A Hybrid NLMS/RLS Algorithm to Enhance the Beamforming Process of Smart Antenna Systems

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***Abstract*—Adaptive beamforming algorithm is a signal processing technique used by smart antenna system to steer the main beam toward the desired signal direction and cancel the interfering signals of other directions. This paper proposes a hybrid non-blind beamforming algorithm that combines the Normalized Least Mean Square (NLMS) algorithm and the Recursive Least Square (RLS) algorithm to exploit the advantages of both algorithms and avoid their drawbacks. The hybrid NLMS/RLS algorithm solves many problems of the other non-blind algorithms. A comparative study between the proposed algorithm and other non-blind beamforming algorithms is introduced to illustrate the points of strength of the proposed algorithm. The hybrid NLMS/RLS algorithm is applied to different types of patch array antenna with resonance frequency 10GHz to demonstrate the performance of the proposed algorithm to each array antenna type.**

***Index Terms*—Adaptive Beamforming; Hybrid Algorithm; Normalized Least Mean Square; Recursive Least Square; Smart Antenna.**

I. INTRODUCTION

A smart antenna system is an integration between array antenna and digital signal processing techniques. The signal processing methods are divided into two processes; a direction of arrival (DOA) process and an adaptive beamforming process. The DOA algorithm computes the directions of arrival of the incoming signals. Then, the adaptive beamforming algorithm is used to choose the convenient weights of each array element to extract the desired source signal from the acquired data of antenna array while canceling interference and noise [1], [2]. The adaptive beamforming algorithms can be classified into blind and non-blind algorithms [3], [4]. In this paper, we use non-blind algorithms which need training phase before to be put in the testing phase.

The most popular non-blind algorithms used for beamforming the radiation pattern of smart antenna are the Least Mean Square (LMS) [5-10], Normalized Least Mean Square (NLMS) [11-15], Sample Matrix Inversion (SMI) [5-7, 16-18], Recursive Least Squares (RLS) [9], [10], [19], [20] and hybrid Least Mean Square Algorithm / Sample Matrix Inversion (LMS/SMI) [4], [17], [21]. Each of these algorithms has strengths and weaknesses as demonstrated in previous work [22], [23]. This paper introduces a new hybrid algorithm that solves many problems of the previous algorithms such as achieving lower side-lobe level, deeper nulls and minimum MSE of the output signal. The proposed algorithm combines Normalized Least Mean Square and Recursive Least Square in cascade and it is called (NLMS/RLS).

The performance of the hybrid NLMS/RLS algorithm will be demonstrated and compared to the other non-blind beamforming algorithms under varying the number of radiating elements and different noise levels. The performance of each algorithm is measured in terms of Half-Power Beam Width (HPBW), maximum Side-Lobe Level (SLL), nulls depth, convergence rate, beamforming stability and Mean Square Error (MSE) of the output signal. Also, the hybrid NLMS/RLS is applied on different types of patch array antenna with resonance frequency 10GHz to demonstrate the performance of the proposed algorithm with these antenna types. The 10 GHz frequency band locates in X-band that is used in modern radar applications especially the military requirements, where the shorter wavelengths of the X-band allow high-resolution imaging radars for target identification and classification.

II. SMART ANTENNA MODELLING

The array factor of spherical angle $AF(\theta)$ for a linear array of N radiating elements and d is the displacement between elements as shown in Figure 1 is given by the following equation [22]:

$$AF(\theta) = \sum_{n=0}^{N-1} w_n e^{j\left(\frac{-(N-1)}{2}+n\right)kd\sin(\theta)} \tag{1}$$

where:
- w_n : Weight of n^{th} radiating element
- k : Wave number ($2\pi/\lambda$)
- λ : Wavelength of the incident wave

Assuming number of incident wave signals M that are incident on the linear array from different directions ($\theta_0, \theta_1, \ldots, \theta_{M-1}$) [3], [23]. The incident signals on n^{th} antenna radiating element are given by the following equation:

$$x_n(t) = \sum_{i=0}^{M-1} S_i(t) e^{j\left(\frac{-(N-1)}{2}+n\right)kd\sin(\theta_i)} + n_n(t) \tag{2}$$

where:
- $S_0(t)$: Desired signal
- $S_{1\to(M-1)}$: Interfering (unwanted) signals
- $n_n(t)$: Additive noise signal at n^{th} element

The array output can be given as follows [24]:

$$y(k) = \overline{w}^{H} \cdot \overline{x}(k) \tag{3}$$

where:

$\overline{w}$: Array weights vector
$\overline{x}(k)$: Vectors of inputs to the array

