Distributed Compressive Data Gathering Framework for Correlated Data in Wireless Sensor Networks

P. Dolas and D. Ghosh
Department of Electronics and Communication Engineering,
Indian Institute of Technology Roorkee, Roorkee (Uttarakhand) INDIA 247 667.
dolasdec@iitr.ac.in

***Abstract*—In wireless sensor network existing spatial (inter) and temporal (intra) correlation causes redundant data in the network. Exploiting the existing spatial (inter) and temporal (intra) correlation to effectively compress data thereby reducing redundancy to reduce data traffic in network is of prime concern. In this paper, we propose a 2-D distributed compressive data gathering framework to reduce redundancy in sensor network data. We also analyze the performance of the proposed scheme with random DCT and DFT measurement matrices on real data sets of sensor readings with different sparsity. Results indicate that high compression can be achieved with negligible mean square error in recovery from far fewer number of samples than the traditional Nyquist rate at the sink thereby enhancing network life time to large extent in large scale wireless sensor networks. Also, the recovery performance improves depending upon the sparsity measure and the measurement matrices used for compressing the data.**

***Index Terms*—Compressive Data Gathering; Data Compression; Joint Sparsity; Sparse Signal; Spatial Correlation; Temporal Correlation.**

I. INTRODUCTION

Wireless sensor networks are now-a-days commonly used in various applications such as physical environment monitoring [1], [2], habitat monitoring [3], health care monitoring [4], infrastructure monitoring, domestic application [5], etc. In all these applications, small sized, low cost, limited battery powered sensor nodes are deployed in ad-hoc fashion. Due to development in fabrication technology these sensor nodes have limited memory, computational power and a short range transceiver capability, in addition to electro-mechanical sensors; all of these fabricated in a matchbox size cassette.

Due to its small size and low cost, these sensor nodes can be deployed in any environmental scenario. Generally, these sensor nodes are deployed in adverse environment where once deployed it is very difficult to either repair or replace these sensor nodes. Therefore, sensor nodes are deployed densely in the area under observation. These sensor nodes are capable to reconfigure automatically. Due to auto-reconfigurable nature of sensor nodes, higher level of fault tolerance capacity may be achieved in wireless sensor networks which is caused either due to hardware or software failure or due to various environmental factors. Also, large area can be monitored with very less human resource.

Beside advantages, major challenges prevailing in wireless sensor network is the limited battery power available in sensor nodes and data redundancy. The redundant data transmission from resource constrained sensor nodes to sink via multi-hop communication consumes maximum amount of energy among various operations performed at any sensor node.

In regular monitoring applications, the closely spaced sensor nodes sense the same physical phenomenon. Therefore, these sensor readings collected at sink exhibit high spatial (inter) correlation. Similarly, due to high sensing frequency the readings collected by any particular sensor node over a short time span exhibit temporal (intra) correlation. Various schemes are proposed in the literature to reduce data traffic in wireless sensor network in order to preserve energy in wireless sensor network thereby enhancing overall network lifetime of the network.

Compressive Sensing [6] is a novel phenomenon for simultaneous sensing and compression for sparse signals with minimal computational and memory requirement. This phenomenon tends to preserve lot of energy with respect to traditional data gathering in which raw data collected by nodes is transmitted to sink and at sink various compression techniques (lossy or lossless) are applied to reduce the amount of data by discarding redundant data. Thus, lot of energy loss is incurred in transmitting data from source to sink. Compressive data gathering based on compressive sensing is an efficient way to compress data in sensor network locally at the sensor node. Compressive data gathering reduces data traffic which preserves lot of energy in wireless sensor network.

Compressive data gathering may be applied efficiently to sensor readings because due to high spatio-temporal correlation existing in sensor data causes data to be represented as sparse signal in some orthonormal basis. Therefore, in this paper, we propose to apply compressive sensing to achieve compression by exploiting both spatial and temporal correlation in real sensor data and study the performance of various measurement matrices on original signal reconstruction at sink.

The rest of the paper is organized as follows. Compressive data gathering and its mathematical formulation is discussed in Section II. Distributed compressive sensing and joint sparsity is discussed in Section III. Proposed distributed compressed data gathering framework is explained in Section IV. Simulations results in support of propose scheme are shown in Section V. Finally, conclusions are drawn in Section VI.

II. COMPRESSIVE DATA GATHERING

Compressive data gathering proposed by Luo et al. in [7] is being widely used in wireless sensor networks to reduce the data traffic thereby distributing energy consumption evenly among all the nodes in the network. Compressive data gathering uses compressive sensing [6] to compress the raw data in wireless sensor network because raw sensor data exhibit high spatial (inter) and temporal (intra) correlation.

