A New Antenna Array Pattern Synthesis Method with Sidelobe Control

Jafar Ramadhan Mohammed
College of Electronics Engineering, Nineveh University, Mosul-41001, Iraq.
jafarram@yahoo.com

***Abstract*—A new method for synthesising antenna array patterns with controlled sidelobes and enhanced directivity is proposed. It relies on the relationship between the array factor and its corresponding element excitations through Fourier transform properties. Unlike the standard array synthesis-based Fourier transform methods, the proposed method enforces some specific constraints to achieve the desired goals. It involves the computation of the radiation pattern from initial element excitations of a linear array using inverse fast Fourier transform (FFT). Once the radiation pattern has been computed, the constraints are applied to modify it by forcing a wide null within the whole main beam region and preserving all the sidelobes unchanged. By doing this, the sidelobe structures of both original and modified patterns are exactly kept same. Then, a new array pattern with ideally no sidelobes can be obtained by subtracting the original array pattern from the modified pattern. Results of applying the proposed method to the uniformly and non-uniformly excited arrays are shown in the final section.**

***Index Terms*—Antenna Arrays; Pattern Synthesis; Fourier Transforms; Constraints.**

I. INTRODUCTION

Generally, the radiation pattern of the most antenna arrays consists of the main lobe and a number of undesirable secondary lobes. In directional antennas, the main goal is to concentrate the radio waves energy in the main lobe direction and minimise the wastes in the secondary lobes region. The effect of high sidelobe levels in such antennas when operating in the transmit mode may cause more interference to other systems, and the desired signal may be picked up by unintended receivers. This effect has also an impact in the receiving antennas where more interfering signals may be picked up by the receiver which leads to an increase in the noise level. Thus, it is very desirable to minimise or cancel the sidelobes in the directional antennas.

Among the various antenna arrays synthesise methods that are capable of providing maximum possible directivity, uniformly excited arrays are of particular interest as they have simplest feeding network in the practical implementation. However, such arrays have a sidelobe level larger than -13.2 dB which is relatively high, and it is not adequate for many applications of directional antennas [1].

In [2], the author suggested an effective solution to the problem of high sidelobes in the antenna array patterns by adding two external elements each spaced by $\lambda/4$ from the end elements of the original array, which together produced a cosine pattern. An extremely low sidelobe was then obtained by specifically designing the element excitations of the original array antenna such that its corresponding radiation pattern was matched to that of the two added elements array. However, the mainbeam of the resultant array pattern has been little distorted, and the pointing direction has been little changed. This is mainly due to the fact that the radiation pattern of the two added elements has relatively high sidelobes within the mainbeam region. Also, an increase in the total number of array elements was noticed.

In [3], the authors suggest reusing the last two end elements instead of adding two extra elements to construct the required cancellation pattern. Although the sidelobes in the resultant array pattern have been considerably reduced and the total number of the array elements was maintained without any additional elements, the distortion in the mainbeam was not solved. Moreover, simple analytical procedures were used. Thus, an optimal solution was not achieved. In [4], an optimal solution was achieved by using global optimisation methods such as genetic algorithm and particle swarm optimisation instead of simple analytical procedures. A sector sidelobe nulling around the interferer direction was achieved by optimising the amplitude and phase excitations of the last two edge elements. These results were obtained without affecting the directivity of the array. However, the number of the optimised elements was only two, and as a result of limiting the degrees of freedom, a single sidelobe was cancelled. A brilliant extension to the method that was presented in [4] with the capability to cancel multiple sidelobes by optimising a minimum number of adjustable elements by means of the genetic algorithm is proposed in [5].