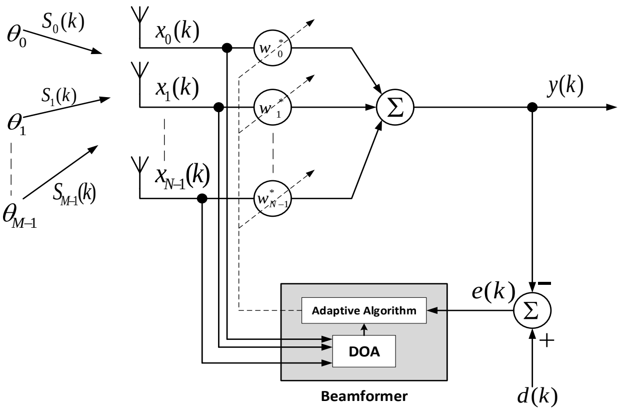

Figure 1: The adaptive beamforming array antenna block diagram [25]

III. ADAPTIVE BEAMFORMING ALGORITHMS

Adaptive beamforming techniques are digital signal processing approaches used to shape the radiation beam in order to steer the main beam toward the wanted signal and reject the interfering and noise signals. The weights are calculated by reducing the error difference between the desired signal and the array output until the weights achieve their optimum values. In this section, the NLMS, RLS and proposed hybrid NLMS/RLS are discussed.

A. Normalized Least Mean Square (NLMS)

The NLMS algorithm is an alternative of the LMS algorithm that solves one of its significant disadvantages, that are, the sensitivity to the inputs scaling. This sensitivity makes a learning rate value selection a difficult task. The stability of the LMS convergence depends on the learning rate value. These drawbacks of the LMS algorithm have been solved by the NLMS algorithm by normalizing the power of the input [11-15], where it updates the connection weights as follows [26]:

$$\overline{w}(k+1) = \overline{w}(k) + \frac{\mu_{opt} e^{*}(k)\overline{x}(k)}{\gamma + \overline{x}^{H}(k)\overline{x}(k)} \tag{4}$$

$$e(k) = d(k) - \overline{w}^{H}(k)\overline{x}(k) \tag{5}$$

where:

$e(k)$: Error signal
μ_{opt} : Optimal learning rate for the NLMS algorithm that is equal to 1 and it is independent of the inputs
γ : Small positive value [11]

B. Recursive Least Square (RLS)

The SMI algorithm is a block-adaptive strategy that gives a faster convergence rate. Although the SMI algorithm has faster convergence rate than the LMS algorithm, many problems can exist due to the potential singularities and computational complexity that is related with computing correlation matrix inversion [16-18]. This problem grows up with increasing the acquisition block size K of incoming signals [9], [10]. Therefore, we can recursively calculate the required correlation matrix and the required correlation vector by using the RLS algorithm as in [19], [20], [24]. The RLS algorithm saves computational complexity with fast conversion rate by computing the correlation matrix inverse iteratively instead of the directly computing. The weights of RLS algorithm are updated as follows [27]:

$$\overline{w}(k) = \overline{w}(k-1) + \overline{g}(k)\left(d^{*}(k) - \overline{x}^{H}(k)\overline{w}(k-1)\right) \tag{6}$$

where:

α : Forgetting factor and it is a positive constant value in range $0 > \alpha \geq 1$
$\overline{g}(k)$: Gain vector and it is defined as:

$$\overline{g}(k) = \frac{\alpha^{-1}\hat{R}_{xx}^{-1}(k-1)\overline{x}(k)}{1 + \alpha^{-1}\overline{x}^{H}(k)\hat{R}_{xx}^{-1}(k-1)\overline{x}(k)} \tag{7}$$

where correlation matrix inverse can be computed iteratively as:

$$\hat{R}_{xx}^{-1}(k) = \alpha^{-1}\hat{R}_{xx}^{-1}(k-1) - \alpha^{-1}\overline{g}(k)\overline{x}^{H}(k)\hat{R}_{xx}^{-1}(k-1) \tag{8}$$

The hybrid LMS/SMI algorithm is a combination of the LMS and SMI algorithms together that is another way to avoid their defects. The weights of LMS algorithm are initialized arbitrarily, hence it takes a large time to reach the optimum weights. Instead of random weights initialization, the weights of LMS algorithms are initialized by SMI algorithm of small (K) block length calculation in [4], [17], [21].

C. Proposed Hybrid (NLMS/RLS) Algorithm

The Hybrid NLMS/RLS is a proposed non-blind adaptive beamforming algorithm that solves many problems of the previous non-blind algorithms. It is a combination of the NLMS and the RLS algorithms together to exploit the merits of both algorithms and avoids their defects. The most considerable of these problems are SLL, nulls depth, and MSE of the output signal. Figure 2 shows the flow chart of the NLMS/RLS algorithm procedures.