Compressive data gathering reduces global energy consumption of the wireless sensor network without additional computational cost. Compression of raw data in wireless sensor network also reduces data communication control overhead in the network.

Compressive data gathering, thus, have twofold advantages over traditional multi-hop data gathering (baseline data gathering). First, it reduces energy overall energy consumption and second, it distributes the load equally throughout the network. These two advantages in turn increase the life time of the network.

Mathematically compressed sensing proposed for one dimensional signal is discussed here briefly. Consider an N-dimensional, discrete time, real-valued signal $\boldsymbol{x}$ such that $\boldsymbol{x} \in \mathbb{R}^N$. Using an $N \times N$ orthonormal basis $\Psi \in \mathbb{R}^N$, the signal $\boldsymbol{x}$ may be represented as

$$\boldsymbol{x} = \Psi \boldsymbol{s} \tag{1}$$

where $\boldsymbol{s}$ is an N-dimensional sparse vector. The signal $\boldsymbol{x}$ is said to have sparsity k iff there are k non-zero elements in $\boldsymbol{s}$, where $k \ll N$. Let, $\boldsymbol{y}$ be an M-dimensional measurement vector formed by M linear projections of the signal $\boldsymbol{x}$, where $M \ll N$. Thus, $\boldsymbol{y}$ may be represented as

$$\boldsymbol{y} = \Phi \boldsymbol{x} = \Phi \Psi \boldsymbol{s} = \boldsymbol{D} \boldsymbol{s} \tag{2}$$

where Φ is an $M \times N$ measurement matrix that is incoherent w.r.t Ψ and satisfies the Restricted Isometry Property (RIP), and $\boldsymbol{D}$= $\Phi\Psi$ is an $M \times N$ matrix known as Dictionary.

As $M \ll N$, solving the under determined system of equation of (2) requires additional constraints. In addition to this, the measurement matrix □□should satisfy the restricted isometry property (RIP) and incoherence w.r.t the orthonormal basis □□in which the signal $\boldsymbol{x}$ is k-sparse. The measurement matrix □□satisfies RIP of order k if there exists a restricted isometry constant $\delta_k \in (0,1)$ such that

$$(1-\delta_k)\left|\left|\boldsymbol{x}\right|\right|^2 \leq \left|\left|\Phi\boldsymbol{x}\right|\right|^2 \leq (1+\delta_k)\left|\left|\boldsymbol{x}\right|\right|^2 \tag{3}$$

where $||.||$ denotes the ℓ_2 norm or the Euclidean norm.

The recovery of $\boldsymbol{x}$ from $\boldsymbol{y}$ by solving the set of equations given by (2) requires additional constraints to be satisfied. These additional constraints may be minimization or maximization of certain parameters subjected to (2). The commonly used constraints given in literature are the minimization of norms such as the ℓ_0, ℓ_1 or ℓ_p norm. The Euclidean norm ℓ_2 minimization results in unique but non-sparse solution because ℓ_2 norm is a convex function and any convex function promises uniqueness. So, ℓ_2 norm is not an appropriate solution to the CS problem. In some literature, ℓ_0 norm minimization is considered. However, it leads to a non-convex optimization problem which is NP-hard to solve. Accordingly, sub-optimal algorithms have been developed to find an approximate solution. Various sub-optimal sparse signal recovery algorithms found in the literature are Orthogonal Matching Pursuit (OMP) [8], Randomized Orthogonal Matching Pursuit (RandOMP) [9], Simultaneous Orthogonal Matching Pursuit (SOMP) [10], OMP for multiple Measurement Vectors (OMPMMV) [11] and so on. As we move from ℓ_2 to ℓ_1 norm, we get a sparser solution as ℓ_1 norm is not strictly convex. The various algorithms based on ℓ_1 norm minimization are Basic Pursuit [12], The FOCul Underdetermined System Solver (FOCUSS) [13], etc.

Figure 1 depicts traditional multi-hop (baseline) transmission in chain type topology. In this scheme as shown the node close to sink is likely to run out of power soon as compared to the node located far from sink. In such scenario entire network will fail.

