Mathematics Base for Navigation Mobile Robot Using Reachability Petri Net

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***Abstract*—Navigation is the journey that brings the precision and accuracy especially for the trip from one place to many destinations. Traveling mobile robot will be challenged when there are plenty of travel options and obstacles; it is necessary for special methods to deal with this. Petri net modeling is one method that can be applied in this navigation. As a model network graph, Petri net can be seen as a model of superior visuals for their token moves that define the process dynamics of a system. Other models that have been provided, such as flow charts, block diagrams or network topology, are using the method of mathematical reachability. Meanwhile Petri net obtained the navigation as a result of marking the value. The final results of this research is that Petri net can be used to obtain mobile robot for navigation.**

***Index Terms*—Petri Net; Marking; Mobile Robot; Navigation.**

I. INTRODUCTION

Robots are used in many industrial fields to make living easy and convenient. They are of great importance and interest in many applications, such as manufacturing, construction, exploration and medical surgery. Motion planning is one of the main tasks in intelligent control of mobile robot. The minimum expectation from robot, therefore, is its ability to plan its motion.

At present research on various algorithms for path planning of mobile robot is a hot topic. Path planning is used in many environments which may be very dangerous for people. To find a safe path in such environment, the path planning research is a fundamental requirement to make the mobile robot moves from a starting point to a goal point. To minimize the processing time and energy consumption, the path is required to be shortest in length.

Navigation is one of the key issues to be solved in the development of mobile robot technology; ability to move and navigate the capital must be owned by the robot to be able to run the task. There are many of the problems encountered in mobile robot navigation - the ability to travel from the beginning to the end, when the robot turns left and right and avoid collisions on the hitch with hitch static condition. The properties are whether the environment is static or dynamic. In static environment, there is no moving obstacle or object except the navigation robot, and in dynamics environment, there are moving objects or obstacles.

Many studies related to navigations including research in path planning. There are many problems in the path planning. Most algorithm models become more complex and longer calculation when the algorithms are applied to real mobile robot navigation. Even when using a mobile robot with sensors, it is difficult to obtain a complete scene information from start to end point. The complete information is needed for the strategy of the robot to plan the movement either forward, turn left or turn right.

Related navigation studies include path planning of line follower robot [1] and the use of computer-controlled line follower robots in public transport [2]. The paper described the navigation movement of the robot using a line follower will get into trouble if found complicate maneuvers for turning left and right. Research such as Optimal robotic cell scheduling with controllers using mathematically based timed Petri nets [3], Modeling and Simulation of Flowers Production Logistics based on Petri net [4] and Task-based Coordination of Flexible Manufacturing Cells using Petri Nets and ISA standards [5] discussed the control on the static robot using petri net. Goal-seeking Behavior-based Mobile Robot Using Fuzzy Particle Swarm Controller [6], Performance Evaluation of Various 2-D Laser Scanners for Mobile Robot Map Building and Localization [7] discusses mobile robot navigation using laser sensor. Research of motion coordination of AGV's in FMS using petri nets [8] validates the use Petri net for a robot to move using PLC programs. Simulation modeling method based on Petri net [9] describes model petri net for computer performance, A Petri net model for analysis, optimisation, and control of railway networks and train schedules [9] describes Petri net model for train scheduling.

Therefore, this research introduces a new framework under Petri net for performance evaluation of mobile robot navigation, by using the Discrete Event Systems modeling. This paper explains the robot modeling journey from the beginning to the its purpose, and the robot behavior in moving forward, turning left and turning right using Petri nets. The movements are demonstrated using a mathematical analysis of reachability petri net.

II. SUPPORTING THEORY

A. Petri Net Structure

Petri net structure is formed by 5 main elements that can be written in the following groups: PN = (**P**, **T**, **I**, **O**, **M**) [10]:

1. **P** = $\{p_1, p_2, p_3 \ldots p_n\}$ is a group of a *place* with $n \geq 0$.
2. **T** = $\{t_1, t_2, t_3 \ldots t_m\}$ is a group of a transition with $m \geq 0$.
3. **I** is *mapping input* **P** x **T** $\rightarrow \{0,1\}$ relating to the group of arrow originating from P to T. this arrow is also called as *input*.
4. **O** is *mapping output* **T** x **P** $\rightarrow \{0,1\}$ relating to the group of arrow originating from **T** to **P**. this arrow is also called as *output*.
5. **M** : **P** is *marking* that places the number of *token* in *place*.

