose of achieving improved voltage profiles and reduction in real power losses. Analytical methods are available to find the optimum locations and size of shunt capacitors in primary radial feeders. Most of these methods assume only uniformly loaded radial feeders while very few methods consider feeders with distributed load. However, these methods have limited application and cannot be applied to complex distribution systems, typically urban systems, where the conductor gradation also varies amongst the various feeders. On load transformer tap settings also change the reactive power distribution in the network and hence it is important to account for them while deciding the reactive power compensation requirements in distribution systems. A novel method has been developed for finding the optimum location of feeding point/reactive power compensation point in distribution systems. Results based on the proposed methods and successfully applied to a complex distribution system and an L.T and an H.T radial distribution system are presented. Results on 32-Bus Urban distribution system showed that the system power loss for initial and optimization for case-A and case-B are 23.0, 15.00, and 14.20 MW, respectively.

Index Terms—Loss Minimization; Power Distribution System; Reactive Power Sources; Voltage Stability.

I. INTRODUCTION

A large complex distribution system, typical of an urban distribution, is generally fed by the main power system at more than one feeding sub-station. Analyses can be carried out for the entire power system representing its distribution systems as equivalent loads and therefore possible active power, reactive power and voltage conditions at the feeding sub-station buses can be obtained. Detailed analysis can then be performed on the distribution systems represented in detail and considering the equivalent generations at the feeding sub-station, as obtained from the entire power system analyses. The method described in [1-4] for optimum allocation of reactive power in transmission systems (optimization method) with some modifications can be applied to distribution systems also.

II. OPTIMIZATION METHOD

In the day-to-day operation of distribution systems, an optimum allocation of reactive power can be obtained for a given in-feed, load and network conditions in the distribution system. The possible reactive power control variables in distribution systems are Transformer tap settings (T) and Switchable VAR Compensator (SVC) settings (Q). The dependent variables are voltages at SVC buses (V) and voltages at other load buses (V).

The approach adopted is similar to the one explained in [1], except the sensitivity matrix S [1], [5-7], relating the control variables and the dependent variables, is evaluated in the following manner. Consider a system where:

n represents-the number of total buses,
 g the number of in-feed sub-station buses,
 t the number of tap-regulating transformers,
 s the number of SVC buses, and
 $r n - (g + s)$, the number of remaining buses.

Assuming that:

$1, 2, \dots, g$ are the in-feed sub-station buses,
 $g+1, g+2, \dots, g+s$ are the SVC buses, and
 $g+s+1, g+s+2, \dots, n$ are the remaining buses,

the linearized control variable vector is defined as,

$$x = [\Delta T_1, \dots, \Delta T_t, \Delta Q_{g+1}, \dots, \Delta Q_{g+s}]^t \quad (1)$$

and the linearized dependent variable vector as,

$$b = [\Delta V_{g+1}, \dots, \Delta V_{g+s}, \Delta V_{g+s+1}, \dots, \Delta V_n]^t \quad (2)$$

As a matter of fact, the small change in voltage phase angle of the bus does not affect the reactive power injections to the bus system. The relation between the transformer tap settings variable, the voltage magnitudes, and the net reactive power change at any bus can be described as,

$$\begin{bmatrix} \Delta Q_S \\ \Delta Q_R \end{bmatrix} = \begin{bmatrix} D_1 & D_2 & D_3 \\ D_4 & D_5 & D_6 \end{bmatrix} \begin{bmatrix} \Delta T_T \\ \Delta V_S \\ \Delta V_R \end{bmatrix} \quad (3)$$

where,

$$[\Delta Q_S] = [\Delta Q_{g+1}, \dots, \Delta Q_{g+s}]^t \quad (3a)$$

$$[\Delta Q_R] = [\Delta Q_{g+s+1}, \dots, \Delta Q_n]^t \quad (3b)$$

$$[\Delta T_T] = [\Delta T_1, \dots, \Delta T_t]^t \quad (3c)$$

$$[\Delta V_S] = [\Delta V_{g+1}, \dots, \Delta V_{g+s}]^t \quad (3d)$$

$$[\Delta V_R] = [\Delta V_{g+s+1}, \dots, \Delta V_n]^t \quad (3e)$$

and the sub-matrices D_1 to D_6 are the corresponding terms of the partial derivatives $\partial Q/\partial T$ and $\partial Q/\partial V$, where,

$$\frac{\partial Q_k}{\partial T_{km}} = \frac{2}{a^3} V_k^2 y_m \sin \alpha_{km} + \frac{1}{a^2} y_{km} V_k V_m \sin(\delta_k - \delta_m - \delta_{km}) \quad (4)$$

$$\frac{\partial Q_m}{\partial T_{km}} = \frac{1}{a^2} y_{km} V_k V_m \sin(\delta_m - \delta_k - \delta_{km}); \quad (5)$$

where k = tap side bus.