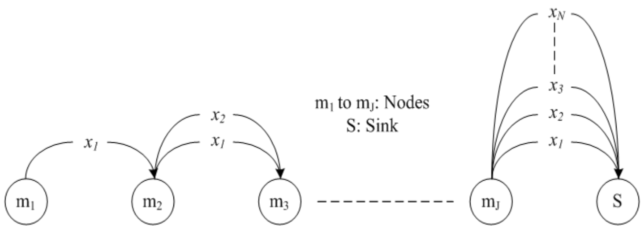

Figure 1: Baseline Data Gathering Scheme

Figure 2 depicts compressive data gathering scheme in chain type topology. In this scheme weighted sensor measurements are forwarded to the next node which is located in the limited transmission range of the node. Every intermediate node in the routing path acts as relay for other node. The intermediate nodes adds its weighted measurement to the weighted measurements received by its preceding node in routing path, finally weighted sum of sensor measurements is received at the sink.

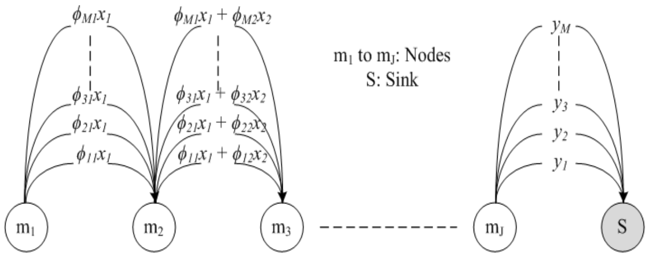

Figure 2: Compressive Data Gathering Scheme

Originally compressed sensing proposed for one dimensional signal vector can be effectively applied to sensor data gathering where each sensor reading is a one dimensional signal. Distributed compressive sensing is an extension of compressed sensing phenomenon to multiple vectors suitable for wireless sensor networks. Distributed compressed sensing is briefly discussed in next Section.

III. DISTRIBUTED COMPRESSIVE SENSING

Distributed compressed sensing was first proposed by Baron et al. in [14]. Distributed compressed sensing is based on the concept of joint sparsity. Joint sparsity can be stated as the ensemble of different signal vectors is sparse or can be sparsely represented in some orthonormal basis. Distributed compressive sensing exploits both spatial and temporal correlation in sensor readings. Three different joint sparsity models have been proposed for sensor networks. These are explained briefly.

A. Joint Sparsity Model – I

In this model each sensor measurement consist of two different components namely the *sparse common component* arising due to global environmental phenomenon and the *sparse innovation* which arises due to some local environmental phenomenon. Mathematically it can be represented as

$$\boldsymbol{x_j} = \boldsymbol{z_c} + \boldsymbol{z_j} \ \ \forall \ j = 1,2,3,\cdots J \tag{4}$$

where $\boldsymbol{z_c}$ is the common component to all $\boldsymbol{x_j}$ and has sparsity k_c in basis for e.g. effect of sun which is common on all sensor nodes in wireless sensor network deployed for environment monitoring and $\boldsymbol{z_j}$ is unique to every $\boldsymbol{x_j}$ having sparsity k_j in the same basis for e.g. effect of shade or wind which is unique to any particular sensor node in wireless sensor network deployed for environment monitoring. Therefore, $\boldsymbol{z_c}$ and $\boldsymbol{z_j}$ may be represented as

$$\boldsymbol{z_c} = \Psi \boldsymbol{s_c}, \quad ||\boldsymbol{s_c}||_{\ell_0} = k_c \tag{5a}$$

and

$$\boldsymbol{z_j} = \Psi \boldsymbol{s_j}, \quad ||\boldsymbol{s_j}||_{\ell_0} = k_j \tag{5b}$$

where $||.||_{\ell_0}$ denotes the ℓ_0 norm which is a measure of cardinality i.e. number of non-zero elements in the vector.

B. Joint Sparsity Model – II

In this model every sensor measurement is constructed from same sparse set of basis vectors, but with different coefficient values. Mathematically expressed as

$$\boldsymbol{x_j} = \Psi \boldsymbol{s_j} \ \ \forall \ j = 1,2,3,\cdots J \tag{6}$$

where $\boldsymbol{s_j}$ have cardinality equal to k. This model may be assumed as a special case of Joint Sparsity Model – I with $k_c = 0$ and $k_j = k$.

C. Joint Sparsity Model – III

In this model each sensor measurement consist of two different components similar to that described in Joint Sparsity Model – I but the common component no longer remains sparse. Thus, in this model the signal consists of *non-sparse common component* and *sparse innovation.* Mathematically it can be represented as

$$\boldsymbol{x_j} = \boldsymbol{z_c} + \boldsymbol{z_j} \ \ \forall \ j = 1,2,3,\cdots J \tag{7}$$

with

$$\boldsymbol{z_c} = \Psi \boldsymbol{s_c}, \tag{8a}$$

and

$$\boldsymbol{z_j} = \Psi \boldsymbol{s_j}, \quad ||\boldsymbol{s_j}||_{\ell_0} = k_j \tag{8b}$$

where $\boldsymbol{z_c}$ is not necessarily sparse.

