Analysis on Euler Angles Rotation of a Rigid Body in Three-Axis Attitude Based on RazakSAT Data

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***Abstract*—Satellite attitude estimation uses different attitude representation either Euler angles, direction cosine matrix, Gibbs vector or quaternion parameters as their kinematic model. The three-axis attitude parameter using quaternion parameter is mostly used to represents the attitude of satellites. As well as in RazakSAT satellite mission, the attitude data is represented by quaternions parameters. However, the quaternions parameters do not have a physical interpretation for the attitude of the satellite. Therefore, quaternion parameters of the satellite are converted to Euler angles rotation for the physical interpretation of its orientation. This paper present and analyse the satellite three-axis Euler angles rotation for its attitude using average rotation, maximum/minimum error and standard deviation. The result shows that the quaternions parameters are successfully represented in Euler angels. The error measurement or noise exists on roll, pitch and yaw of Euler angles. For verification of the Euler angles error, the angular velocity from satellite gyroscopes is used as references. Residual analysis at low frequency is 0.00515 [degree/ second] which considered as Euler angles measurement error or noise.**

***Index Terms*—Attitude Determination System; Euler Angles Rotation; Quaternion Parameters; RazakSAT Satellite.**

I. INTRODUCTION

The Malaysian own satellite name RazakSAT which is the world first remote sensing satellite launched into Near Equatorial Orbit (NEqO). It is a mini-satellite with 180 kg mass, orbiting in Low Earth Orbit (LEO) of altitude 680 km with an inclination of 9 degrees from the equatorial plane. The imaging satellite will provide a high-resolution image of Malaysia which will be used for various applications [1-6]. The satellite attitude location is crucial in determining the quality of image acquired as shown in Figure 1.

The satellite attitude is an orientation in space relative to the inertial frames such as Earth, Moon, Sun or any other celestial object. The attitude analysis is an important characteristic in satellite operations such as for Earth observation, communication and military. The determination technique uses different attitude representation either Euler angles, direction cosine matrix, Gibbs vector or quaternion parameters as their kinematic model. The three-axis attitude parameter using Euler angles is the most suitable technique due to its straightforward physical interpretation [7-9].

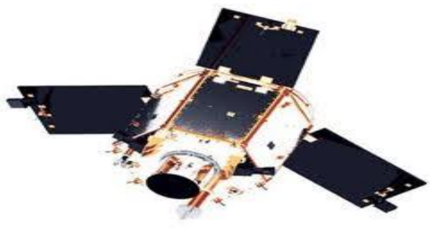

Figure 1: The RazakSAT satellite.

The accuracy of RazakSAT satellite attitude is influenced by errors such as sensor noise, bias, and misalignment. The prediction of attitude using Attitude Determination System (ADS) sensor would involve errors. The space phenomena such as eclipse, dazzling and earth albedo would cause misalignment in estimation and ADS sensor measurement [10-13]. The accuracy of attitude estimation will be improved by applying the ADS sensor measurement. The satellite is using the gyroscope, sun sensor and magnetometer as the inertial sensor and references sensor as shown in Figure 2, 3 and 4.

Figure 2: The RazakSAT magnetometer

Figure 3: The RazakSAT gyroscopes

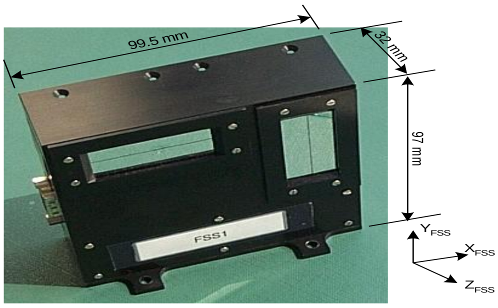

Figure 4: The RazakSAT sun sensor

In this paper, three-axis Euler angles rotation for RazakSAT satellite attitude is presented and analyzed in terms of average rotation, minimum/maximum error and standard deviation. The satellite attitude is represented as three-axis Euler angel's rotation that being converted from quaternion parameters. The quaternions parameters are being represented in three-axis Euler angels parameters. The Euler angles errors are being verified by measurement of angular velocity from gyroscope and torque from the magnetometer.