$$\frac{\partial Q_k}{\partial V_k} = \frac{Q_k}{V_k} - B_{kk} V_k \quad (6)$$

$$\frac{\partial Q_k}{\partial V_m} = Y_{km} V_k \sin(\delta_m - \delta_k - \theta_{km}) \quad (7)$$

Rearrange Equation 3, all the control variables shift to the right-hand side and the dependent variables to the left-hand side,

$$\begin{bmatrix} \Delta V_S \\ \Delta V_R \end{bmatrix} = \begin{bmatrix} -D_2 & -D_3 \\ -D_5 & -D_6 \end{bmatrix} \begin{bmatrix} D_1 & -I \\ D_4 & 0 \end{bmatrix} \begin{bmatrix} \Delta T_T \\ \Delta Q_S \end{bmatrix} \quad (8)$$

or,

$$\begin{bmatrix} \Delta V_S \\ \Delta V_R \end{bmatrix} = [S] \begin{bmatrix} \Delta T_T \\ \Delta Q_S \end{bmatrix} \quad (9)$$

where (I) is an identity matrix of ($s \times s$) size.

III. SYSTEM STUDIED AND RESULTS

A computer program based on the proposed method has been developed and applied to a few complex distribution systems. Results obtained for a typical 32-bus urban distribution system are presented. The single line diagram of the system is shown in Figure 1.

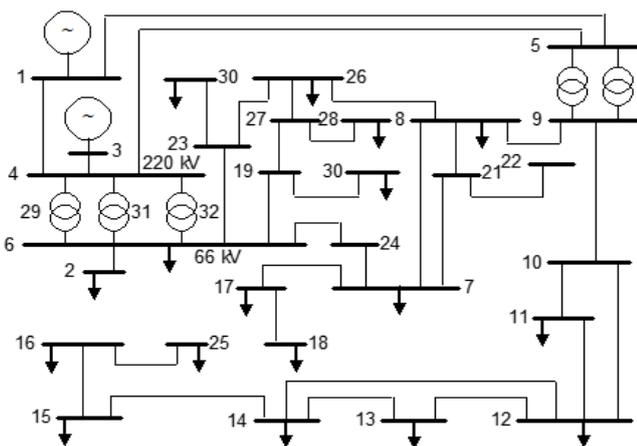


Figure 1: A 32-Bus complex distribution system

The transformer, line data, and load data are given in Table 1, Table 2, and Table 3, respectively. Buses 1 and 3 are the in-feed sub-station points of the system. An initial power flow solution is obtained for the system with nominal 1.0 tap setting for all the transformers and 1.0 per unit voltage assumed at the feeding sub-station buses. The results of voltage profile are given in Figure 2, which show a poor voltage profile in the system and real power losses amounting to 23.3 MW.

Reactive power optimization method with the objective of power loss minimization has been applied to improve the situation of the system. The minimum and maximum limits on the transformer tap settings are considered as 0.95 and 1.05 respectively. Some of the load buses are considered as SVC buses with initial settings equal to 0.0 MVAR. The minimum and maximum limits and step size of SVC-settings are considered as 0.0, 30.0 and 5.0 MVAR respectively. At the end of the 3rd iteration (Case-A) of the VAR optimization study, the situation has improved. The voltage at all the buses was around 0.95 p.u. The system real power losses reduced to 15.0 MW.

