Kinematic Analysis of a Screw Wheeled Omnidirectional Mobile Robot

M.J.A.Safar, Y.Chandradekaran, S.N.Basah, K.S.Basaruddin and M.S.M.Hashim
School of Mechatronic Engineering, Universiti Malaysia Perlis,
Pauh Putra Campus, 02600 Arau, Perlis
juhairi@unimap.edu.my

***Abstract*—This paper presents a study of an omnidirectional mobile robot based on a screw wheels mechanism. A concept design of an omnidirectional mobile robot propelled by four screw wheels approach is introduced. The forward and inverse kinematics equations for the proposed screw wheeled omnidirectional mobile robot are derived and verified via simulation. The results show the effectiveness of both forward and inverse kinematics to deliver 3-DOF omnidirectional movements.**

***Index Terms*—Holonomic; Kinematics; Mobile Robot; Omnidirectional; Screw Wheels.**

I. INTRODUCTION

Service robots applications in real human life are increasing rapidly. The number of service robots between 2016 and 2019 is projected to increase 25% of the service robot population in the year 2015, where the projection of 42 million units are contributed only from personal and domestic use [1]. Most of the service robots rely on the capability of mobile robot technology to move and navigate autonomously in human environments. A service robot with a 3 degrees of freedom (3-DOF) capability is desirable in many cases to deal with challenging environment mainly related to the crowd and restricted spaces. Several existing applications are developed based on the omnidirectional mobile platforms [2]. However, most of the mechanisms introduced are inappropriate for the outdoor applications due to the disadvantages of the special wheels design. Most of the unique wheels are sophisticated in design and consist of several mini rollers or gaps which are exposed to dirt clog in outdoor environments. The primary objective of this paper is to propose a novel omnidirectional mechanism for the mobile robot based on the screw-propelled mechanism which is feasible for outdoor applications. In this study, the kinematics solution of the proposed design is derived and simulated to evaluate the performance of the proposed omnidirectional mobile robot.

II. LITERATURE REVIEW

Screw wheel mechanism is introduced as early in 1899 for hauling a plough in agricultural machines [3]. However, the first vehicle with screw wheels locomotion was introduced later as transportation on snow [4]. A pair of screw wheels with a different direction of the helix is attached to a tractor to give a propulsive, driving force. There are few applications which later introduced by many developers to deal with different soil properties such as in mud, sandy area and even on water surfaces [5-6]. Most of the vehicles are designed purposely for soft terrain applications where the screw wheels are efficiently propelled due to sufficient reaction forces at the screw thread surface. Nagaoka et al. [7] introduce the mathematical modelling and the propulsive characteristic of a rover for the sandy ground application. The rover is developed with a pair of Archimedean screws. A similar architecture with moderate screw structure is proposed for self-reconfigurable robots [8]. The robot provides most of the necessary movements such as forward, backward and sideways motion as well as rotational movements. A study based on computer simulation is presented in [9]. Although this mechanism is more superior to the common differential drive mechanism by the existing of additional 1-DOF for sideway locomotion, this mechanism only allows one individual task at a time. By increasing the number of screw wheels to four units in a particular arrangement, a 3-DOF omnidirectional and holonomic movement is achieved [10]. The previous work presented in [10] only focused on the experimental works, where the proposed movement is limited to a basic movement as shown in Figure 1. There is no evidence of mathematical modelling which solve the kinematics or dynamics were presented. Thus, this study aims to solve the mathematical modelling of the same kind omnidirectional mobile robot by introducing the forward and inverse kinematics equations. By implementing the kinematics equations, the path planning problems for the omnidirectional mobile robot is easier and capable of performing more complex movements than the movements shown in Figure 1.