II. THREE-AXIS ATTITUDE OF RIGID BODY

In space, a rigid body is a collection of mass particles (component of a satellite) that maintain a fixed relationship with one another in a reference frame. Figure 5 illustrates the orientation of satellite axis $\hat{u}, \hat{v}, \hat{w}$ in the reference 1, 2, 3 frame [7].

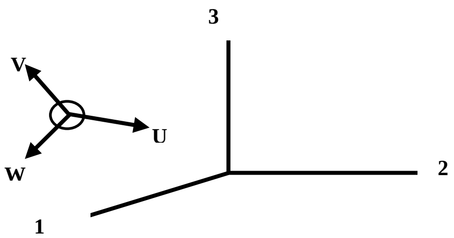

Figure 5: Satellite orientation axis $\hat{u}, \hat{v}, \hat{w}$ in references 1, 2, 3 frames

Assume that there exists an orthogonal, right-handed triad $\hat{u}, \hat{v}, \hat{w}$ of unit vector fixed in the body with Equation (1),

$$\hat{u} \quad x \quad \hat{v} = \hat{w} \tag{1}$$

From Figure 5, components of $\hat{u}, \hat{v}$ and $\hat{w}$ along the three axis of coordinate frame will fix the orientation completely [7]. This requires nine parameters which regarded as elements of a 3 x 3 matrix is called as attitude matrix or Direction Cosine Matrix (DCM), A with Equation (2),

$$A \equiv \begin{bmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{bmatrix} \tag{2}$$

With Equation (3),

$$\hat{u} = \left(u_1 \quad u_2 \quad u_3\right)^T, \hat{v} = \left(v_1 \quad v_2 \quad v_3\right)^T, w = \left(w_1 \quad w_2 \quad w_3\right)^T \tag{3}$$

III. DIFFERENT TYPE OF ATTITUDE REPRESENTATION

Direction Cosine Matrix (DCM) is considered as the fundamental orientation of the rigid body of a satellite. However, DCM also has advantages and disadvantages which depends on the specific application. Table 1 shows the alternatives of attitude parameters that represent the three-axis attitude [7].

Table 1
Alternative Representations of Three-Axis Attitude

Parameterization	Advantages	Disadvantages
Direction Cosine Matrix	No singularities, no trigonometric functions, and convenient product rule for successive rotation.	Six redundant parameters.
Quaternions	No singularities, no trigonometric functions, and convenient product rule for successive rotation.	One redundant parameter, and no obvious physical interpretation.
Gibbs Vector	No singularities, no trigonometric functions, and convenient product rule for successive rotation.	Infinite for 180 degree rotation.
Euler Angles	No redundant parameters, and has clear physical interpretation.	Trigonometric functions, singularity at some pitch, no convenient product rule for successive rotations.

IV. QUATERNIONS AND EULER ANGLES

The quaternion representation of rigid rotation leads to a convenient kinematical expression involving the Euler symmetric parameters [14]. Unit quaternion is defined by Equation (4),

$$q = \left[q_1 \quad q_2 \quad q_3 \quad q_4\right]^T \tag{4}$$

In term of Euler axis $e = \left[e_x \quad e_y \quad e_z\right]^T$ and angle θ. The element of unit quaternion can be expressed as Equation (5) [14],

$$\begin{aligned} q_1 &= e_x \sin\frac{\theta}{2} \\ q_2 &= e_y \sin\frac{\theta}{2} \\ q_3 &= e_z \sin\frac{\theta}{2} \\ q_4 &= \cos\frac{\theta}{2} \end{aligned} \tag{5}$$

The quaternion parameterizations obey the constraint in Equation (6),

$$q_1 + q_2 + q_3 + q_4 = 1 \tag{6}$$

The last unit of quaternion is called scalar which has its origin in quaternion and represented as a mathematical extension of the complex number as in Equation (7),

$$a + bi + cj + dk \quad \text{with} \quad a, b, c, d \in \Re \tag{7}$$

where *i*, *j* and *k* are hypercomplex numbers that are satisfying Equation (8),

$$\begin{aligned} i^2 = j^2 = k^2 &= -1 \\ ij = -ji &= k \\ jk = -kj &= i \\ ki = -ik &= j \end{aligned} \tag{8}$$