Marking is the placement of the number of token in *place* cycle. The *token* is used for describing the running or execution in the Petri net, where the number or position of *token* can change and move when the Petri net is executed. The early marking of petri net is usually marked with **M**o. The above five elements are known as the mathematic topology of petri net. The five topologies can be written in mathematic forms.

B. *Description of Petri Net*

Petri net graphics consist of four components, namely:
1. Place (P) described with a cycle.
2. Transition (T) described with a bar.
3. *Token* described with a *dot* used for describing the movement of the *token* from one *place* to another *place* when the petri net is executed.
4. Arrow used as the connection between place and transition. This arrow is labeled in accordance with its weight (k) meaning that they are many parallel arrows. If there is a weight in an arrow, the weight of the arrow is usually assumed to be one.

Based on the foregoing, modeling systems with petri net are used on the following grounds:
1. Petri net has a mathematic base enabling us to analyze the system qualitatively and quantitatively.
2. Petri net can determine which relationship can be given priority from the synchronization or non-synchronization events.
3. Logic model obtained from the knowledge of how the system works to obtain an understandable model. It is also supported by the visualization of a clear graphic to properly observe the system even though it is in the form of simulation.
4. Knowing whether there is a deadlock in the system.

A simple or complex system analysis can be conducted by developing a petri net-based model.

C. *Algorithm Design*

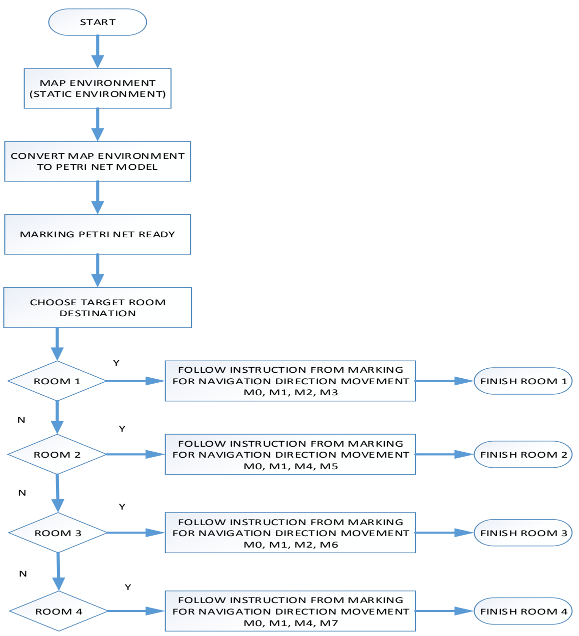


Figure 1: Algorithm design navigation mobile robot using petri net

Figure 1 describes the motion planning from start to the target (room). The travelling mobile robot obtained the map area that was converted into a Petri net and have resulting value marking of M0 to M7. The value of the marking can be interpreted to plan the movement of the mobile robot.

D. *Case Study*

A case study is developed with the following illustration models shown in Figure 2. The map has 4 paths and movements of the robot and three obstacles. The mobile robot's movement includes:
1. From Start go to the room 1
2. From Start go to the room 2
3. From Start go to the room 3
4. From Start go to the room 4

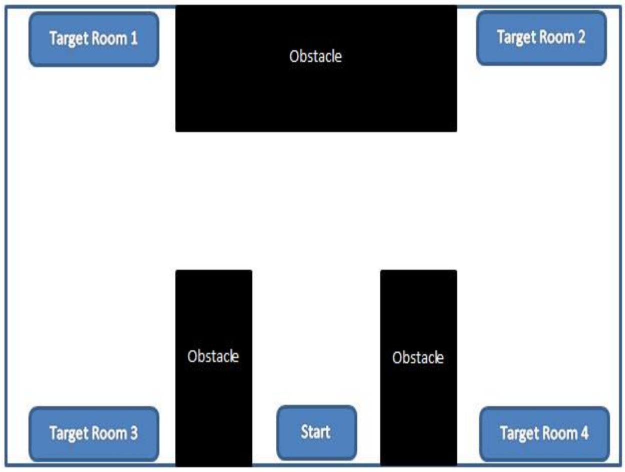


Figure 2: Map environment

1) *Design Analysis*

Firstly, the map environment is converted to petri net model.