Table 1
Transformer data on 100 MVA Base

Bus		R (pu)	X (pu)
From	To		
Tap-regulating transformers:			
4	6	0.0016	0.0318
5	9	0.0028	0.0550
Fixed-tap transformers:			
4	29	0.0027	0.0541
4	31	0.0024	0.0480
4	32	0.0071	0.1424
6	29	0.0007	0.0147
29	21	0.0012	0.0247
31	6	0.0001	0.0010
32	6	0.0001	0.0010

Table 2
Line impedance data on 100 MVA Base

Bus		R (pu)	X (pu)	B/2 (pu)
From	To			
1	4	0.0213	0.1058	0.3543
1	5	0.0484	0.2398	0.2044
2	4	0.0284	0.1359	0.1098
3	4	0.0239	0.1219	0.0933
4	5	0.0058	0.0282	0.0248
6	16	0.0137	0.0198	0.0020
6	17	0.0367	0.0528	0.0003
6	19	0.1025	0.1480	0.0009
6	23	0.2123	0.3066	0.0019
6	24	0.2200	0.0317	0.0002
7	8	0.0631	0.0799	0.0001
7	17	0.0220	0.0317	0.0002
7	21	0.0439	0.0555	0.0007
7	24	0.0367	0.0528	0.0003
8	9	0.0110	0.0219	0.0007
8	21	0.0193	0.0244	0.0003
8	26	0.1535	0.3061	0.0023
9	10	0.0058	0.0253	0.0009
10	11	0.1025	0.1480	0.0009
10	12	0.0684	0.1102	0.0008
11	12	0.0439	0.0634	0.0004
12	13	0.0264	0.0381	0.0009
13	14	0.0199	0.0321	0.0009
14	15	0.0323	0.0465	0.0011
15	16	0.0244	0.0392	0.0010
16	25	0.0139	0.0321	0.0004
17	18	0.0073	0.0106	0.0001
19	20	0.0129	0.0437	0.0004
19	27	0.1538	0.2220	0.0014
21	22	0.0193	0.0244	0.0003
23	26	0.0805	0.1163	0.0007
23	30	0.0263	0.0524	0.0005
26	27	0.0367	0.0528	0.0003
27	28	0.0220	0.0317	0.0002

Table 3
Line impedance data on 100 MVA Base

Bus No.	P-load (MW)	Q-load (MVAR)
2	10.00	7.50
6	74.00	55.50
7	18.00	13.50
8	16.00	12.00
9	22.00	16.50
10	10.00	7.50
11	9.00	6.75
12	17.00	12.75
13	13.00	9.75
14	14.00	10.50
15	10.00	7.50
16	21.00	15.75
17	7.00	5.25
18	8.00	6.00
20	7.00	5.25
25	16.00	12.00
26	25.00	18.75
8	5.00	3.75
29	0.00	-50.00
30	3.00	2.25

At the end of the 5th iteration (Case-B) of VAR optimization study, the situation further improved. The voltages at all the buses came close to 1.00 per unit. The system real power losses reduced to 14.2 MW. The system voltage profile, transformer tap settings and the compensation settings for Cases A and B are given in Figure 2, Table 4 and Table 5.

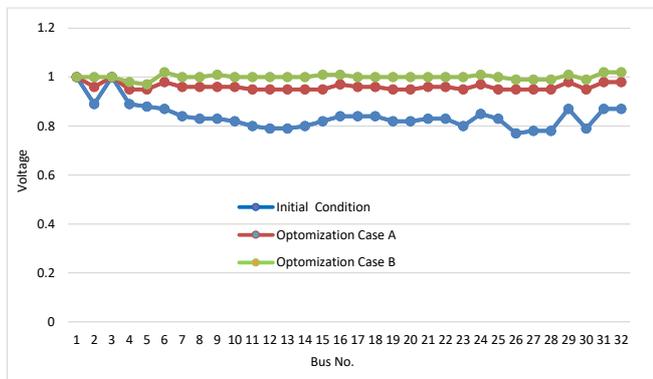


Figure 2: Voltage profiles

Table 4
Transformer tap settings

Transformer between buses	Initial condition	Optimization method	
		Case-A	Case-B
4-6	1.00	0.95	0.95
5-9	1.00	0.95	0.95

Table 5
Compensation settings for case-A and case-B

Bus No.	Initial condition	Optimization method	
		Case-A	Case-B
2	0.0	10.0	15.0
11	0.0	5.0	15.0
12	0.0	15.0	15.0
13	0.0	15.0	15.0
14	0.0	15.0	15.0
15	0.0	5.0	15.0
25	0.0	5.0	15.0
26	0.0	15.0	25.0
28	0.0	5.0	15.0

IV. RADIAL DISTRIBUTION SYSTEMS

Generally, high-tension (H.T.) radial feeders (typical of 11 kV) are spread like a tree, emerging from the feeding sub-station. Reactive power compensation is provided on such feeders preferably at a single location. Low-tension (L.T.) radial feeders (typical of 415 Volts) are also spread like a tree emerging from the distribution transformers. Generally, reactive power compensation is not provided in such low-tension radial feeders. But the location of the distribution transformers (feeding-source) in such networks plays an important role on the voltage profile and real power losses in the system.