In the NLMS/RLS algorithm, the adapting of the weights values divide into two intervals. Firstly, the weights are initialized and updated by the NLMS algorithm until the absolute of error value $e(k)$ in Equation (5) reach the error threshold limit e_{th} as illustrated in Figure 3. The error threshold limit e_{th} is depending on the noise value on the received signal, mutual coupling between elements and also the errors on the system that cause another noise [2], [23], [18]. The error threshold limit e_{th} should be increased at low SNR values. Secondly, the RLS algorithm handles the weights updating from the error threshold limit e_{th} to the convergence limit as illustrated in Figure 3. In the RLS algorithm interval, the weights should be initialized by the last weights values of the NLMS algorithm interval as in Equation (9).

$$\overline{w}_{RLS}(k) = \overline{w}_{NLMS}(k-1) \qquad \text{at } |e(k)| \leq e_{th} \tag{9}$$

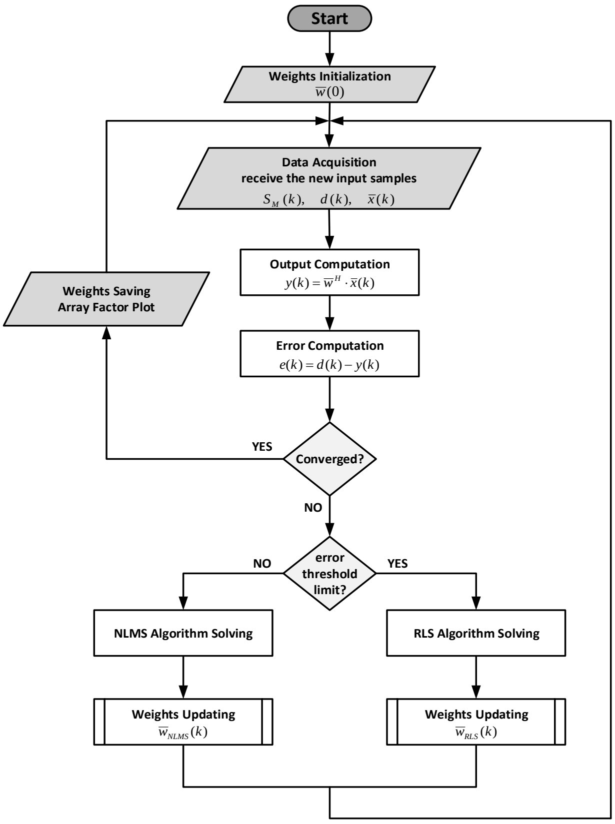

Figure 2: The flow chart of the proposed NLMS/RLS algorithm procedures

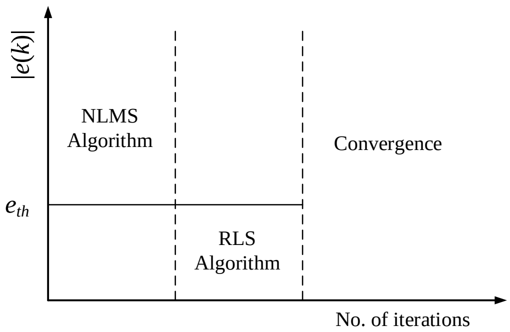

Figure 3: The convergence sections of the proposed NLMS/RLS algorithm

IV. SIMULATION RESULTS AND DISCUSSION

In this simulation, assuming the signal-of-interest is coming from the direction of 30° and two random interfering signals are coming from the direction of 0° and -60°. Figures 4 to 9 show the simulation results of the normalized array factor for linear array using the LMS, NLMS, SMI, RLS, LMS/SMI, and NLMS/RLS algorithms respectively using 8, 16, 24, 32, and 51 antenna radiating elements, while the displacement between radiating d elements is fixed at $\lambda/2$ and 30 dB SNR of interfering noise on each radiating element. Figures 10 to 13 indicate the performance of each algorithm (HPBW, Max. SLL, and nulls depth), where each result represents the average of 100 simulation results at different cases for each algorithm. Each case has different conditions, where the characteristics of each interfering signals are changed by making them random signals in addition to the random noise. Figures 4 to 9 show only one case from these cases.

The radiation HPBW of the antenna narrows by rising the number of radiating elements N as illustrated in Figures 4 to 10. The SMI, RLS, and LMS/SMI algorithms introduce high side-lobe levels, while the LMS, NLMS, and NLMS/RLS give low SLLs. Moreover, The SMI algorithm has the highest SLLs on the other side, the NLMS/RLS algorithm has the lowest SLLs as illustrated in Figure 11. The SMI, RLS, and NLMS/RLS algorithms have deeper nulls compared to the LMS, NLMS, and LMS/SMI algorithms as shown in Figures 12 and 13. The deepest nulls are given by the SMI algorithm followed by the NLMS/RLS algorithm, then the RLS algorithm, while the LMS and NLMS have the lowest nulls depth. As presented in Figures 4 to 9, the LMS, NLMS, and NLMS/RLS algorithms have more beamforming stability than the SMI, RLS, and LMS/SMI algorithms. The beamforming of the adaptive algorithm is called stable when the beamforming is independent on the received signals values but upon their directions. In another word, the algorithm has stable beamforming when it is only sensitive to the incident signals directions.