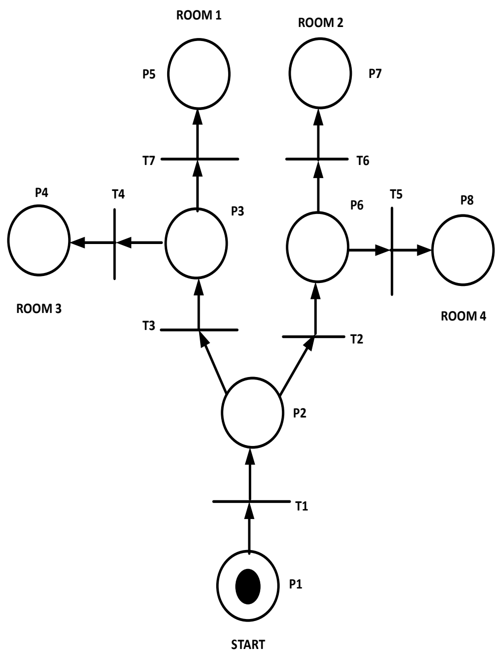


Figure 3: Petri net model

Figure 3 describes the position of the mobile robot from the starting position (P1) to room 1 (P5) or room 2 (P7) or room 3 (P4) or room 4 (P8).

Petri net modeling is obtained as follows:

Table 1
Information about Place and transition

Place	Information	Transition	Information
P1	Robot available at start and move straight	T1	Start to move forward
P2	Robot select move turn left or turn right	T2	Robot select turn right
P3	Robot select move turn left or turn right	T3	Robot select turn left
P4	Robot finish at room 3	T4	Robot select turn left
P5	Robot finish at room 1	T5	Robot select turn right
P6	Robot select move turn left or turn right	T6	Robot select turn left
P7	Robot finish at room 2	T7	Robot select turn right
P8	Robot finish at room 4		

E. Reachability Tree

The reachability tree can also be found by firing transition until all the state of the system have been reached. This yields the reachability tree of the net as shown in Figure 4. This procedure is performed to identify the reachable states.

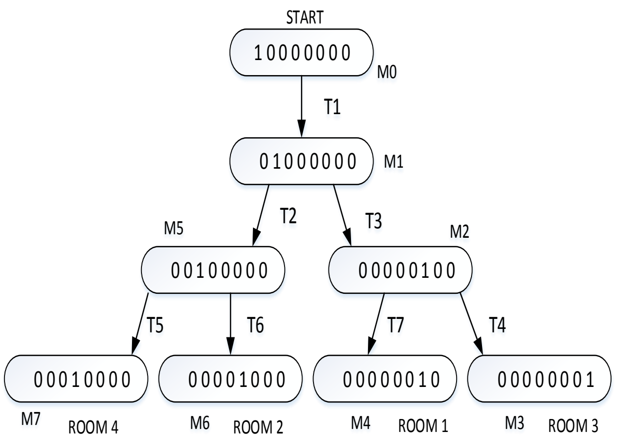


Figure. 4: Reachability tree form Petri net

The important thing of reachability is the marking value that navigates the movement of the mobile robot.

Table 2
Information about Marking

Marking	Value	Information
M0	10000000	Move forward
M1	01000000	Move turn left or turn right
M2	00100000	Move turn left or turn right
M3	00010000	Finish at room 3
M4	00001000	Finish at room 1
M5	00000100	Move turn left or turn right
M6	00000010	Finish at room 2
M7	00000001	Finish at room 4