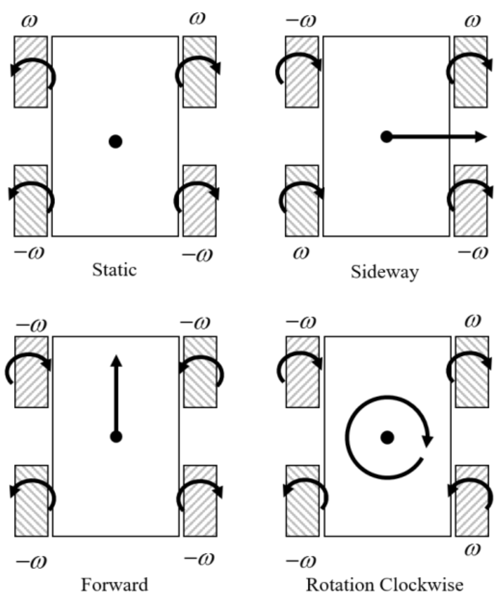


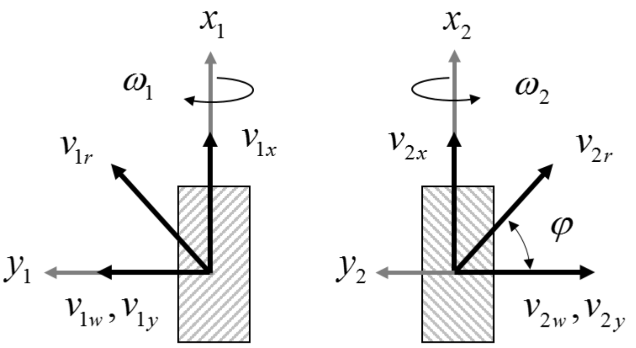
Figure 1: Omnidirectional Mobile Platform with Screw Wheels [10]

III. RESEARCH METHODOLOGY

In this section, the basic concept of the omnidirectional mobile robot with screw wheels is presented. Based on the screw mechanism principles, the mathematical modelling in term of forwarding and inverse kinematics are derived for a four-wheeled screw wheels mobile robot.

A. Screw Wheel

Figure 2 shows the basic principles of the screw wheels used in this study. The screw wheel is designed as a cylindrical body with outer helical structure, where φ is the helix angle. Assuming an angular velocity, ω_2 is provided to the screw wheel as shown in Figure 2(b), the resultant velocity, v_{2r} which perpendicular to the helix is produced. This resultant velocity also can be described in term of the velocity component in *x*- and the *y*-axis direction. At least two units of screw wheels with opposite helix as shown in Figure 2 are necessary for producing motion as mentioned in [7-9].



(a) (b)

Figure 2: Forces acting at the screw wheels

With reference to the *i*-th (i=1,2) screw wheel, the general form of lateral and longitudinal velocities of the screw wheel can be obtained by;

$$v_{ix} = v_{ir} / \cos\varphi \tag{1}$$

$$v_{iy} = v_{ir} / \sin\varphi \tag{2}$$

Meanwhile, the pure rolling movement of the screw wheel is given by;

$$v_{iw} = \omega_i R_w \tag{3}$$

where, R_w is the radius of the screw wheels.

B. Omnidirectional Mobile Robot

Figure 3 shows the kinematical design of the proposed omnidirectional mobile robot. Four units of screw wheels are arranged so that the diagonal wheels have a similar helical angle and direction. This arrangement is almost similar to an omnidirectional mobile robot with mecanum wheels [11] except the rotation axis of the wheels. With this combination, the velocity vectors are arranged so that a 3-DOF movement in term of $(v_x \quad v_y \quad \omega_z)^T$ can be generated at the centre of the mobile robot. These all three velocities can be delivered simultaneously for more flexible movements.

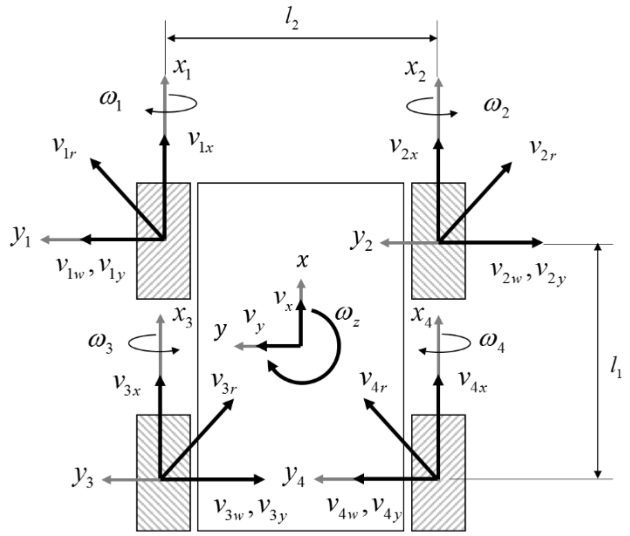


Figure 3: Proposed Omnidirectional Mobile Robot

C. Kinematics

By setting the helical angle, φ = 45°, the velocity components of each wheel in the direction of x_i and y_i-axis (*i*=1,2,3,4) yields;