IV. Performance Analysis of proposed scheme

Sensor data in a WSN is highly correlated and exhibit both spatial and temporal correlations. Compressive data gathering can be used to exploit both spatial correlation and temporal correlations to achieve high compression. In this paper, we propose to exploit spatial as well as temporal correlation, similar approach was proposed in [15], but in this paper we also compare the reconstruction performance using different measurement matrices *viz.:* DCT and DFT matrices in terms of MSE of the proposed scheme and percentage of exact recovery of measurements. For recovery of original sensor measurement at sink we have used an extension of FOCUSS recovery algorithm to multiple measurement vectors termed as MFOCUSS [16].

Let $\boldsymbol{X} = [x_{nj}]$ be the $N \times J$ matrix composed of sensor readings collected by J sensor nodes over N time instances, x_{nj} is j^{th} sensor reading at n^{th} time instance. The, rows of $\boldsymbol{X}$ exhibit spatial correlation and the columns of $\boldsymbol{X}$ exhibit temporal correlation. As rows are spatially correlated, there exists a $J \times J$ orthonormal basis $\Theta \in \mathbb{R}^J$ in which all rows of $\boldsymbol{X}$ are sparse with support size of k_s such that $k_s \ll J$. Similarly, columns are temporally correlated so there exists an $N \times N$ orthonormal basis $\Psi \in \mathbb{R}^N$ in which all columns of $\boldsymbol{X}$ are sparse with support size of k_t such that $k_t \ll N$. This scenario is similar to JSM-II described in Section III. Therefore, we may write,

$$\boldsymbol{X} = \Psi \boldsymbol{S} \Theta^T \tag{9}$$

where $\boldsymbol{S}$ is sparse matrix representation of $\boldsymbol{X}$.

Since $\boldsymbol{X}$ is sparse, we choose a measurement matrix Φ of dimension $M \times N$ where $M \ll J$ and is incoherent with an orthonormal basis Ψ for temporally correlated data. Similarly, we choose a measurement matrix Ω of dimension $L \times J$ where $L \ll N$ and is incoherent with an orthonormal basis Θ for spatially correlated data. Therefore,

$$\boldsymbol{Y} = \Phi \boldsymbol{X} \Omega^T = \Phi \Psi \boldsymbol{S} \Theta^T \Omega^T = \boldsymbol{D_t S D_s} \tag{10}$$

where $\boldsymbol{Y}$ is a matrix of dimension $M \times L$ formed by linearly projected measurements of $\boldsymbol{X}$. $\boldsymbol{D_t}$ and $\boldsymbol{D_s}^{\boldsymbol{T}}$ are the over-complete dictionaries for temporal and spatial correlated data respectively. As observed, NJ number of samples are compressed to only ML number of samples by applying DCS.

In this paper, we propose to evaluate the performance of DCS applied to jointly sparse signals in terms of mean square error (MSE) of the recovered signal with varying compression ratio (CR) using different random measurement matrices *viz*: DCT and DFT matrices. MSE is chosen as the metric for performance evaluation as exact recovery of sparse signal from an over complete dictionary is NP-hard to solve as discussed in Section II. The MSE is defined as

$$MSE \triangleq \frac{1}{NJ} \sum_{n=1}^{N} \sum_{j=1}^{J} \frac{(x_{nj} - \hat{x}_{nj})^2}{x_{nj}^2} \tag{11}$$

where $\hat{x}_{nj}$ is the reconstructed j^{th} sensor node measurement at n^{th} time instance. The compression ratio (CR) is defined as the ratio of number of uncompressed samples to that of compressed samples.

$$CR \triangleq \frac{NJ}{ML} \tag{12}$$

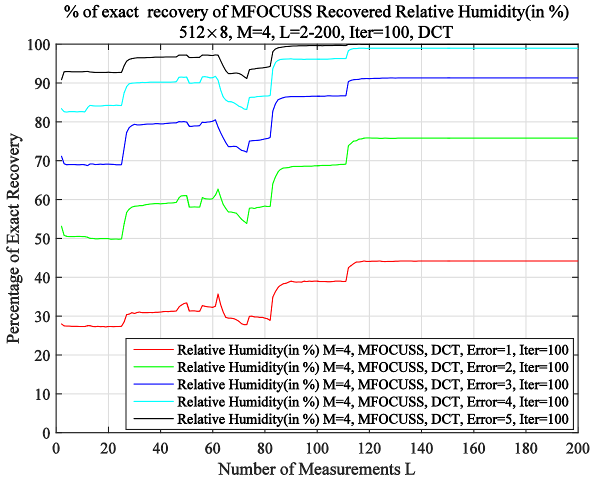

Figure 3 : Relative Humidity (in %) using DCT measurement matrix.