Other array synthesis methods which offer sidelobe nulling at pre-specific angular direction(s) were discussed in [6-11]. In [12], a certain number of the selected elements were chosen to be adjustable, and the author shows that the concept of sidelobe nulling is equivalent to subtracting a cancellation pattern from the quiescent pattern. A similar concept was also discussed in [13], where the authors assume that all the number of array elements is adjustable. They constructed a specific function that contains only the sidelobes of the uniformly excited array. Then it was subtracted from the array factor of the original uniformly excited array. However, the formulated function was not similar enough to the sidelobe structure of the original uniformly excited array. Thus, the sidelobes in the resulting array pattern are reduced down by only 10 dB with respect to that of the corresponding uniformly excited arrays.

In this paper, the author introduces a new method for synthesising antenna arrays with high directivity in the desired direction and low sidelobes in the other undesirable directions. The synthesis method which is presented here is of particular interest in the communication systems (especially for the fifth-generation mobile communications to meet the growing demand for cancelling the interfering signals and reducing the power consumption). Also, it is interested in radars, and high-resolution imaging systems. To achieve such goals, the proposed method should form a specific cancellation pattern with two main features: first, its sidelobe structure should be an exact replica of that of the original array pattern. Second, to maintain the main lobe of the resultant array undistorted, it should have a very low level or zero in the main lobe direction. These required specifications can be met by exploring the Fourier transform properties where a relationship exists between any array pattern and its corresponding element excitations. Note that the key feature of the proposed method lies in the designing of high-resolution cancellation pattern which can be satisfied by choosing the Fourier to transform points much higher than the number of the array elements. Consequently, the corresponding element excitations for such highly accurate cancellation pattern will require to be distributed over a larger number of elements (theoretically same number as the Fourier transform points). Finally, to get an array pattern without sidelobes, the designed cancellation pattern is then subtracted from the original array pattern. Moreover, it is also shown that, in practice, such an ideal cancellation pattern cannot meet. Thus, an approximate cancellation pattern is considered and the sidelobes can be reduced down but cannot be completely eliminated.

II. METHODOLOGY

Consider a linear array of N isotropic elements which are separated uniformly by a distance d. The relation between antenna's radiation and its corresponding element excitations can be written as

$$AF\,(u) = \sum_{n=0}^{N-1} a_n e^{jn\psi} \tag{1}$$

where $\psi = (2\pi d/\lambda)u$, $u = sin(\theta)$, θ represents the angle with respect to the direction normal to the array axis, and a_n is the excitation coefficients of the array elements. The above-mentioned equation forms a finite Fourier series that relates the antenna's radiation pattern and its corresponding element excitations a_n through a discrete inverse Fourier transform (IFT). Alternatively, the element excitation can be obtained by applying a discrete Fourier transform on the given array factor (AF).

These two Fourier transform relationships are repeatedly exploited with some constraints to construct the required cancellation pattern which is used for sidelobe cancellation or reduction.

The construction process starts with the calculation of the array factor (AF), according to Equation (1), using an initial set for the *N* element excitation coefficients (for example, uniform amplitude distribution). This calculation is carried out with a K-point inverse FFT. To achieve better resolution, the value of K is usually chosen to be much larger than the number of the array elements *N*. In order to perform such calculations correctly, zero padding with the initial element excitations, should be used. For example, a vector of size K should be generated with its first *N* values being the initial excitations for *n=1...N*, and the rest of the elements are chosen to be zero (zero padding) for *n=N+1...K*. By this way, an exact cancellation pattern is computed in K points with the inverse FFT. This is followed by the first-null-to-null-beam-width (FNNBW) calculation and enforcing the magnitude of the cancellation pattern to be zero only within the whole range of the FNNBW, while the sidelobe levels are left unchanged. After notching out the main lobe, a direct K-point FFT is performed on the resultant pattern of notched mainlobe to get a new set of excitation coefficients. Figure 1 shows the exact cancellation pattern and the corresponding element excitations for used parameters *N=20* and K=512. For comparison, the uniformly excited array pattern, i.e., initial pattern, under the same parameters is also shown in Figure 1.