Quaternion multiplication involves the multiplication of complex numbers. In matrix notation, the quaternion multiplication is written as Equation (9),

$$\tilde{q} \otimes q = \begin{bmatrix} q_4 & q_3 & -q_2 & q_1 \\ -q_3 & q_4 & q_1 & q_2 \\ q_2 & -q_1 & q_4 & q_3 \\ -q_1 & -q_2 & -q_3 & q_4 \end{bmatrix} \begin{bmatrix} \tilde{q}_1 \\ \tilde{q}_2 \\ \tilde{q}_3 \\ \tilde{q}_4 \end{bmatrix} \tag{9}$$

The direction cosine matrix can be expressed in term of Euler symmetric parameters is written as Equation (10),

$$R_b^o = \begin{bmatrix} 1-2(q_3^2+q_4^2) & 2(q_2q_3-q_1q_4) & 2(q_2q_4+q_1q_3) \\ 2(q_2q_3+q_1q_4) & 1-2(q_2^2-q_4^2) & 2(q_3q_4-q_1q_2) \\ 2(q_2q_4-q_1q_3) & 2(q_1q_2+q_3q_4) & 1-2(q_2^2+q_3^2) \end{bmatrix} \tag{10}$$

Hence, the conversion equation from quaternion to Euler angles becomes Equitation (11),

$$\begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix} = \begin{bmatrix} \arctan\left(\dfrac{2(q_1q_2+q_3q_4)}{1-2(q_2^2+q_3^2)}\right) \\ \arctan 2(q_1q_3+q_2q_4) \\ \arctan\left(\dfrac{2(q_1q_4+q_2q_3)}{1-2(q_3^2+q_4^2)}\right) \end{bmatrix} \tag{11}$$

V. Results and Discussion

The Euler angles position of the roll, pitch and yaw converted from RazakSAT satellite quaternion parameters are shown in Figures 6, 7 and 8. Table 2 shows the accuracy of Euler angles from satellite rigid body. Average of Euler angle for roll, pitch, yaw are 27^0, 4.03^0 and -68.41^0 respectively. Figure 6 and 7 show that there are some errors of roll and pitch, which have been detected during satellite orientation at *t* equal to 1, 2, 3, 139, 237 and 263 minutes.