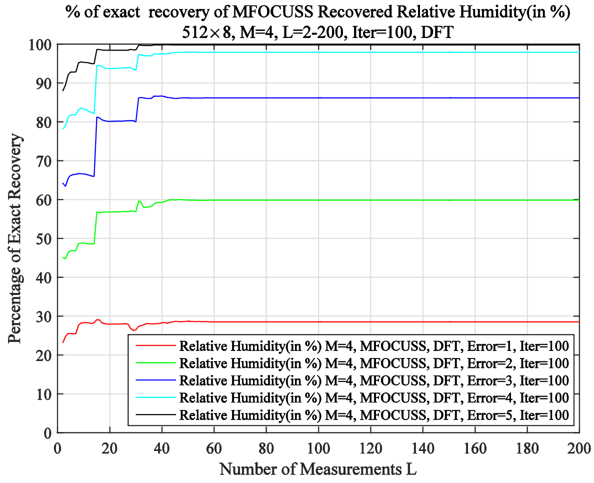

Figure 4 : Relative Humidity (in %) using DCT measurement matrix.

V. Simulation Results

In our simulations, we used real set of sensor readings of environmental monitoring data taken from LUCE deployment of EPFL sensor scope WSN database available at [2]. This database consists of few key environmental quantities with high spatial and temporal correlation. In our work, we used readings of relative humidity (in %) recorded by $J = 8$ closely located SNs at an approximately constant distance d from the sink at $N = 512$ successive time instances. The readings were spatially and temporally correlated and were sparsely representable in Hadamard basis Ψ and Θ. The temporal sparsity of signal $k_t = 3 \ll N = 512$ and spatial sparsity of signal $k_s = 1 \ll J = 8$. The two measurement matrices Φ and Ω are randomly generated. The number of measurements M is fixed at 4 while L is varied from 2 to 200. Thus, compression ratio ranges from 512:1 to 5.12:1 is achieved. For recovery, we used MFOCUSS algorithms.

The performance of the proposed 2-D compressed sensing is evaluated for two different choices of measurement matrices *viz.* DCT and DFT matrices. The entries of DCT measurement matrix Φ are randomly sampled rows of $N \times N$ DCT matrix, and the entries of DCT measurement matrix Ω are randomly sampled L rows and J the columns of DCT matrix. The entries of $N \times N$ DCT matrix are given by

$$\alpha_{p,q} = \begin{cases} \frac{1}{\sqrt{N}} & \forall\, p = 0, 0 \le q \le N-1 \\ \sqrt{\frac{2}{N}} \cos\frac{\pi(2q+1)p}{2N} & \forall 1 \le p \le N-1, 0 \le q \le N-1 \end{cases} \qquad (13)$$

Similarly, DFT measurement matrices are generated by randomly sampling rows of $N \times N$ DFT matrix $\boldsymbol{W} \in \mathbb{C}^N$, given by

$$\boldsymbol{W} = \frac{1}{\sqrt{N}} \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & \omega & \cdots & \omega^{N-1} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \omega^{N-1} & \cdots & \omega^{(N-1)(N-1)} \end{bmatrix} \qquad (14)$$

where $\omega = e^{-2\pi i/N}$ is the primitive n^{th} root of unity.

The recovery algorithm used in our simulations is MFOCUSS which is an extension of FOCUSS algorithm to multiple measurement vectors (MMV). It is based on principle of weighted least square that minimizes the weighted ℓ_2 norm of $\boldsymbol{x}$.

The simulation results depicting percentage of exactly recovered signal vs Number of measurement (L) using DCT measurement matrix is shown in Figure 3. Similarly, Figure 4 depicts percentage of exactly recovered signal vs Number of measurement (L) using DFT measurement matrix. RMSE plot of both DCT and DFT matrix is shown in Figure 5.