Now the goal is to obtain the element excitations that generates this exact cancellation pattern. Since the Fourier transform is a linear operator, it is easy to apply the direct K-point FFT to this exact cancellation pattern whose excitations are shown in Figure 1. Note that the new excitation vector has K non-zero values, while the actual array has only *N* elements. By retaining only *N* out of K excitations and putting the values from *n=N+1...K* all to zero, then the resulting pattern will not be same as the exact cancellation pattern. In this case, the resulting pattern will be called approximate cancellation pattern. Figure 2 shows the approximate cancellation pattern and the corresponding element excitations for the same parameters as in the previous example that was shown in Figure 1.

Then, the overall array pattern can be obtained by subtracting the original array pattern from the approximate cancellation pattern as follows.

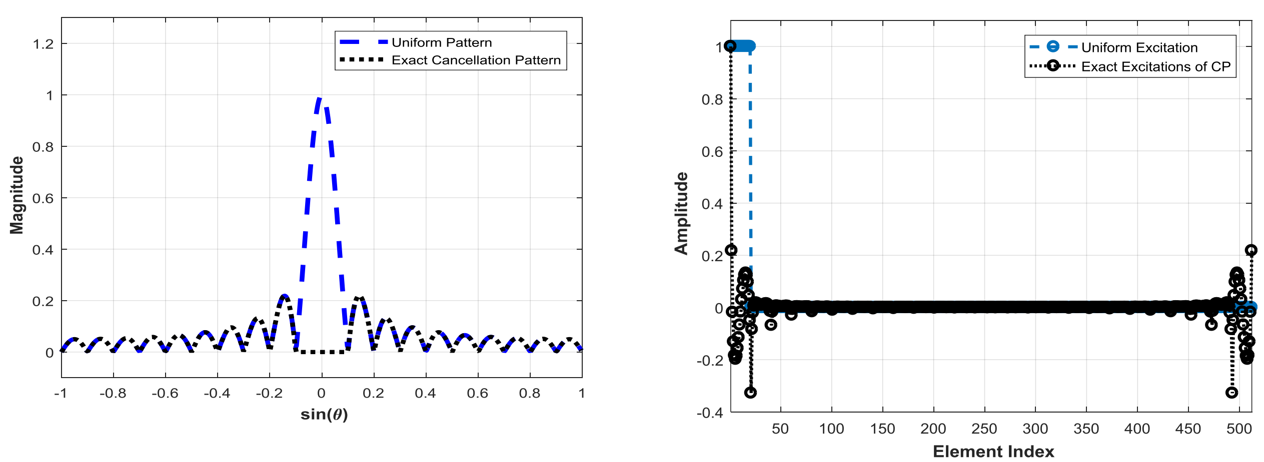


Figure 1: The Exact Cancellation Pattern (left) and its Corresponding Element Excitations (right) For N=20 and K=512.