$$v_{1x} = v_{1r}/\sqrt{2} \quad \text{and} \quad v_{1y} = v_{1w} + v_{1r}/\sqrt{2} \tag{4}$$

$$v_{2x} = v_{2r}/\sqrt{2} \quad \text{and} \quad v_{2y} = -v_{2w} - v_{2r}/\sqrt{2} \tag{5}$$

$$v_{3x} = v_{3r}/\sqrt{2} \quad \text{and} \quad v_{3y} = -v_{3w} - v_{3r}/\sqrt{2} \tag{6}$$

$$v_{4x} = v_{4r}/\sqrt{2} \quad \text{and} \quad v_{4y} = v_{4w} + v_{4r}/\sqrt{2} \tag{7}$$

By combining the equations for both x_i and y_i components shown in Equations (4)–(7), the wheels velocity can be described by;

$$v_{1w} = -v_{1x} + v_{1y} \tag{8}$$

$$v_{2w} = -v_{2x} - v_{2y} \tag{9}$$

$$v_{3w} = -v_{3x} - v_{3y} \tag{10}$$

$$v_{4w} = -v_{4x} + v_{4y} \tag{11}$$

Based on the relationship of velocities induced at the centre of the mobile robot, the following equations are obtained:

$$v_{1x} = v_x - l_1\omega_z \quad \text{and} \quad v_{1y} = v_y + l_2\omega_z \tag{12}$$

$$v_{2x} = v_x + l_1\omega_z \quad \text{and} \quad v_{2y} = v_y + l_2\omega_z \tag{13}$$

$$v_{3x} = v_x - l_1\omega_z \quad \text{and} \quad v_{3y} = v_y - l_2\omega_z \tag{14}$$

$$v_{4x} = v_x + l_1\omega_z \quad \text{and} \quad v_{4y} = v_y - l_2\omega_z \tag{15}$$

By comparing the Equations (8)–(11) and Equations (12)–(15), yields;

$$v_{1w} = -v_x + v_y + L\omega_z \tag{16}$$

$$v_{2w} = -v_x - v_y - L\omega_z \tag{17}$$

$$v_{3w} = -v_x - v_y + L\omega_z \tag{18}$$

$$v_{4w} = -v_x + v_y - L\omega_z \tag{19}$$

where, $L = l_1 + l_2$. These equations (Equation (16)–(19)) can be summarised into inverse kinematics equations in matrix form as;

$$\omega_w = \frac{1}{R_w} J V_0 \tag{20}$$

where, $\omega_w = [\omega_1 \quad \omega_2 \quad \omega_3 \quad \omega_4]^T$, $V_0 = [v_x \quad v_y \quad \omega_z]^T$ and the Jacobian, J is obtained as;

$$J = \begin{bmatrix} -1 & 1 & L \\ -1 & -1 & -L \\ -1 & -1 & L \\ -1 & 1 & -L \end{bmatrix}. \tag{21}$$

Since $J \in R^{4\times3}$, the forward kinematics equations can be obtained by solving the inverse problem of Pseudo matrix as;

$$V_0 = J^+ R_w \omega_w + \left(I - J^+ J\right)\varpi \tag{22}$$

where, $J^+ = \left(J^T J\right)^{-1} J^T \in R^{3\times4}$. Assuming the $\varpi = 0$, the forward kinematics can be simplified and rewritten into;

$$V_0 = \frac{R_w}{4} J^+ \omega_w \tag{23}$$

where,

$$J^+ = \begin{bmatrix} -1 & -1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1/L & -1/L & 1/L & -1/L \end{bmatrix}. \tag{24}$$

IV. Results and Discussion

Simulation is conducted to verify the effectiveness of the proposed mathematical modelling using MATLAB software. In the simulation, the displacement of l_1 and l_2 is set to 0.2 m, the radius of the wheel is 0.02 m, and the length of the wheel is 0.1 m. The initial position and orientation of each simulation are set to $(0,0,0)^T$. The simulation time is 1 s.

A. Basic Operations

This simulation was conducted based on the inverse kinematics solution to compare the basic movements of the omnidirectional mobile robot to the results presented in [10]. Based on Figure 1, the movement of the mobile robot can be concluded into Table 1. The positive value of the rotation indicates the same direction as shown in Figure 1, while the negative is the opposite direction.