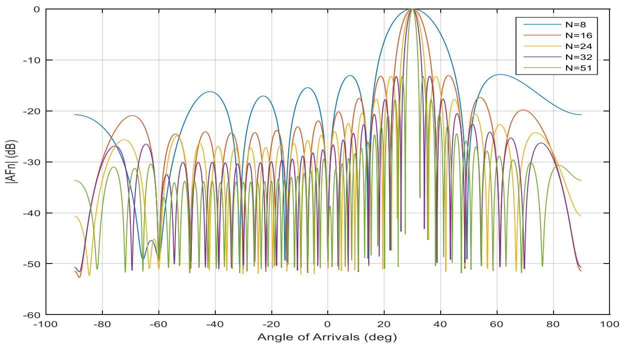

Figure 4: Normalized array factor of linear array under different radiating elements number using the LMS algorithm at $d = \lambda/2$ and SNR=30 dB

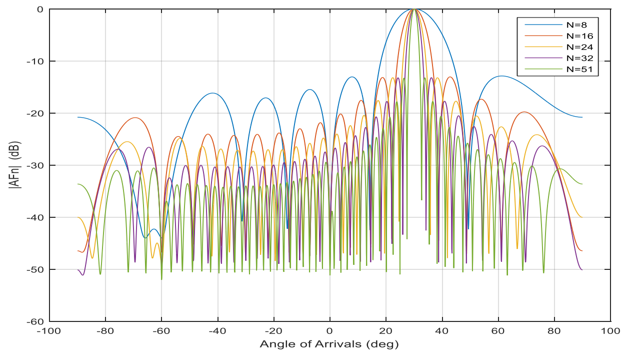

Figure 5: Normalized array factor of linear array under different radiating elements number using the NLMS algorithm at $d = \lambda/2$ and SNR=30 dB

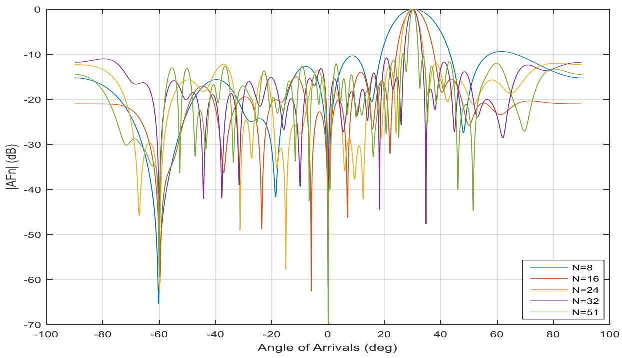

Figure 6: Normalized array factor of linear array under different radiating elements number using the SMI algorithm at $d = \lambda/2$ and SNR=30 dB

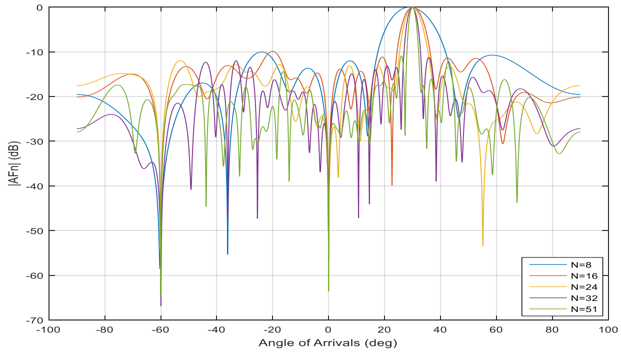

Figure 7: Normalized array factor of linear array under different radiating elements number using the RLS algorithm at $d = \lambda/2$ and SNR=30 dB

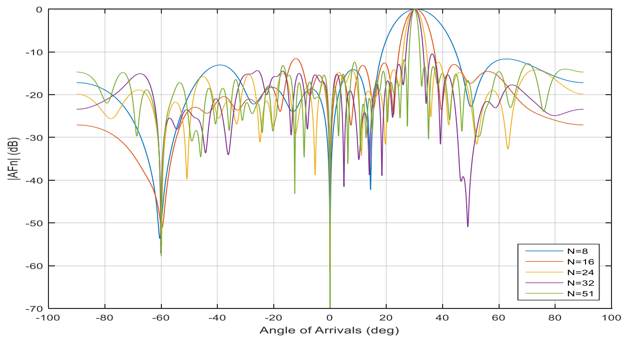

Figure 8: Normalized array factor of linear array under different radiating elements number using the LMS/SMI algorithm at $d = \lambda/2$ and SNR=30 dB

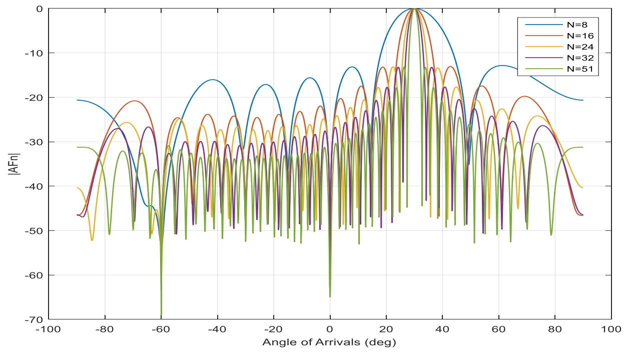

Figure 9: Normalized array factor of linear array under different radiating elements number using the NLMS/RLS algorithm at $d = \lambda/2$ and SNR=30 dB