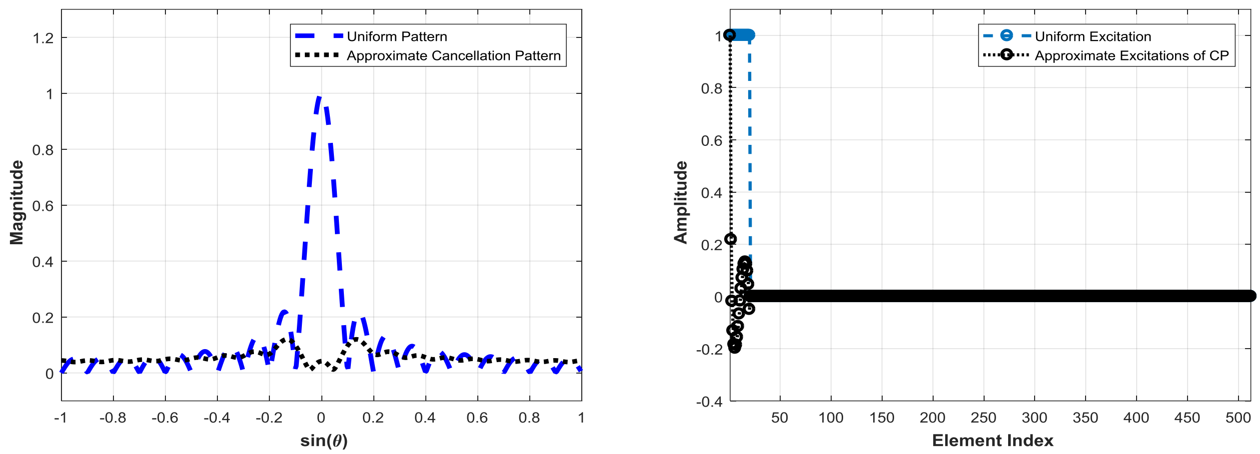


Figure 2: The Approximate Cancellation Pattern (left) and its Corresponding Element Excitations (right) For N=20 and K=512.

$$AF_{new}(u) = \underbrace{\sum_{n=0}^{N-1} a_n e^{jn\psi}}_{Original\ array} - \underbrace{\sum_{n=0}^{N-1} \Delta_n e^{jn\psi}}_{Cancellation\ pattern} \quad (2)$$

Comparing Equation (2) and Equation (1), it can be noted that the approximate excitation coefficients of the new array w_n for $n = 1,2, \ldots N$ are

$$w_n = a_n - \Delta_n \quad (3)$$

Here, a_n, as mentioned before, represents the original element excitation and Δ_n represents the required amount of perturbation on the element excitations to achieve the approximate cancellation pattern. Consequently, the new array pattern according to (2) will have secondary lobes. However, the levels of these secondary lobes are much lower than that of the original uniformly excited array pattern. To obtain an array pattern without secondary lobes, one needs an array with K elements instead of N so that none of the recovered excitations of the cancellation pattern is set to zero. Thus, the proposed method is most effective for arrays that consist of a large number of elements.

III. Simulation Results

To illustrate the effectiveness of the proposed method, three different examples are presented where the first example is related to the uniformly excited arrays, while the other two examples are related to the non-uniformly excited arrays. In all examples, an equally spaced linear arrays with $d = \lambda/2$ and a certain number of elements (N) ranging from small to large values are considered. Also, in all cases, an amplitude-only synthesis is performed and the phases of the retained (N) excitation coefficients according to (2) are made equal to the phases of the initial element excitations at the starting process. Figure 3 shows the Matlab code that used to obtain the showed results.

Figure 4 shows the radiation pattern of the original uniform array and the resulting array pattern for both exact and approximate cases and N=10 elements and K=512. For the exact case, it can be seen that the sidelobes of the original array and that of the exact cancellation patterns are completely matched over the whole sidelobe region. Thus, the sidelobes in the resulting pattern with such exact case are completely eliminated. In addition, the magnitude of the cancellation pattern is zero within the range of the FNNBW. Thus, the main beam shape and the HPBW of the resulting array are exactly same as those of the original uniform array, while the directivities of the original and the resulting arrays are 8.1790 and 8.4909 respectively (an improvement about 0.3119).