Table 1
Combination of Wheel's Angular Velocity

Movement	ω_1	ω_2	ω_3	ω_4
Static/ No locomotion	ω	ω	$-\omega$	$-\omega$
Sideway (Right)	$-\omega$	ω	ω	$-\omega$
Forward	$-\omega$	$-\omega$	$-\omega$	$-\omega$
Rotation (Clockwise)	$-\omega$	ω	$-\omega$	ω

The results obtained in the simulation are shown in Figures 5–8. The dotted line represents the initial position of the omnidirectional mobile robot. Figure 4 shows the result of the input $\omega_1 = \omega_2 = 20$ rad/s and $\omega_3 = \omega_4 = -20$ rad/s. No movements are observed from the results due to the zero resultant velocity, where the velocities are cancelled on each other although all wheels were driven.

$$[v_x \quad v_y \quad \omega_z]^T = [0 \quad 0 \quad 0]^T.$$

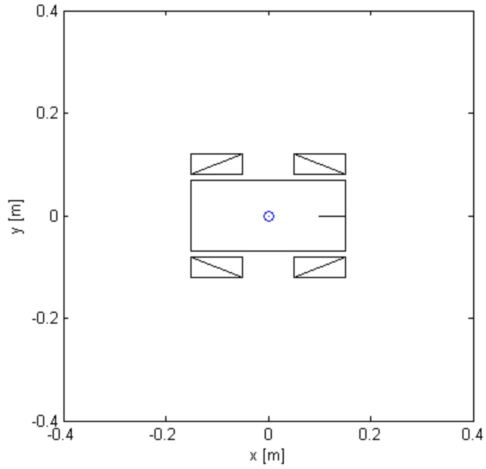


Figure 4: Static or no locomotion

Figure 5 shows the result of the input $\omega_1 = \omega_4 = -20$ rad/s and $\omega_2 = \omega_3 = 20$ rad/s. The mobile robot has moved in the sideways direction (right direction). It is also confirmed that the velocity only produced in the direction of *y*-axis with a value of 0.4 m/s, where the overall output is;

$$[v_x \quad v_y \quad \omega_z]^T = [0 \quad -0.4 \quad 0]^T.$$

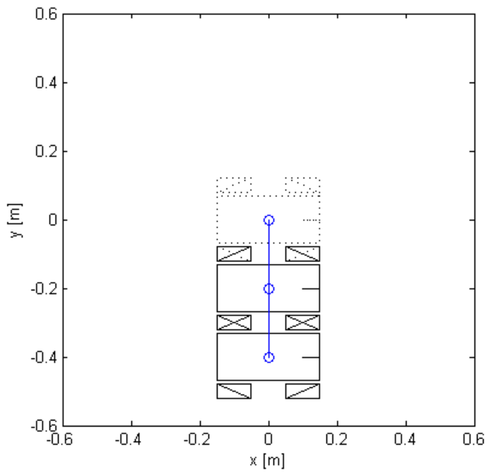


Figure 5: Sideway movement (Right)

Figure 6 shows the result for the wheel's angular velocities input of $\omega_1 = \omega_2 = \omega_3 = \omega_4 = -20$ rad/s. The mobile robot has moved forward in the direction of the *x*-axis. It is also confirmed that the velocity only produced in the direction of *x*-axis with a value of 0.4 m/s. The remaining velocities are zero, where the overall output is as follows;

$$[v_x \quad v_y \quad \omega_z]^T = [0.4 \quad 0 \quad 0]^T.$$

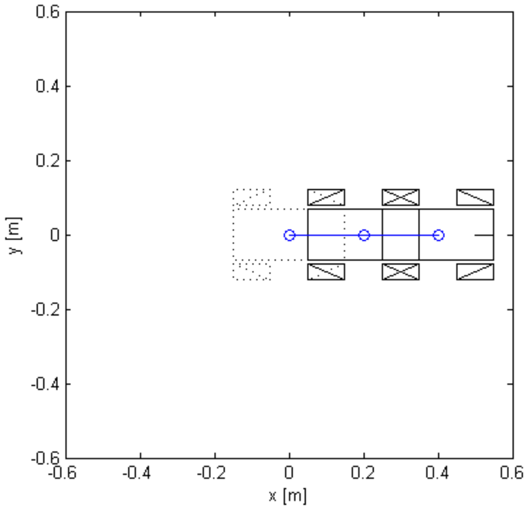


Figure 6: Forward movement

Meanwhile, Figure 7 shows the result for the input $\omega_1 = \omega_3 = -20$ rad/s and $\omega_2 = \omega_4 = 20$ rad/s to produce a pure rotation at its centre. As a result, the mobile robot has rotated clockwise at the centre of its body. This is due to the existence of the same magnitude of angular velocities at the clockwise direction. The angular velocity induced at the centre of the body, $\omega_z = -1$ rad/s is obtained. The remaining components are zero.