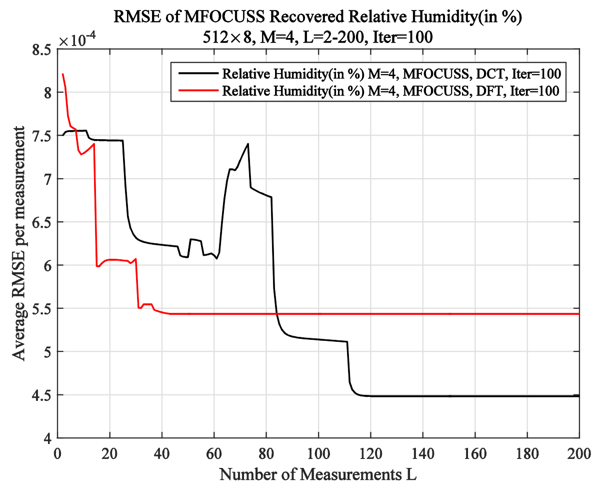

Figure 5 : RMSE using DCT and DFT measurement matrices of relative humidity.

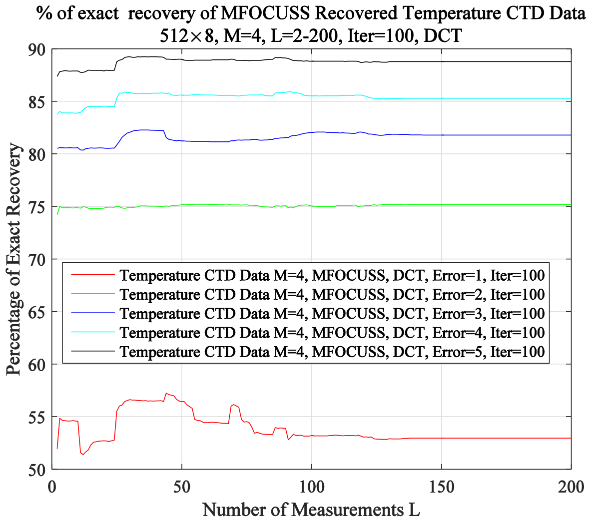

Figure 6 : Temperature (in ˚C) of CTD sensors using DCT measurement matrix.

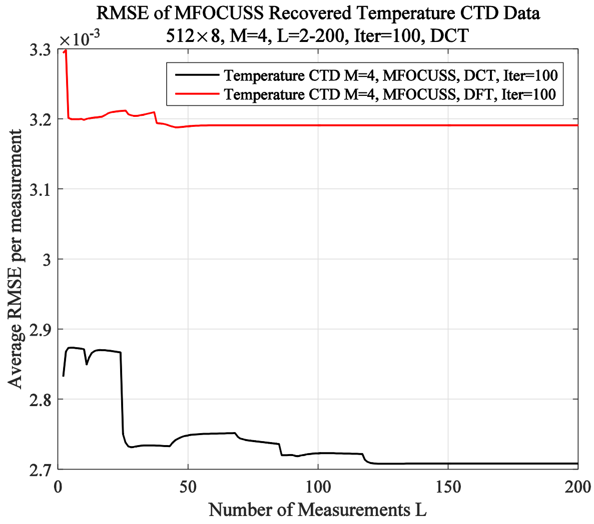

Figure 8 : RMSE using DCT and DFT measurement matrices of temperature measured using CTD sensors.

Further, this proposed scheme was also tested with another real sensor data readings obtained from National Data Buoy Center's (NDBC), Tropical atmosphere ocean (TAO) deployment in Pacific Ocean [17], for monitoring of Equatorial Pacific Ocean Climate Studies (EPOCS) which measures various parameters such as temperature, salinity, humidity, wind direction, wind speed and many more marine geology parameters. In our simulations we used readings of temperature of ocean water (in ˚C) and salinity of ocean water measured using CTD (conductivity, temperature and depth) sensors.

Again, we used readings from $J = 8$ closely spaced sensor nodes for $N = 512$ time instances. These readings were spatially and temporally correlated. The Oceanic temperature readings exhibited temporal correlation with sparsity $k_t = 7 \ll N = 512$ and spatial correlation with sparsity $k_s = 1 \ll J = 8$. Now, we used DCT and DFT measurement matrices for compressing the effective data to be transmitted to sink and M-FOCUSS algorithm is used to recover the data.

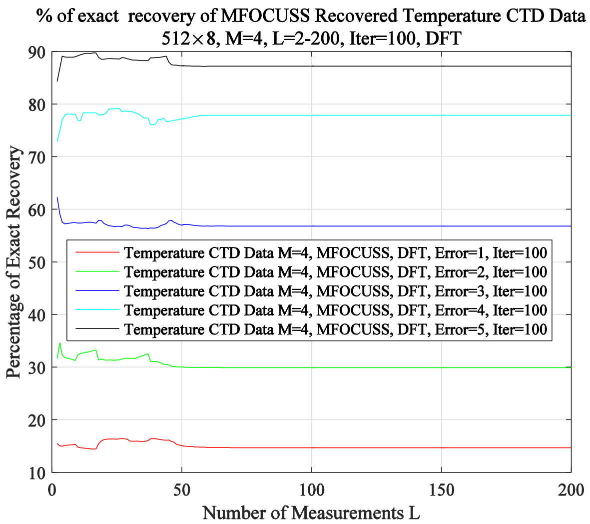

Figure 7 : Temperature (in ˚C) of CTD sensors using DFT measurement matrix.