On the other hand, the sidelobe level in the resulting pattern with the approximate case is reduced down from -13.2 dB to more than -20 dB (an improvement of about -7 dB). Figure 5 shows the corresponding element excitations for the patterns mentioned above. From this figure, it can be observed that the exact and the approximate excitation coefficients of the new array elements are nearly equal. This means that it is possible to reach and accomplish the exact case. More importantly, the excitation coefficients are reliable for practical implementation especially after introducing, very recently, programmable variable attenuators that able to implement any excitation coefficient with high accuracy. Figure 6 shows the results for N=80 and K=512, while Figure 7 shows the corresponding element excitations. Figure 8(a) shows the radiation patterns of the original Chebychev array (the designed sidelobe level of the original Chebychev array was SLL= -20dB), the exact cancellation pattern and the resulting array pattern for N=20 elements, while Figure 8(b) shows the corresponding excitation coefficients. Here, in this case, the directivities of the original Chebychev and the resulting arrays are 10.9949 and 11.2268 respectively (an improvement of about 0.2319). Also, note that the sidelobes of the resulting Chebychev array have been completely cancelled and its main lobe is not affected. Figure 9 shows the results of applying the proposed method to the Taylor array for N=20 elements, sidelobe level (SLL)= -20 dB, and npar=3 (this parameter represents the number of approximately constant-level sidelobes next to the mainlobe). The directivities of the original Taylor and the resulting arrays are 10.8748 and 10.9677 respectively. These results fully confirm the effectiveness of the proposed method and it may provide tremendous advantages to the current and future radar and communication systems.

```
%=========================================================================
%                    Input Parameters
%=========================================================================
d      = 0.5;
noEl   = 10;                           % Number of elements
noFF   = 512;                          % # of far-field (FF) directions
u      = (-noFF/2:noFF/2-1)/d/noFF;    % u-coordinates FF
Illum = ones(1,noEl);                  % Initial Uniform illumination;
%=========================================================================
%           For given initial excitation, compute AF by using Inverse FFT
%=========================================================================
CP = ifftshift(ifft(Illum,noFF));     % Compute Cancellation pattern Using IFFT
CP=abs(CP)/(max(abs(CP)));            % Normalize the Cancellation Pattern
AF_uniform=CP;
uu=asin(1/(d*noEl));                  % angular location of first null
CP(u >= -uu & u <= uu)= 0;            % Exact Cancellation pattern with main
beam notched
IllumCP = fft(ifftshift(CP));         % Illumination derived from exact CP
IllumCPLimit = IllumCP(1:noEl);       % Retain only first N values
CP_Limit = ifftshift(ifft(IllumCPLimit,noFF)); % Approximate cancellation
pattern
AF_result=(AF_uniform)-(CP_Limit); % Resulting Array Factor for Approximate
case
IllumR = fft(ifftshift(AF_result));% Modified illumination derived from
AF_result
%=========================================================================
%                             Plots
%=========================================================================
figure(1)
plot(u,abs(AF_uniform),'--b','LineWidth',3);hold on;
%plot(u,abs(CP),':g','LineWidth',3);hold on;
plot(u,abs(CP_Limit),':k','LineWidth',3);hold on;
%plot(u,abs(AF_result),'k','LineWidth',3);hold on;
axis([-1 1 -0.1 1.3]);box on;
xlabel('\bf sin(\theta)');ylabel('\bf Magnitude');grid on;
```

Figure 3: Matlab code for the proposed method.