$$[v_x \quad v_y \quad \omega_z]^T = [0 \quad 0 \quad -1]^T$$

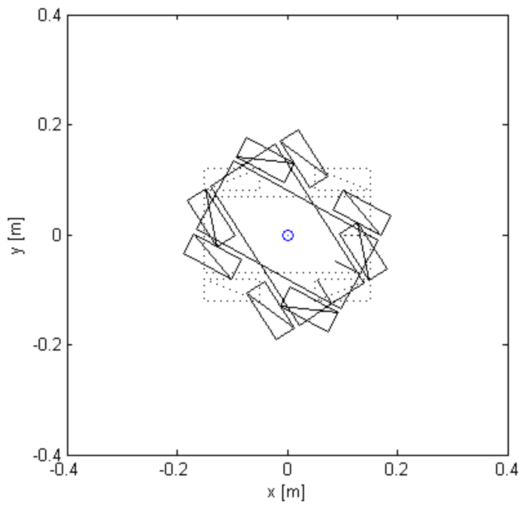


Figure 7: Rotation (Clockwise)

Based on these results, it is confirmed that the estimated movement of the omnidirectional mobile robot is identical to the experimental results presented in [10].

B. Advanced Movements

In this simulation, other possible movements for an omnidirectional mobile robot are evaluated through the implementation of both forward and inverse kinematics. Using this approach, the omnidirectional mobile robot can be controlled to travel in any arbitrary direction and orientation which are challenging to cater through the experimental solution. The desired velocities for the omnidirectional mobile robot are set to $V_{0d} = [0.2 \quad 0.5 \quad 2]^T$ with the simulation time of 1 s. Figures 8–9 show the comparison between output velocities and the desired velocities. It is observed that the output is obtained precisely as the desired velocities input. Figure 10 shows the angular velocities of the wheel that are obtained from inverse kinematics solution. These velocities are required to produce the desired velocity, V_{0d}. Figure 11 shows the trajectory and the robot's pose. The omnidirectional mobile robot is observed moving towards a trajectory with the angle of 68.2 deg at a velocity of 0.54 m/s. At the same time, the mobile robot also is rotating at the speed of 2 rad/s. Based on this result, it is verified that the developed forward and inverse kinematics are working correctly. However, the current simulation did not consider any errors which may involve during actual robot.