F. Reachability Mathematic Analysis

The marking changes when a transition fires. A transition fires when its input states are marked. More formally, a transition tj is enabled in marking M if $M(pi) \sim I(pi, tj)$. When a transition tj fires, it results in a new marking, M', which occurs by removing $I(pi, tj)$ tokens from each of its input places, and adding $O(pi, tj)$ tokens to each of its output places. More formally, M' is reachable from M according to the equation:

$$M'(pi) = M(pi) + O(pi\, tj) - I(pi\, tj) \qquad (1)$$

The following is obtained from the figure:

1. There are 8 *places*
2. And 7 transitions

This analytical mathematics for illusion mobile robot move from:

START TO ROOM 1

$$I = \begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&1&0&0&0&0\\ 0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0\\ 0&0&0&0&1&1&0\\ 0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0 \end{pmatrix} \quad O = \begin{pmatrix} 0&0&0&0&0&0&0\\ 1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0\\ 0&0&0&1&0&0&0\\ 0&0&0&0&0&0&1\\ 0&1&0&0&0&0&0\\ 0&0&0&0&0&1&0\\ 0&0&0&0&1&0&0 \end{pmatrix}$$

WHEN T1 IS *FIRED*

$\mathbf{M}_k = \mathbf{M}_{k-1} + \mathbf{O}u_k - \mathbf{I}u_k \Rightarrow \mathbf{M}_1 = \mathbf{M}_{1-1} + (\mathbf{O} - \mathbf{I})$ T1

M1 = M0+(O-I)T1

$$\begin{pmatrix}0\\1\\0\\0\\0\\0\\0\\0\end{pmatrix} = \begin{pmatrix}1\\0\\0\\0\\0\\0\\0\\0\end{pmatrix} + \begin{pmatrix} -1&0&0&0&0&0&0\\ 1&-1&-1&0&0&0&0\\ 0&0&1&-1&0&0&-1\\ 0&0&0&1&0&0&0\\ 0&0&0&0&0&0&1\\ 0&1&0&0&-1&-1&0\\ 0&0&0&0&0&1&0\\ 0&0&0&0&1&0&0 \end{pmatrix} \begin{pmatrix}1\\0\\0\\0\\0\\0\\0\\0\end{pmatrix} \Rightarrow \begin{pmatrix}0\\1\\0\\0\\0\\0\\0\\0\end{pmatrix} = \begin{pmatrix}0\\1\\0\\0\\0\\0\\0\\0\end{pmatrix}$$

⇒ Token moves from P1 to P2 is **reachable**

⇒ Mean robot can move from M0 to M1

⇒ Robot move forward

WHEN T3 IS *FIRED*

M2 = M1+(O-I)T3

$$\begin{pmatrix}0\\0\\1\\0\\0\\0\\0\\0\end{pmatrix} = \begin{pmatrix}0\\1\\0\\0\\0\\0\\0\\0\end{pmatrix} + \begin{pmatrix} -1&0&0&0&0&0&0\\ 1&-1&-1&0&0&0&0\\ 0&0&1&-1&0&0&-1\\ 0&0&0&1&0&0&0\\ 0&0&0&0&0&0&1\\ 0&1&0&0&-1&-1&0\\ 0&0&0&0&0&1&0\\ 0&0&0&0&1&0&0 \end{pmatrix} \begin{pmatrix}0\\0\\1\\0\\0\\0\\0\\0\end{pmatrix} \Rightarrow \begin{pmatrix}0\\0\\1\\0\\0\\0\\0\\0\end{pmatrix} = \begin{pmatrix}0\\0\\1\\0\\0\\0\\0\\0\end{pmatrix}$$

⇒ Token moves from P2 to P3 is **reachable**

⇒ Mean robot can move from M1 to M2

⇒ Robot move turn right 90° and then move forward

WHEN T7 IS *FIRED*

M4 = M2+(O-I)T7

$$\begin{pmatrix}0\\0\\0\\0\\1\\0\\0\\0\end{pmatrix} = \begin{pmatrix}0\\0\\1\\0\\0\\0\\0\\0\end{pmatrix} + \begin{pmatrix} -1&0&0&0&0&0&0\\ 1&-1&-1&0&0&0&0\\ 0&0&1&-1&0&0&-1\\ 0&0&0&1&0&0&0\\ 0&0&0&0&0&0&1\\ 0&1&0&0&-1&-1&0\\ 0&0&0&0&0&1&0\\ 0&0&0&0&1&0&0 \end{pmatrix} \begin{pmatrix}0\\0\\0\\0\\0\\0\\0\\1\\0\end{pmatrix} \Rightarrow \begin{pmatrix}0\\0\\0\\0\\1\\0\\0\\0\end{pmatrix} = \begin{pmatrix}0\\0\\0\\0\\1\\0\\0\\0\end{pmatrix}$$