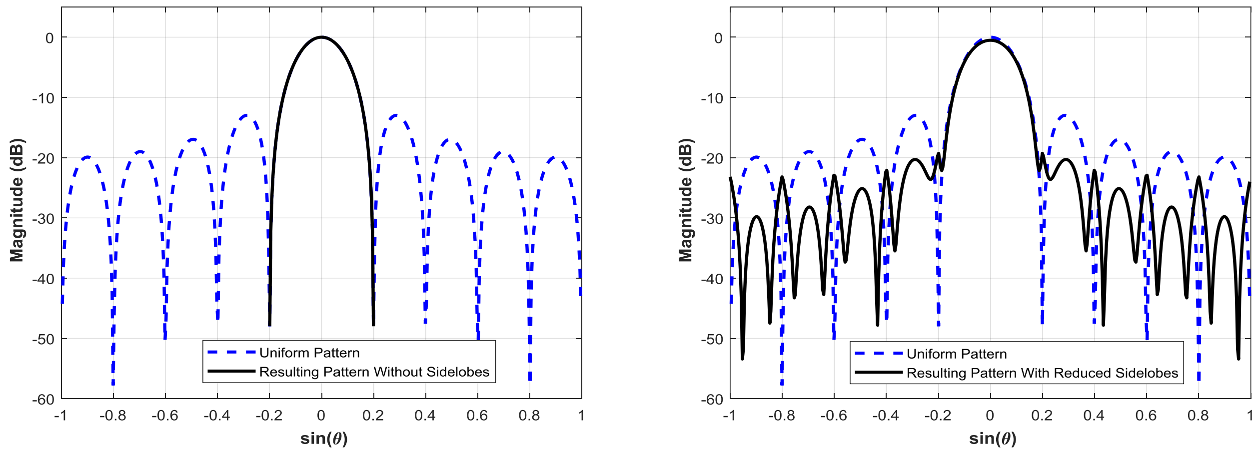


Figure 4: The resulting patterns for N=10 and K=512. Exact case (left) and Approximate case (right).

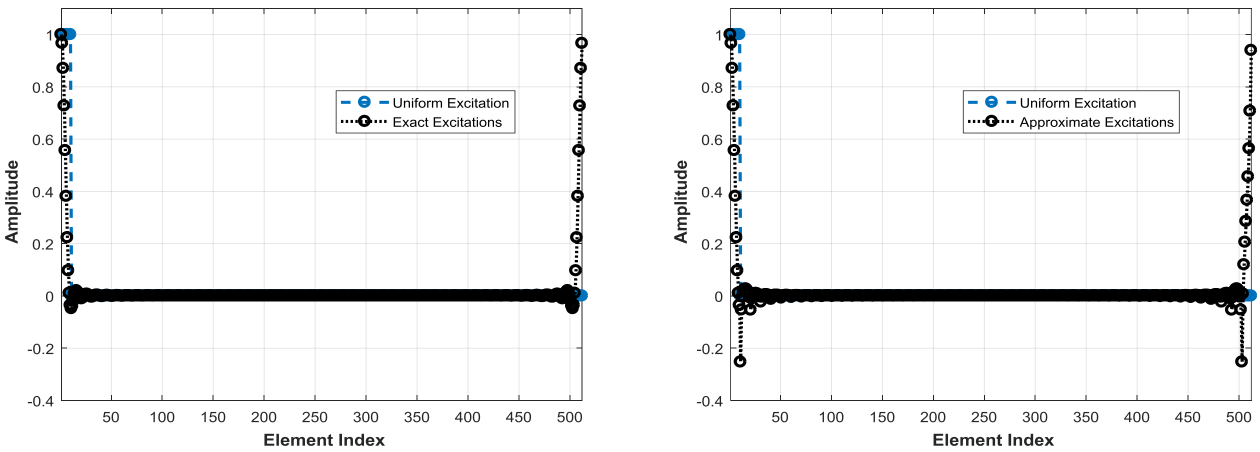


Figure 5: Amplitude excitations for patterns that shown in Figure 4.