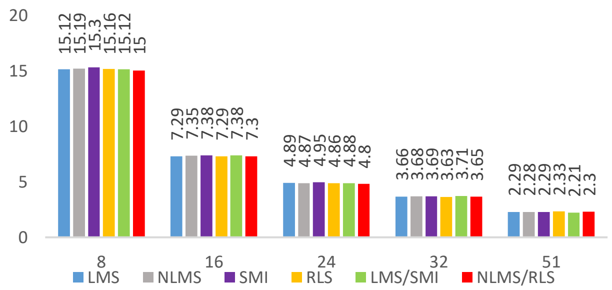

Figure 10: The average HPBW of 100 simulations result under a different number of antenna elements

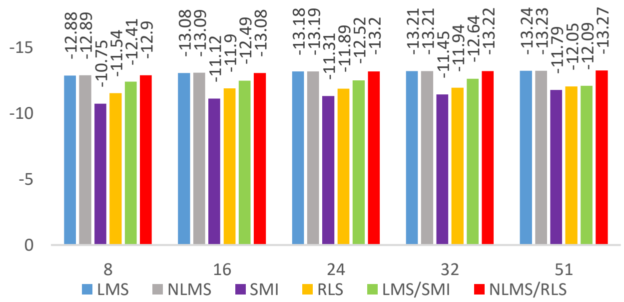

Figure 11: The average max. SLL of 100 simulations results under a different number of antenna elements

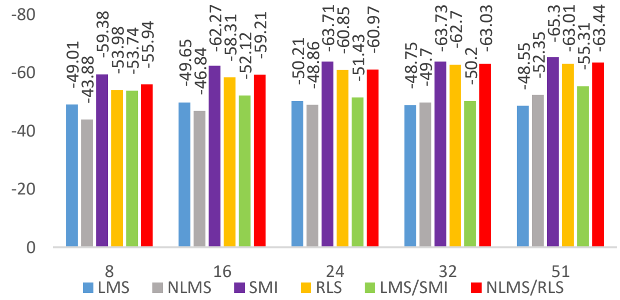

Figure 12: The average nulls depth at 0° of 100 simulations result under a different number of antenna elements

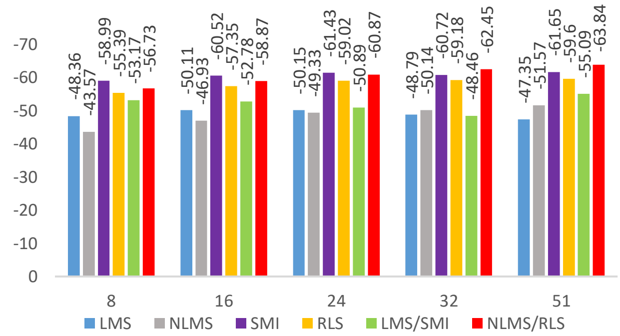

Figure 13: The average nulls depth at 60° of 100 simulations result under a different number of antenna elements

Figures 14 to 18 illustrate the simulation results of the MSE versus number of iterations for the linear array using the LMS, NLMS, RLS, LMS/SMI, and NLMS/RLS algorithms respectively using 8, 16, 24, 32, and 51 antenna radiating elements at $d = \lambda/2$ and SNR = 30 dB. The convergence rate of the LMS algorithm speeds up by rising the number of radiating elements N. At $N = 8$, LMS converges after 45 iterations, while at $N = 51$, it converges after 7 iterations as illustrated in Figure 14. On the other hand, the convergence rate of the NLMS algorithm is insensitive to N because of the normalized power of the inputs. It converges after 6 iterations for each value of N as illustrated in Figure 15. In the RLS algorithm, the convergence occurs after 3 iterations and the convergence rate is insensitive to N as illustrated in Figure 16. The hybrid LMS/SMI algorithm has sped convergence because of the weights initialization by the SMI algorithm where weights values are near to the optimum solution. The error decreases by raising the number of antenna elements, where the maximum MSE at $N = 8$ is 4.2×10^{-4} and at $N = 51$ reaches 0.45×10^{-4} as depicted in Figure 17. In the NLMS/RLS algorithm, the convergence occurs after 7 iterations for all values of N as illustrated in Figure 18. The convergence rate of the NLMS/RLS algorithm is independent on the number of antenna elements N since this algorithm is a combination of the NLMS and RLS algorithms which their convergence rates are independent on the number of antenna radiating elements N.

Figure 19 presents the average of MSE difference between the reference signal and the output of tested signal at different noise levels (SNR value of the received signal on each radiating element is changed among 30, 20, 15, 10 and 5 dB) using the different algorithms at N = 16 and $d = \lambda/2$.