In this paper, an elegant method has been developed, which is useful for finding the optimum location of feeding point/reactive power compensation point, for a given radial distribution network and its load demands. The location which leads to minimum losses in the system is decided as the optimum location. The fact that the poor voltage conditions and increased losses are exploited in this approach. Supply of load from the electrical center of the load area provides optimum voltage profile in the system and the losses will be minimum as the conductors will be carrying minimum current and will be of the minimum length. Therefore the optimum feeding point for a given radial distribution network with its specified load demand can be defined as the location which gives the minimum losses in the system. Once the optimum feeding point is found a reactive power feeding source can be installed at this location in order to improve the voltage profile and to minimize the system losses. For a given distribution network with its load demands, the total losses are computed corresponding to each node as the feeding point. The node for which the total loss in the system is minimum, is selected as the optimum location for feeding point reactive power compensation. A merit order of nodes based on their corresponding total losses in the system is obtained. Due to some geographical or other reasons, if the nodes selected for feeding/reactive compensation are not suitable, the nodes which give next minimum losses are preferred. The method for power flow solution of radial networks presented in [1], [8] is employed.

While performing the power flow analysis, it was observed that the available conventional power flow methods fail to give a converged power flow solution corresponding to some of the nodes considered as feed-points. When there is a poor voltage profile in the radial distribution systems most of the available power flow methods either fail to provide the solution require more number of iterations and computer time to give a solution. Thus, to overcome this problem, a power flow method based on forward-backward current flow voltage drops presented in [1], has been successfully used.

V. OPTIMUM FEEDING POINT USING LOAD FLOW SOLUTION METHOD

This is an extension of forward-backward load flow solution technique proposed in [9]. The optimum feeding point can be defined as the node which gives minimum losses. In this method, total system active and reactive power losses are calculated at each node as a possible feeding node. The node at which the total losses are minimum is selected as the optimum feeding point. Due to some geographical or other reason, if the nodes selected are not suitable, the nodes which give next minimum losses are preferred.

In some feeding points, the load flow solution will not converge due to large voltage drop and losses. Therefore, the number of iterations in the load flow have to be limited. However, the advantage of this method is that the same merit order of the feeding nodes can be obtained without performing the load flow with higher tolerance value for convergence. In other words, the optimum feeding point can be obtained by performing one iteration of the load flow at each node as a possible candidate location. Once the optimum feeding point is found a reactive power source can be installed at this location.

The size of the capacitor is decided by many factors such as voltage level, feeder over all power factor, economic returns on the investment of capacitor installation etc.

The method described above for finding the optimum location of feed-point/reactive power compensation point in radial distribution systems has been applied to a few systems and the results obtained are compared by performing the rigorous power flow analysis corresponding to all the nodes as feed-point/reactive power compensation point

In day-to-day operation, the capacitors switching (in-service/out-of-service) for varying loading conditions (peak load to light load) can be carried out effectively with suitable criteria based on the capacitor-locations merit order obtained by the proposed method.

VI. TYPICAL SYSTEM ANALYSIS AND SUMMARY OF RESULTS

Two distribution feeders of 18-node L.T and 19-node H.T radial distribution system are selected for system analysis.

A. 18-Node L.T System

The 18-node L.T system single line is shown in Figure 3; the network and load data are given in Table 6 and Table 7. The conductor type is 4 ACSR with $R = 1.578 \Omega/\text{km}$ and $X = 0.354 \Omega/\text{km}$. The results obtained with fully converged load flow and with approximately converged (with low value of tolerance of 10^{-2}) load flow solution are tabulated in Table 8.

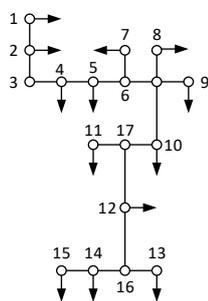


Figure 3: 18-Node L.T System

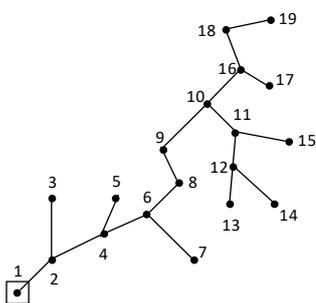


Figure 4: 19-Node H.T System

It can be inferred from the results that the load flow had not converged within 50 iterations with a convergence tolerance of 10^{-5} for the nodes 2, 15 and 1 of the feeder. However, the merit order of the feeding points obtained with fully converged load flow and with approximately converged load flow were almost same and both gave the same optimum feeding point. The summary results of 18-node L.T system of merit order for optimum location is shown in Table 8.