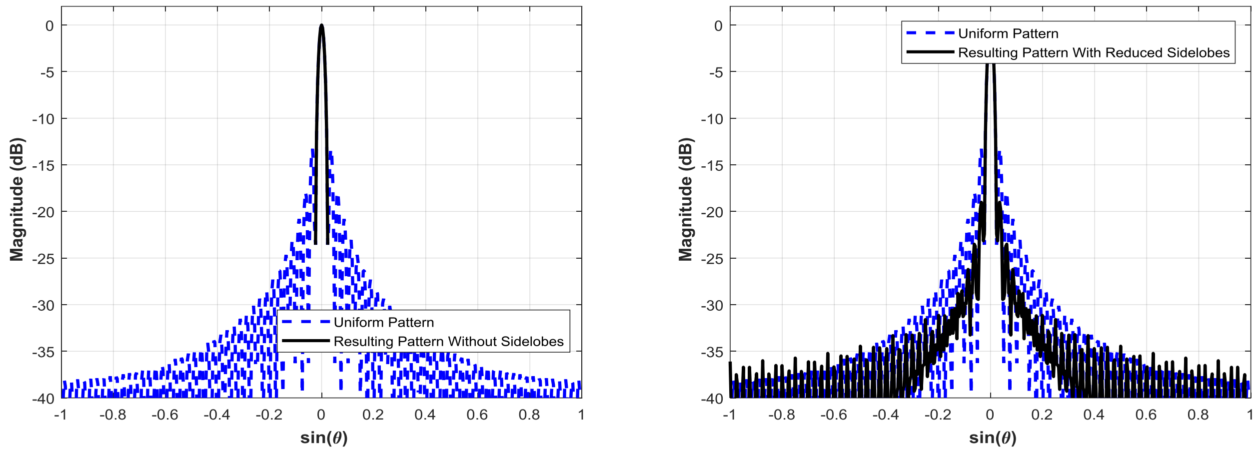


Figure 6: The resulting patterns for N=80 and K=512. Exact case (left) and Approximate case (right).

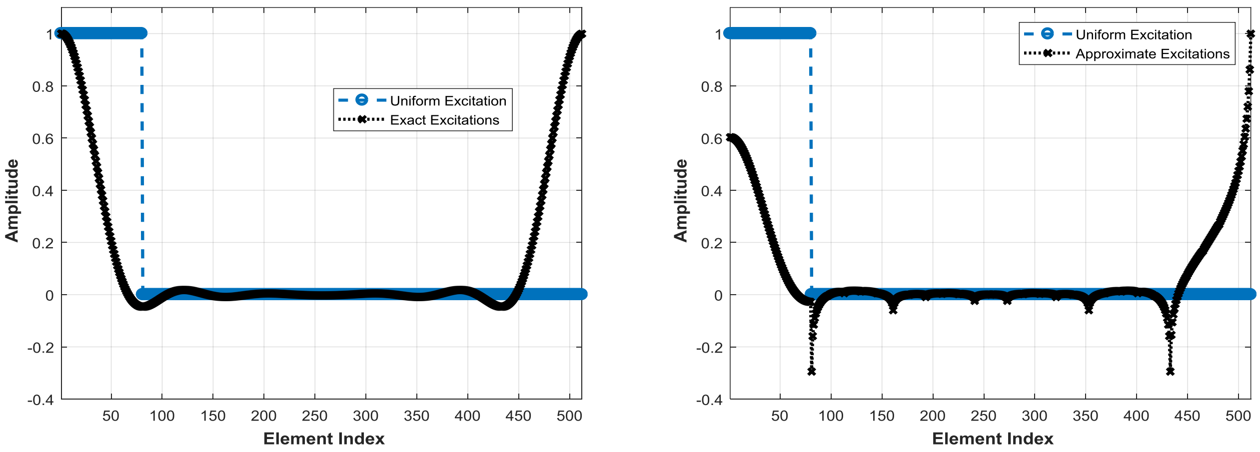
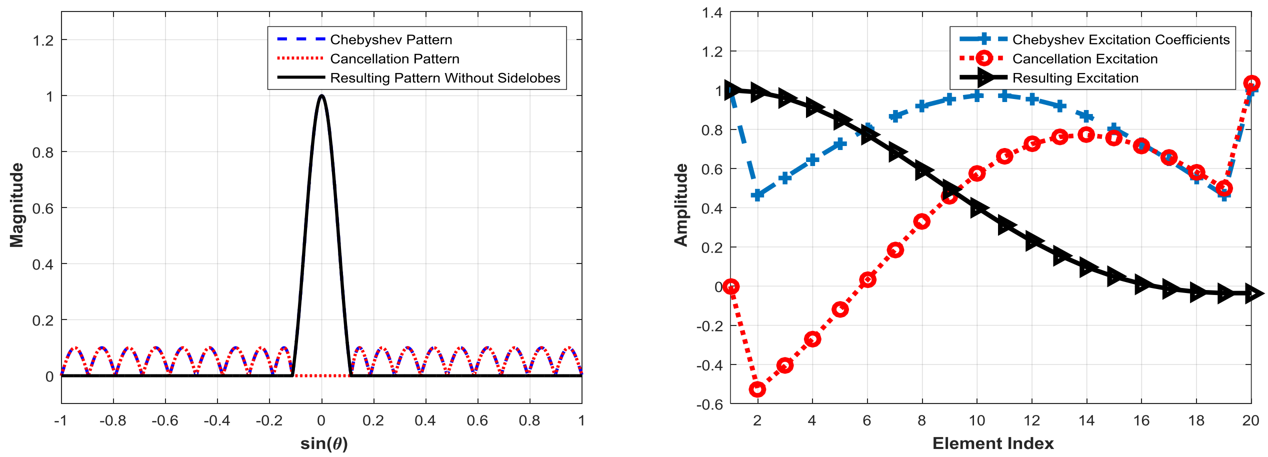