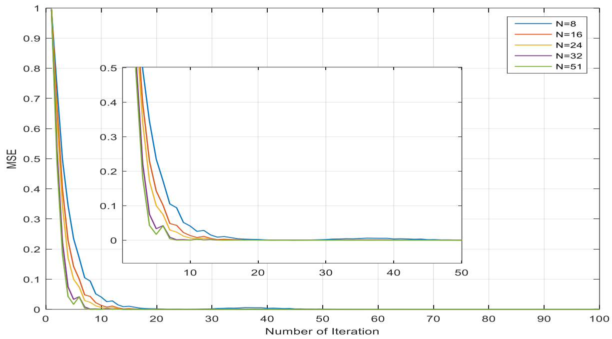

Figure 14: Mean square error of the LMS algorithm for linear array under different radiating elements number at $d = \lambda/2$ and SNR=30 dB

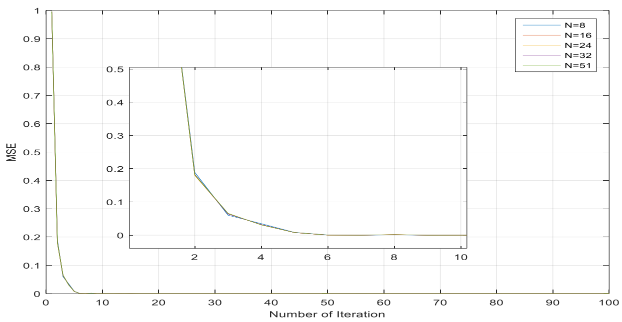

Figure 15: Mean square error of the NLMS algorithm for linear array under different radiating elements number at $d = \lambda/2$ and SNR=30 dB

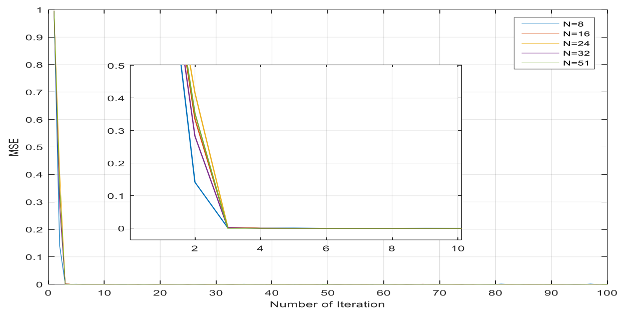

Figure 16: Mean square error of the RLS algorithm for linear array under different radiating elements number at $d = \lambda/2$ and SNR=30 dB

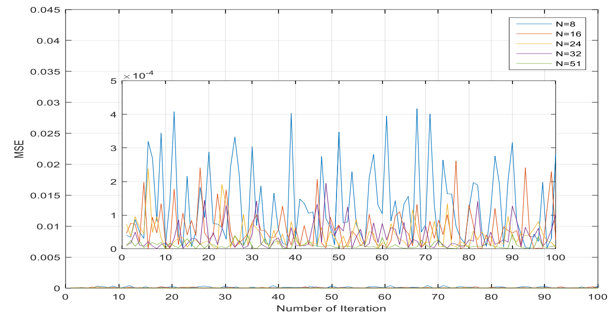

Figure 17: Mean square error of the LMS/SMI algorithm for linear array under different radiating elements number at $d = \lambda/2$ and SNR=30 dB

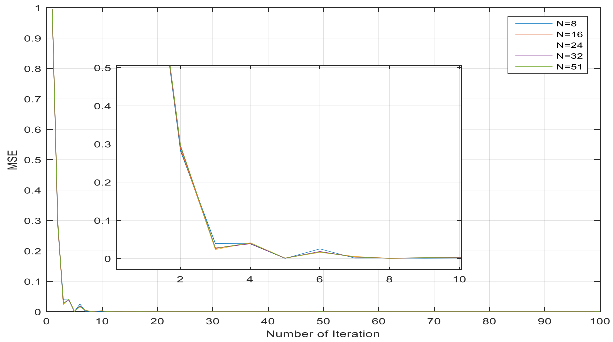

Figure 18: Mean square error of the NLMS/RLS algorithm for linear array under different radiating elements number at $d = \lambda/2$ and SNR=30 dB

The MSE values rise by increasing the noise level in all algorithms. Furthermore, the error difference at 5 dB SNR is very large compared to the other higher SNR values in all previous algorithms. Therefore, the smart antenna performance drops in an unacceptable manner. From Figure 19, it is clear that the SMI algorithm has the lowest MSE value at different noise levels, followed by the NLMS/RLS algorithm, the RLS algorithm, LMS/SMI algorithm, the LMS algorithm, and the NLMS algorithm, respectively. The noise influence on each algorithm is determined according to how deep the nulls can be given by the beamforming algorithm in the direction of interfering signals. In other words, the MSE value is inversely proportional to the nulls depth. Therefore, the NLMS algorithm has high MSE value because it introduces the lowest nulls depth compared to the other algorithms at different noise levels. For the NLMS/RLS algorithm, the threshold limit e_{th} is 0.1 at 30, 20 and 15 dB SNR. At 10 dB SNR, e_{th} is 0.3 and at 5 dB SNR, e_{th} is 0.5 to achieve satisfied performance.