⇒ Token moves from P3 to P5 is **reachable**

⇒ Mean robot can move from M2 to M4

⇒ Robot move forward and get the goal room 1

G. Results

From the results of the mathematical calculations, the robot’s path can be drawn as follows.

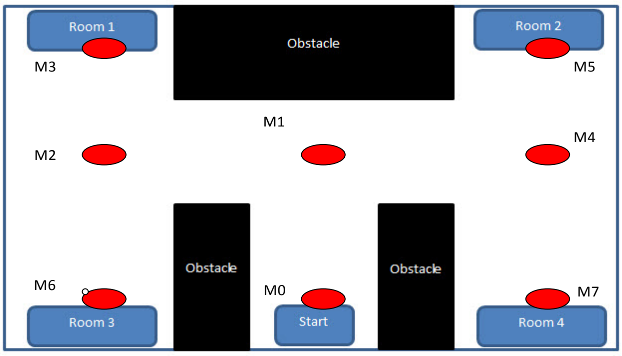
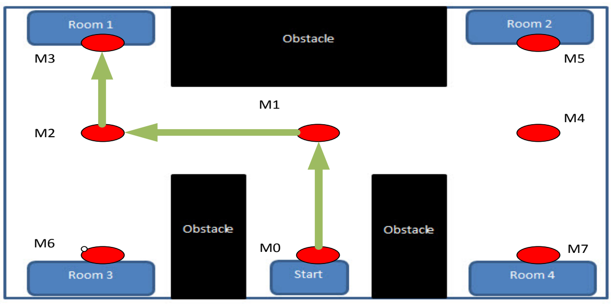


Figure 5: Illustrates placement of marking for navigation mobile robot

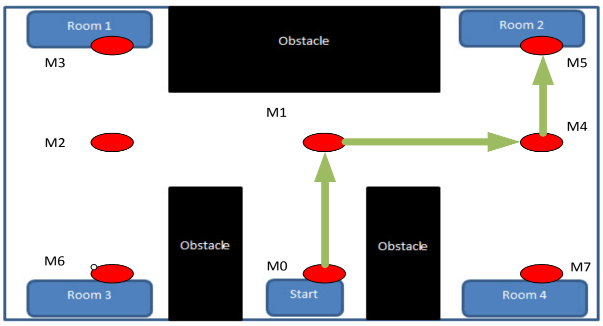
Figure 5 illustrates the placement of signs for the mobile robot navigation. The mobile robot behavior will follow marking direction. For example, when in position M0, mobile robot will move forward. Reaching position M1, the mobile robot will turn left. At marking M2, the mobile robot will turn right and arrived at M3 to finish the path to Room 1.

Figure 6 describes the mobile robot movements:

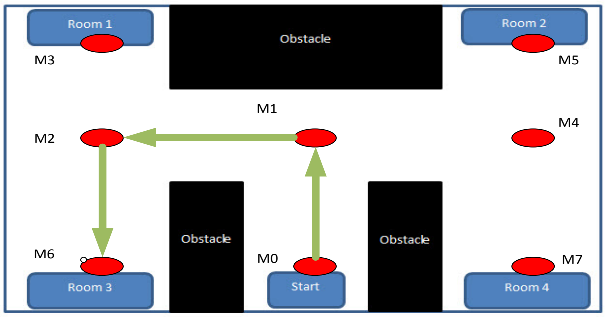
- From start to the Room 1, marking sequence M0, M1, M2 and M3 is generated.
- From start go to the Room 2 marking sequence M0, M1, M4 and M5 is generated.
- From start go to the room 3 marking sequence M0, M1, M2 and M6 is generated.
- From start go to the room 4 marking sequence M0, M1, M4 and M7 is generated.