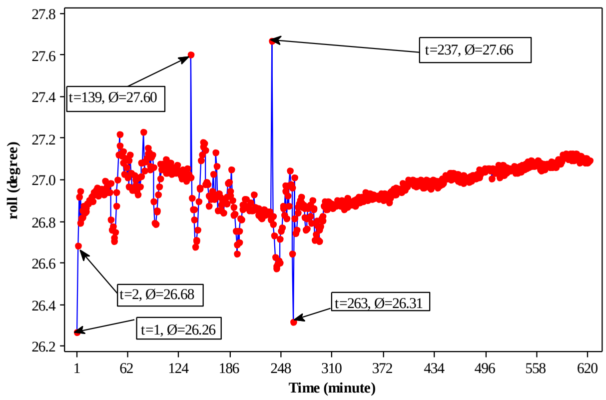

Figure 6: The Euler angle position for Roll

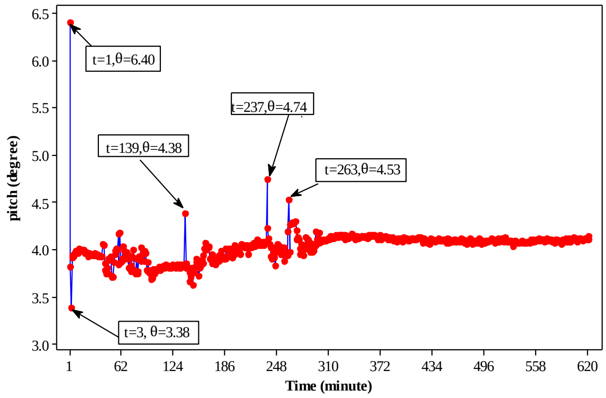

Figure 7: The Euler angle position for Pitch

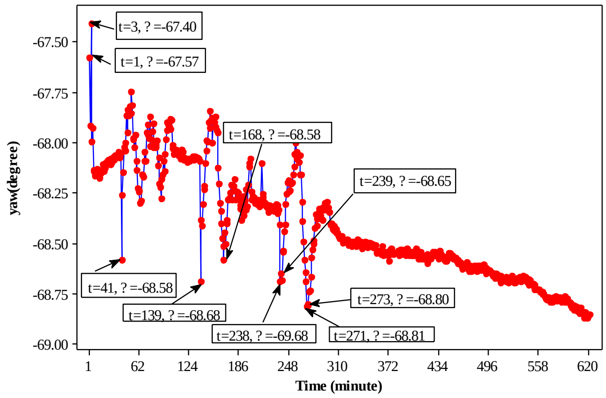

Figure 8: Euler angle position for yaw

Table 2
Accuracy of Euler angles from rigid body of RazakSAT

Euler Angle	Average	Maximum Error	Minimum Error	Standard Deviation
Roll	27	27.7	26.6	0.122
Pitch	4.03	6.4	3.38	0.158
Yaw	-68.41	-67.4	-68.9	0.273

Figure 8 shows that the measurement error for yaw has been detected on more than five points compared to roll and pitch position. The errors are detected at *t* equal to 1, 3, 41, 139, 168, 238, 239, 271 and 273 minute. Table 3 shows the detail analysis for measurement error of Euler angles from satellite rigid body for roll, pitch and yaw.

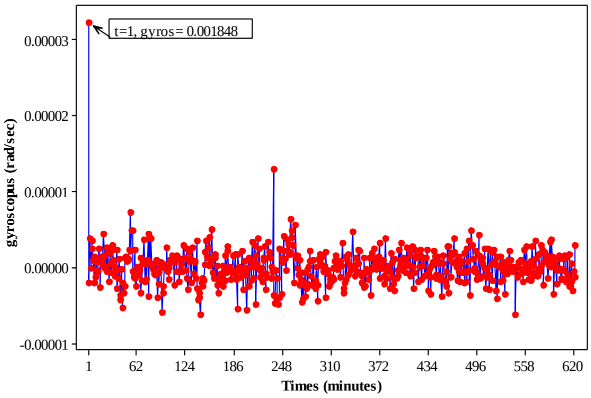

Figure 9: The RazakSAT gyroscopes measurement

Figure 6 shows the measurement of angular velocity from gyroscopes used for verification the Euler angles analysis. The data Euler angles of roll on Figure 3(a) at t equal to 1, Ø is 26.26˚, t equal to 2, Ø is 26.68˚is used in Equation (11) that has produce Equation (12),

$$\omega = \frac{\partial \phi}{\partial t} = \frac{26.68° - 26.26°}{120 - 60} = 0.007[\text{degree/sec}] \qquad (12)$$

Result by Equation (12) is compared with gyroscope measurement as shown in Figure 9, where at t equal to 1, w is 0.001848 [degree/sec]. As the residual analysis for these data is small, so the result shown by Table 3, 4 and 5 is considered as noise and disturbances or measurement error.

Table 3
Error of Euler Angles: Roll

t	1	2	139	237	263
roll	26.26	26.68	27.6	27.66	26.31

Table 4
Error of Euler Angles: Pitch

t	1	3	139	237	263
pitch	6.4	3.38	4.38	4.74	4.53

Table 5
Error of Euler Angles: Yaw

t	1	3	41, 168	139	238	239	271,273
yaw	-67.5	-67.4	-68.6	-68.7	-69.7	-68.7	-68.8

VI. CONCLUSION

The Euler angles of a rigid body in three axis attitude spaces based on RazakSAT satellite data has been analysed accordingly. The satellite Euler angles rotation is produced by using the satellite quaternion data to represent the roll, pitch and yaw. The average of Euler angles rotation of the roll, pitch, and yaw are 27^0, 4.03^0, and -68.41^0 respectively. The error of Euler angles has been detected from several data. The angular velocity from satellite gyroscopes is used as a reference for the Euler angles verification. The error from Euler angles is considered as disturbances or measurement noise.

For future work, the analyses on attitude estimation and actuator controller are needed to stabilize the satellite attitude is proposed to overcome the measurement error or noise from the acquired data.

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