Table 6
18-Node LT Radial distribution system: Line data

Node			Node		
From	To	Length (m)	From	To	Length (m)
1	2	60	10	17	100
2	3	250	10	18	60
3	4	60	11	17	200
4	5	120	12	16	200
5	6	60	12	17	200
6	7	300	13	16	250
6	18	120	14	15	500
8	18	60	14	16	180
9	18	450			

Table 7
18-Node L.T. Radial distribution system: Load data

Node no.	Lload (kW)	Qload (kVar)	Node no.	Lload (kW)	Qload (kVar)
1	0.200	0.176	9	3.730	3.290
2	0.200	0.176	10	3.730	3.290
3	0.250	0.220	11	3.730	3.290
4	0.200	0.176	12	3.730	3.290
5	0.250	0.220	13	3.730	3.290
7	3.730	3.290	14	3.730	3.290
8	2.238	1.974	15	2.238	1.974

Table 8
Merit order of feeding points for the 18-node L.T network

Merit order	Fully converged load flow (with tolerance 10^{-5})			Approximately converged load flow (with tolerance 10^{-5})		
	Node	P_{Loss} (kW)	Q_{Loss} (kVar)	Node	P_{Loss} (kW)	Q_{Loss} (kVar)
1	17	2.04	0.46	17	2.02	0.45
2	10	2.27	0.51	10	2.27	0.51
3	12	2.68	0.60	12	2.64	0.59
4	18	2.72	0.61	18	2.71	0.61
5	8	3.87	0.87	8	3.86	0.87
6	16	4.70	1.06	16	4.67	1.05
7	6	4.72	1.06	6	4.69	1.05
8	11	5.66	1.27	11	5.62	1.26
9	5	6.22	1.40	5	6.15	1.38
10	14	8.46	1.90	14	8.38	1.88
11	4	10.08	2.26	4	9.97	2.24
12	13	11.66	2.62	13	11.41	2.56
13	3	12.71	2.85	3	12.43	2.79
14	7	13.93	3.13	7	13.72	3.08
15	9	16.11	3.61	9	15.69	3.52
16	2	*42.85	*9.61	2	*42.85	*9.61
17	15	*45.86	*10.29	15	*45.86	*10.29
18	1	*46.40	*10.41	1	*46.40	*10.41

* The load flow does not converge within the iteration limit (50).

B. 19-Node High Tension System

The 19-node H.T system single line diagram is shown in Figure 4, network and load data are given in Table 9 and Table 10. The feeder shown in Figure 4 also analyses for merit order and the results are tabulated in Table 11. It can be observed from the results that the merit order of the feed-points obtained from the fully converged load flow results and approximately converged results are similar.

Table 9
19-Node H.T Radial distribution system: Line data

From	Node To	R (pu)	X (pu)	Length (km)
1	2	0.02580	0.01110	3.0
2	3	0.04300	0.01850	5.0
2	4	0.01290	0.05555	1.5
4	5	0.01290	0.05555	1.5
4	6	0.00860	0.00370	1.0
6	7	0.01720	0.00740	2.0
6	8	0.02150	0.00925	2.5
8	9	0.02580	0.01110	3.0
9	10	0.04300	0.01850	5.0
10	11	0.01290	0.00555	1.5
11	12	0.00860	0.00370	1.0
11	15	0.04300	0.01850	5.0
12	13	0.03010	0.00555	3.5
12	14	0.02440	0.01295	4.0
10	16	0.01290	0.01480	1.5
16	17	0.05160	0.00555	6.0
16	18	0.04300	0.02220	5.0
18	19	0.03440	0.01850	4.0

Table 10
19-Node H.T. Radial distribution system: Load data

Node No.	kVA	P _{Load} (kW)	Q _{load} (Kvar)
2	300	108.00	52.307
3	40	144.00	69.742
4	100	36.00	17.436
5	250	90.00	43.589
6	100	36.00	17.436
7	400	144.00	69.742
8	300	108.00	52.307
9	600	216.00	104.614
10	150	54.00	26.153
11	350	126.00	61.025
12	100	36.00	17.436
13	250	90.00	43.589
14	300	108.00	52.307
15	450	162.00	78.460
16	400	144.00	69.742
17	400	144.00	69.742
18	300	108.00	52.307
19	40	144.00	69.742