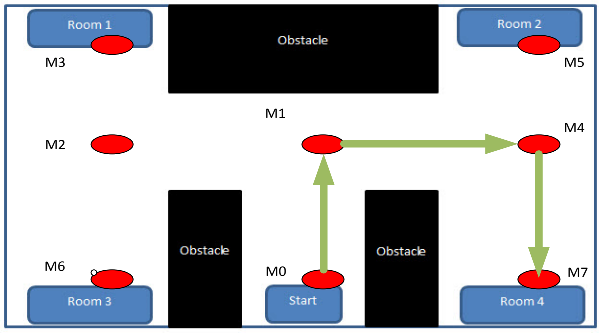
(a)



(b)



(c)



(d)

Figure 6: (a) Robot move from start to room 1; (b) Robot move from start to room 2; (c) Robot move from start to room 3; (d) Robot move from start to room 4

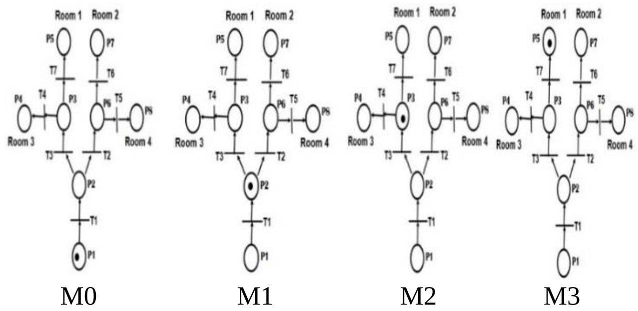


Figure 7: Illustration Robot move from start to room 1

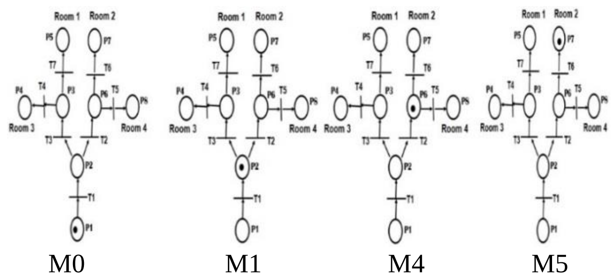


Figure 8: Illustration Robot move from start to room 2

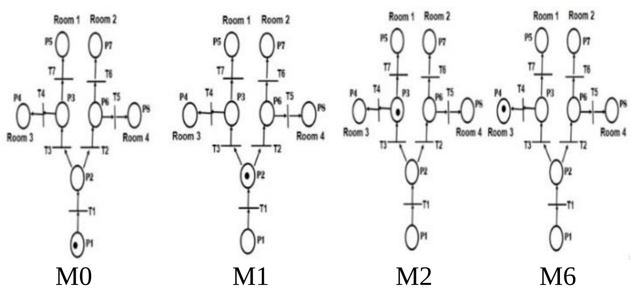
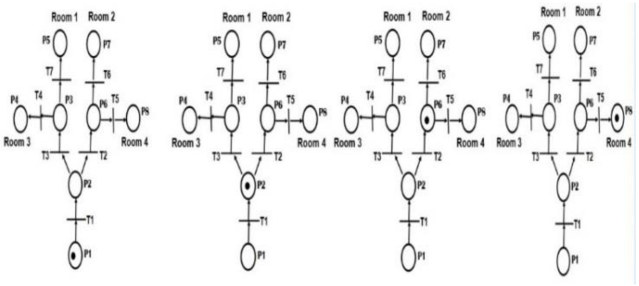


Figure 9: Ilustration of robot movement from start to Room 3



M0 M1 M4 M7

Figure 10: Illustration Robot move from start to Room 4

III. CONCLUSION

In this method, framework of mathematical reachability for petri net is proposed for mobile robot performance navigation. This method explains how modeling navigation and robot's path planning from the beginning to the target room. It also determined when the robot move forward, turning left or turning right.

The work presented in this research has many possible real-life applications. By using RFID sensor, writing and reading procedures to detect initial marking of M0, M1, M3, M4, M5, M6, M7 and M8 can be performed and executed as the mobile robot's movement.

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