The simulation results depicting percentage of exactly recovered signal vs Number of measurement (L) using DCT measurement matrix for oceanic temperature readings is shown in Figure 6. Similarly, Figure 7 depicts percentage of exactly recovered signal vs Number of measurement (L) using DFT measurement matrix. RMSE plot of both DCT and DFT matrix is shown in Figure 8 for oceanic temperature readings.

The proposed scheme is also applied to readings of salinity measurements of Pacific Ocean obtained from CTD sensors. The salinity readings exhibited temporal correlation with sparsity $k_t = 1 \ll N = 512$ and spatial correlation with sparsity $k_s = 1 \ll J = 8$. Now, we used DCT and DFT measurement matrices for compressing the effective data to be transmitted to sink and M-FOCUSS algorithm is used to recover the data.

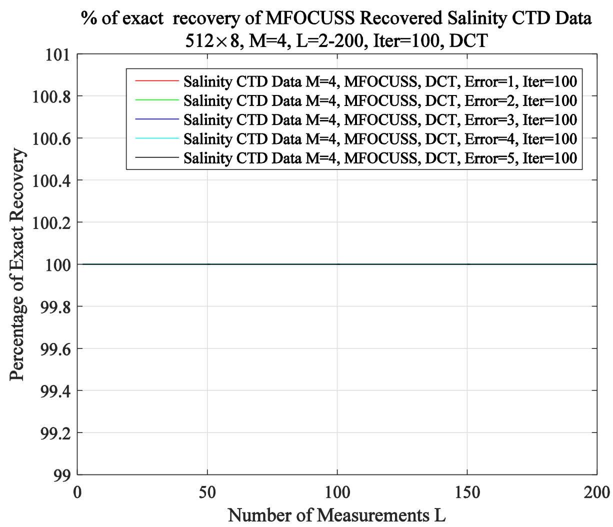

Figure 9 : Salinity by CTD sensors using DCT measurement matrix.

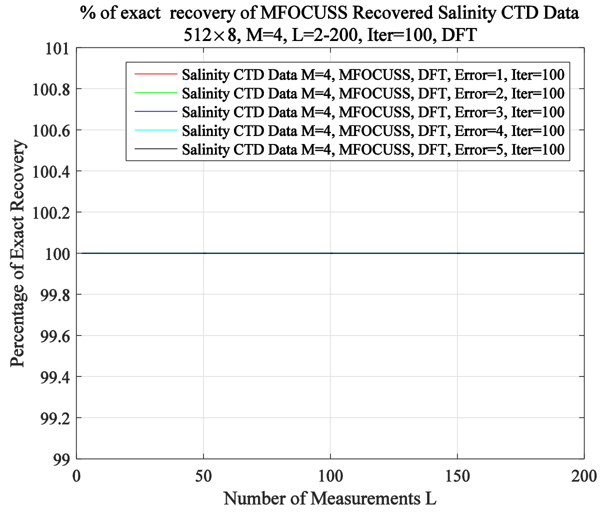

Figure 10 : Salinity by CTD sensors using DFT measurement matrix.

The simulation results depicting percentage of exactly recovered signal vs Number of measurement (L) using DCT measurement matrix for salinity readings is shown in Figure 9. Similarly, Figure 10 depicts percentage of exactly recovered signal vs Number of measurement (L) using DFT measurement matrix. RMSE plot of both DCT and DFT matrix is shown in Figure 11 for oceanic temperature readings.

VI. Conclusion

Our simulation results indicate that as the number of measurements increases percentage of exactly recovered samples increases with negligible RMSE. In our simulations, for wireless sensor network with just 8 sensor nodes, energy preservation as high as 99.8% is achieved with compression ratio of 512:1 which indicates that in large scale wireless sensor network our proposed scheme helps to achieve huge energy conservation. Thus, enhancement in overall network lifetime proportional to energy saving is also achieved. Results also indicate that as sparsity decreases the better is the recovery with negligible RMSE and DFT measurement matrix gives better recovery performance for signal with less sparsity.