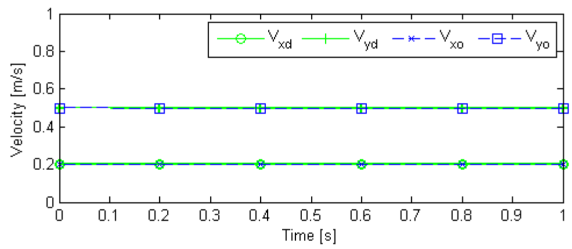


Figure 8: Desired and Response Velocity

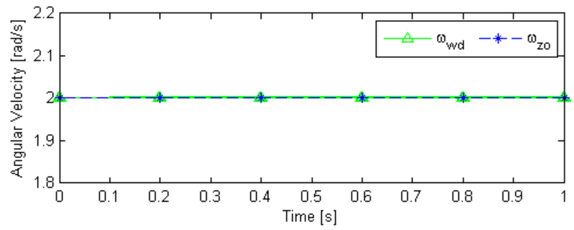


Figure 9: Desired and Response Angular Velocity

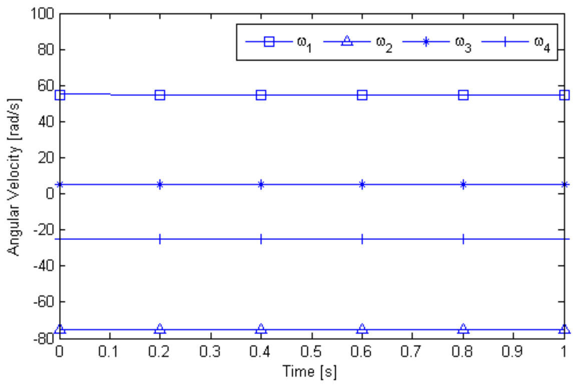


Figure 10: Required Angular Velocities of the Wheels

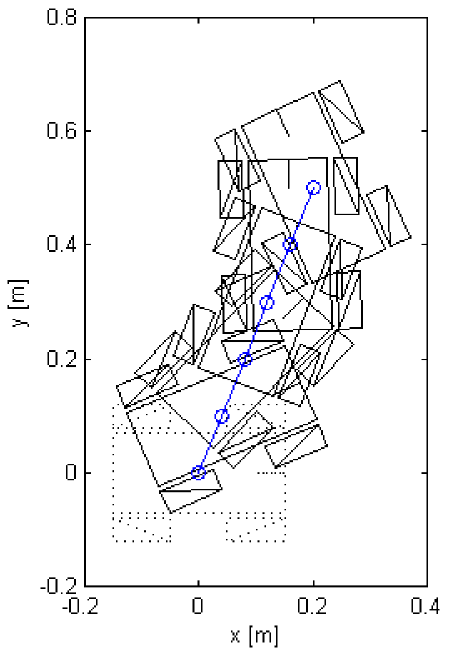


Figure 11: Trajectory of the Omnidirectional Mobile Robot

V. CONCLUSION

In this paper, we have successfully derived the mathematical modelling for forward and inverse kinematics of an omnidirectional mobile robot with screw wheels. The simulation shows that the mobile robot is capable to deliver an omnidirectional and holonomic motion. A few basic movements of the omnidirectional mobile robot were verified using the result presented in [9]. By using the combination of forward and inverse kinematics, the path planning for the omnidirectional mobile robot is easily achieved.

REFERENCES

[1] Int. Federation of Robotics. (2016, Oct 12). *Service Robotics: Sales up 25 percent – 2019 boom predicted* [Online]. Available: https://ifr.org/ifr-press-releases/news/service-robotics

[2] M. J. A. Safar, "Holonomic and Omnidirectional Locomotion Systems for Wheeled Mobile Robots: A Review," *Jurnal Teknologi*, vol. 77, no. 28, pp. 91–97, Sept. 2015.

[3] J. A. Morath, "Agricultural Machine," US Patent 635501, May 18, 1899.

[4] F. R. Burch, "Snow Motor Vehicle," US Patent 1431440 A, Nov. 27, 1920.

[5] Z. J. Hollis, "Steering Device for Amphibious Vehicle," US Patent 3395671 (A), Aug. 6, 1968.

[6] D. T. Beagley, S. J. Peraldini, "A Screw Propelled Vehicle," WO 2012162750 A1, June 3, 2011.

[7] K. Nagaoka, M. Otsuki, T. Kubota, and S. Tanaka, "Terramechanics-based Propulsive Characteristics of Mobile Robot Driven by Archimedean Screw Mechanism on Soft Soil," Proc. IEEE/RSJ Int. Conf. on Intell. Robots and Systems, pp. 4946–4951, 2010.

[8] J. Liedke, L. Winkler and H. Wörn, "An alternative locomotion unit for mobile modular self-reconfigurable robots based on archimedes screws," 2013 9th Int. Symp. on Mechatronics and its Applications (ISMA), Amman, 2013, pp. 1–6.

[9] D. Osinski and K. Szykiedans, "Small Remotely Operated Screw-propelled Vehicle," in *Progress in Automation, Robotics and Measuring Techniques, Advances in Intelligent Systems and Computing*, vol. 351, Springer, 2015, pp. 191–200.

[10] J. T. Freeberg, "A Study of Omnidirectional Quad-Screw-Drive Configurations for All-Terrain Locomotion," M.S. Thesis, Dept. of Mechanical Engineering, College of Engineering, Univ. of South Florida, 2010.

[11] M. O. Tătar, C. Popovici, D. Mândru, I. Ardelean and A. Pleşa, "Design and development of an autonomous omni-directional mobile robot with Mecanum wheels," 2014 IEEE Int. Conference on Automation, Quality and Testing, Robotics, Cluj-Napoca, 2014, pp. 1-6.