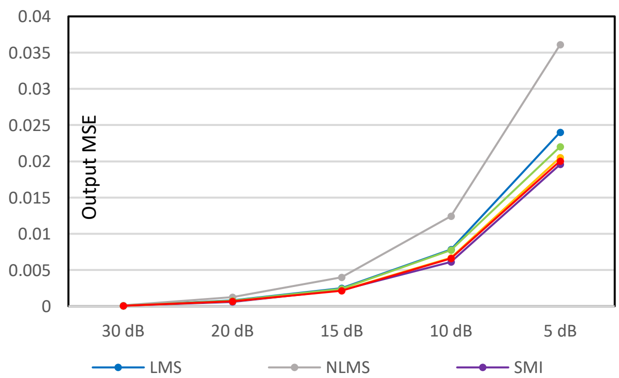

Figure 19: The average of MSE difference between the reference signal and the tested signal using different algorithms at $N=16$ and $d=\lambda/2$

In this section, the hybrid NLMS/RLS algorithm will be applied to an array of planar patch dipole, rectangular microstrip (probe fed – inset fed) and quasi-Yagi antennas to investigate the performance of the proposed algorithm at real radiating elements. The planar dipole, probe-fed microstrip, inset-fed microstrip antennas with the dimension considerations as shown in Figures 20 to 22 respectively have the narrow bandwidth at 10 GHz resonance frequency [25]. The quasi-Yagi antenna with the dimension as shown in Figure 23 covers the wide frequency band from 7.74 to 12 GHz at 10 GHz resonance frequency [28].

Figures 24 and 25 show the simulation results of the normalized gain and MSE respectively for the linear array using different types of the radiating antenna at N equals to 16, the SNR is 20 dB which represents interfering noise, mutual coupling between elements and the other physical errors on the smart antenna system that cause another noise. The spacing between elements is $0.6\ \lambda$. where the best performance of the smart antenna system is achieved at $d = 0.6\ \lambda$ and it is better than the performance at $d = \lambda/2$ for each algorithm [22] according to beamwidth and the most cases of the nulls depth as shown in Table 1.

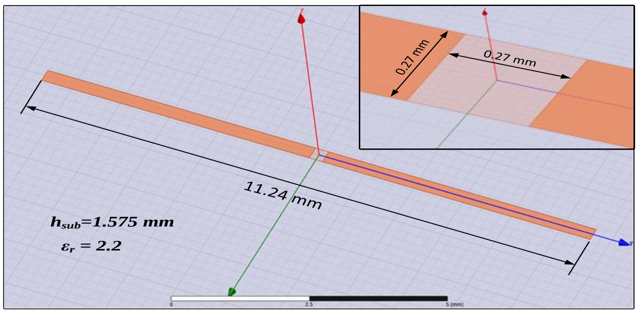

Figure 20: The dimension considerations of a planar dipole antenna for 10 GHz resonance frequency

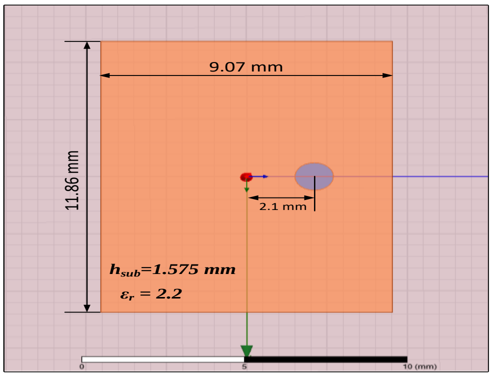

Figure 21: The dimension considerations of probe-fed microstrip antenna for 10 GHz resonance frequency

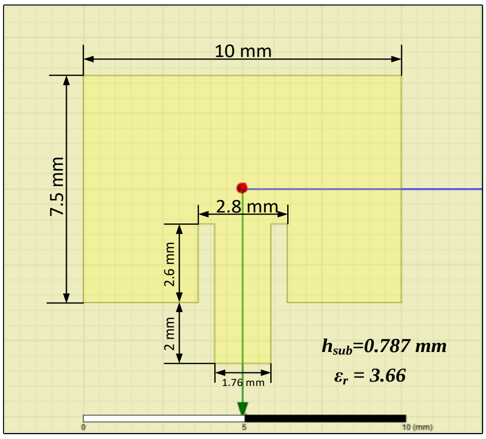

Figure 22: The dimension considerations of inset-fed microstrip antenna for 10 GHz resonance frequency

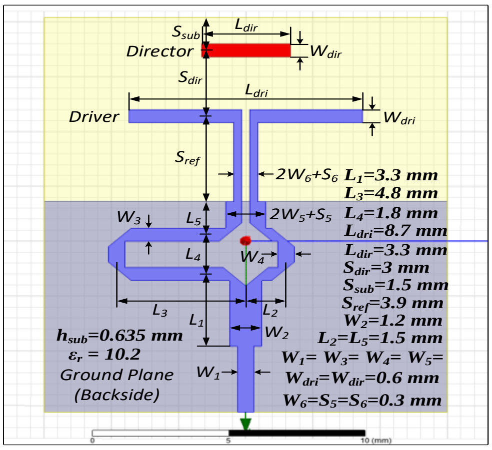

Figure 23: The dimension considerations of the quasi-Yagi antenna for 10 GHz resonance frequency

The results of the isotropic source antenna array introduce the best performance (SLL, nulls depth and convergence rate) which represents the ideal case as shown in Figures 24 to 25, and Table 2. The results of planar patch dipole, rectangular microstrip (probe fed – inset fed) and quasi-Yagi antennas array are compared with the isotropic source antenna array to illustrate the performance of the algorithms when applied on signals received by real elements and taking into interfering noise, mutual coupling between the real elements and the other physical errors on the smart antenna system that cause another noise.