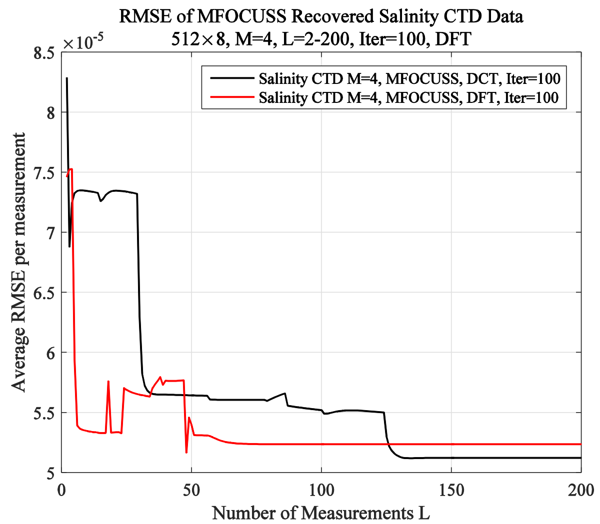

Figure 11 : RMSE using DCT and DFT measurement matrices of salinity measured using CTD sensors.

References

[1] "CitySee, Wu Xi City, China." [Online]. Available: http://www.greenorbs.org/all/citysee.htm.

[2] "Sensor Scope EPFL Switzerland." [Online]. Available: http://lcav.epfl.ch/sensorscope-en.

[3] "Habitat Monitoring on Great Duck Island." [Online]. Available: http://www.greatduckisland.net.

[4] L. Schwiebert, S. K. S. Gupta, and J. Weinmann, "Research challenges in wireless networks of biomedical sensors," Proc. 7th Annu. Int. Conf. Mob. Comput. Netw. - MobiCom '01, vol. 9, pp. 151–165, 2001.

[5] M. Srivastava, R. Muntz, and M. Potkonjak, "Smart kindergarten," Proc. 7th Annu. Int. Conf. Mob. Comput. Netw. - MobiCom '01, pp. 132–138, 2001.

[6] D. L. Donoho, "Compressed sensing," IEEE Trans. Inf. Theory, vol. 52, no. 4, pp. 1289–1306, 2006.

[7] C. Luo, F. Wu, J. Sun, and C. W. Chen, "Compressive data gathering for large-scale wireless sensor networks," Proc. 15th Annu. Int. Conf. Mob. Comput. Netw. (MobiCom '09), no. 800, p. 145, 2009.

[8] G. Davis, S. G. Mallat, and M. Avellaneda, "Adaptive Greedy Approximations," Constr. Approx., vol. 13, no. 1, pp. 57–98, 1997.

[9] M. Elad and I. Yavneh, "A Plurality of Sparse Representations is Better than the Sparsest One Alone," IEEE Trans. Inf. Theory, vol. 55, no. 10, pp. 1–14, 2009.

[10] J. A. Tropp, A. C. Gilbert, and M. J. Strauss, "Algorithms for simultaneous sparse approximation. Part I: Greedy pursuit," Signal Processing, vol. 86, no. 3, pp. 572–588, 2006.

[11] J. Chen and X. Huo, "Theoretical Results on Sparse Representations of multiple Measurement vectors," IEEE Trans. Signal Process., vol. 54, no. 12, pp. 4634–4643, 2006.

[12] S. S. Chen, D. L. Donoho, and M. A. Saunders, "Atomic Decomposition by Basis Pursuit," SIAM J. Sci. Comput., vol. 2, no. 1, pp. 158–178, 2003.

[13] I. F. Gorodnitsky and B. D. Rao, "Sparse signal reconstruction from limited data using FOCUSS: A re-weighted minimum norm algorithm," IEEE Trans. Signal Process., vol. 45, no. 3, pp. 600–616, 1997.

[14] D. Baron, M. F. Duarte, M. B. Wakin, S. Sarvotham, and R. G. Baraniuk, "Distributed Compressive Sensing," Arxiv, vol. abs/0901.3, p. 42, 2009.

[15] C. Agrawal and D. Ghosh, "Distributed Compressive Data Gathering in Wireless Sensor Networks," in International Conference on Signal Processsing, 2012, pp. 2110–2115.

[16] S. F. Cotter, B. D. Rao, and K. Kreutz-Delgado, "Sparse solutions to linear inverse problems with multiple measurement vectors," IEEE Trans. Signal Process., vol. 53, no. 7, pp. 2477–2488, 2005.

[17] "National Data Buoy Center." [Online]. Available: http://tao.ndbc.noaa.gov/