Table 11
Merit order of feeding points for the 19-node H.T network

Merit order	Fully converged load flow (with tolerance 10 ⁻⁵)			Approximately converged load flow (with tolerance 10 ⁻⁵)		
	Node	P _{Loss} (kW)	Q _{Loss} (kVar)	Node	P _{Loss} (kW)	Q _{Loss} (kVar)
1	10	99.64	42.87	10	98.24	42.26
2	9	127.72	54.95	9	126.23	54.31
3	16	140.53	60.46	16	140.13	60.29
4	11	142.38	61.26	11	141.97	61.08
5	12	189.60	81.57	12	188.72	81.20
6	8	196.71	84.63	8	195.73	84.21
7	6	291.02	125.21	6	286.59	123.30
8	4	349.39	150.32	4	341.29	146.83
9	18	438.38	188.60	18	434.64	187.00
10	13	445.74	191.77	13	442.52	190.39
11	7	451.72	194.35	7	446.71	192.19
12	2	470.38	202.37	2	463.67	199.49
13	15	479.35	206.23	15	474.65	204.21
14	5	488.84	210.31	5	481.65	207.22
15	14	490.12	210.86	14	485.20	208.75
16	17	593.86	255.50	17	582.43	250.58
17	1	1047.93	450.85	1	991.26	426.47
18	19	1186.66	*510.54	19	*1087.74	*467.98
19	3	1674.44	*720.40	3	*2569.56	*1105.50

VII. CONCLUSIONS

Computer programs have been developed based on the proposed algorithm for optimum allocation of reactive power in complex distribution systems with a combination of both radial and ring-main load factors. The reactive compensation (capacitor installation) is definitely beneficial for both the utility and the consumer.

However, in these analyses an important observation made is that the optimum location of the feeders having different voltage levels and also for finding the optimum location of feed-point/ reactive power compensation point in L.T/H.T radial distribution systems.

The results obtained for a complex distribution system, an L.T. radial distribution system and an H.T. distribution system have been presented. It is shown that the application of the proposed algorithm for radial distribution systems provides satisfactory results comparable to those obtained by rigorous and time-consuming optimization techniques.

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REFERENCES

- [1] A. Lomi and D. Thukaram, "Optimum reactive power dispatch for alleviation of voltage deviations", *TELKOMNIKA Journal*, vol. 10, no. 2, 2012, pp. 257-264.
- [2] A. Lomi and D. Thukaram, "Selection of Static VAR Compensator Location and Size for System Voltage Stability Improvement", *Int'l Journal of Electric Power Systems Research*, USA, 2000.
- [3] D. Thukaram, A. Lomi and S. Chirarattananon, "Analysis of Static VAR Compensator Location On System Voltage Stability", *Proceeding of Third International Conference on Digital Power System Simulators*, Vasteras, Sweden, May 23-28, 1999.
- [4] R. Iyer, S.K. Ramachandran and S. Hariharan, "Optimal Reactive Power Allocation for Improved System Performance", *IEEE Trans. On PAS*, vol. 103, no. 6, 1984, pp. 1509-1515.
- [5] O. Crisan and M. Liu, "Voltage Collapse Prediction Using Improved Sensitivity Approach", *Electric Power Systems Research*, 28(3), 1994, pp.181-190.
- [6] A. Gomez, J.L.M. Ramos, J.L.R. Macias, and Y.C. Salinas, "Sensitivity-Based Reactive Power Control for Voltage Profile", *IEEE Trans. On Power Systems*, vol. 8, no. 3, 1993, pp. 937-945.
- [7] N. Flatabo, R. Ognedal and T. Carlsen, "Voltage Stability Condition in a Power Transmission System Calculated by Sensitivity Methods", *IEEE Trans. On Power Systems*, vol. 5, no. 4, 1990, pp. 1286-1293.
- [8] A. Lomi, "Planning and Operation of Reactive Power Sources for Voltage Stability Improvement and Harmonics Minimization", *Dr. Eng. Diss.*, AIT, 2000.
- [9] A. Lomi and F.Y. Limpraptono, "Implementation of expert system for power system voltage stability improvement", *International Journal of Electrical Energy*, Vol. 5, No. 1, pp. 65-70, 2017.