Table 1
The Beamforming Average Results of a Linear Array at $d = 0.6\ \lambda$, $N = 16$ and SNR = 30 dB [22]

	HPBW (Deg.)	Max. SLL (dB)	Null Depth at 0°(dB)	Null Depth at -60°(dB)
LMS	6.12°	-13.16	-49.74	-49.62
NLMS	6.12°	-13.18	-47.54	-47.07
SMI	6.14°	-11.95	-59.59	-59.42
RLS	6.11°	-12.49	-59.42	-58.54
LMS/SMI	6.13°	-12.82	-53.27	-52.86
NLMS/RLS	6.1°	-13.18	-59.39	-59.18

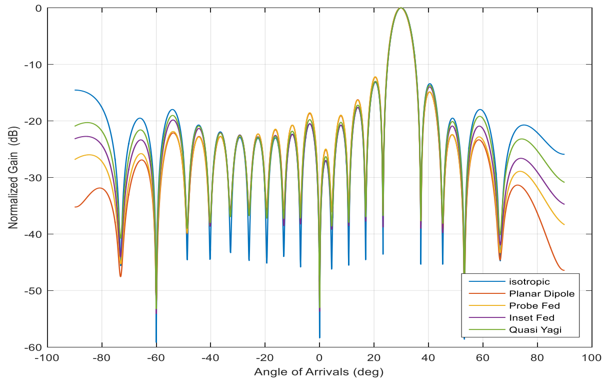

Figure 24: Normalized gain at different types of antenna using NLMS/RLS algorithm at N = 16, d = 0.6 λ and SNR = 20 dB

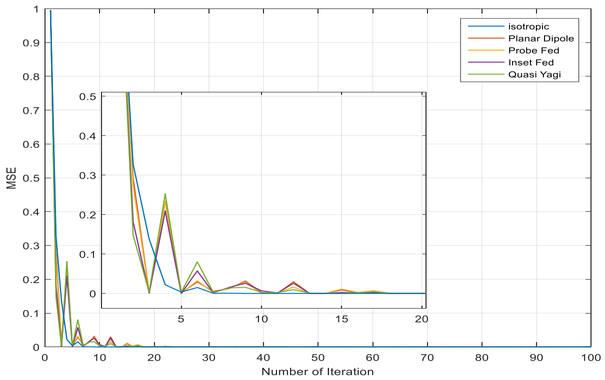

Figure 25: The MSE at different types of antenna using NLMS/RLS algorithm at N = 16, d = 0.6 λ and SNR = 20 dB

As presented Figure 24 and Table 2, the HPBW is insensitive to radiating elements type. The deepest nulls and lowest SLL are given by the isotropic antenna followed by the inset-fed microstrip, quasi-Yagi, planar dipole and probe-fed microstrip array antenna. The inset-fed microstrip and quasi-Yagi antennas converge faster than the planar dipole and probe-fed microstrip antennas as shown in Figure 25 and Table 2.

Table 2
Beamforming Results for the NLMS/RLS Algorithm at N = 16, d = 0.6 λ and SNR = 20 dB

	HPBW (Deg.)	Max. SLL (dB)	Null Depth at 0°(dB)	Null Depth at -60°(dB)	Convergence
Isotropic	6.1°	-13.03	-53.11	-54.18	7 iterations
Planar Dipole	6.1°	-12.27	-47.55	-47.61	18 iterations
Probe Fed	6.1°	-12.19	-46.92	-46.29	18 iterations
Inset Fed	6.1°	-13.03	-48.76	-48.69	13 iterations
Quasi Yagi	6.1°	-12.99	-48.58	-48.33	13 iterations

V. Conclusion

Through simulation experiments and comparison among different algorithms, it is found that the hybrid NLMS/RLS algorithm has the best performance (fast convergence, stable pattern beamforming, low side-lobe level, deep nulls and low MSE values). The hybrid NLMS/RLS algorithm gives the lowest SLLs in most cases of the different conditions. The convergence of the SMI, RLS, and LMS/SMI algorithms is improved at the expense of high side-lobe level and instability of array factor beamforming. The SMI algorithm has the minimum MSE values and the deepest nulls at the directions of interfering signals, followed by the NLMS/RLS algorithm, the RLS algorithm, the LMS/SMI algorithm, the LMS algorithm and the NLMS algorithm, respectively.

The performance of different types of patch array antenna using the hybrid NLMS/RLS algorithm is demonstrated. The inset-fed microstrip array antenna has the best performance followed by quasi-Yagi, planar dipole and probe-fed microstrip array antenna, respectively.

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