Figure 7: Amplitude excitations for patterns that shown in Figure 6.



(a) (b)

Figure 8: (a) The resulting patterns for Chebychev array and N=20, (b) the corresponding element excitations.

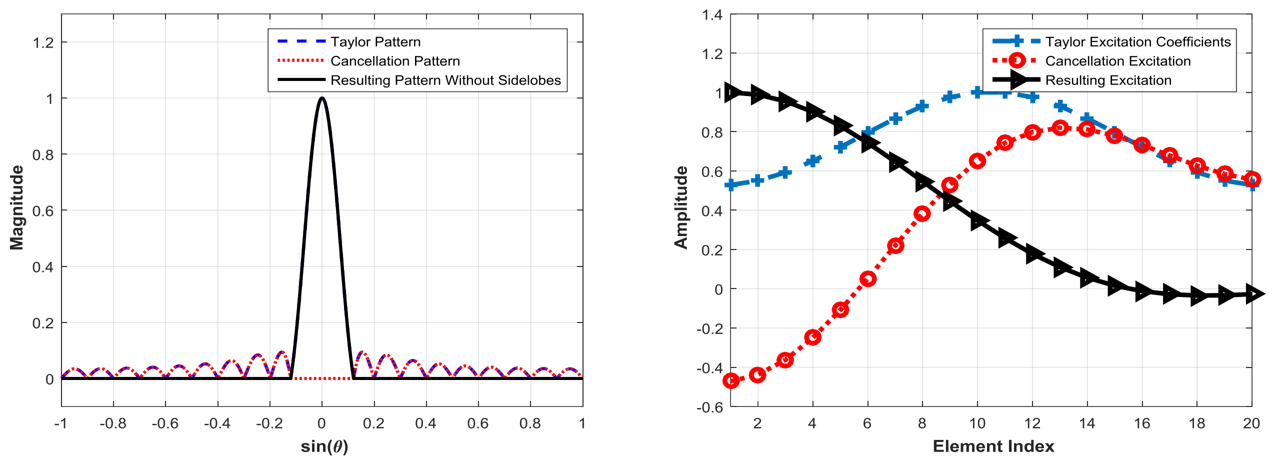


Figure 9: (a) The resulting patterns for Taylor array and N=20, (b) the corresponding element excitations.

IV. Conclusions

It is clear that the cancellation pattern which used for sidelobe elimination or reduction can be synthesised with two main constraints. First, it should have a sidelobe structure that is exactly or nearly identical to that of the original array pattern. Second, its main lobe should be completely notched out. The excitation coefficients of such a specifically designed pattern can be obtained by simply using the Fourier transform algorithm. The sidelobes of the resulting array pattern can be completely eliminated by considering all the K values at the output of inverse FFT. Also, it can be only reduced by truncating the vector K and retaining only first N values. In order to practically realise the first case, i.e., complete elimination of the sidelobes, the corresponding element excitations will require to be distributed over a larger number of elements (theoretically same number as the Fourier transform points K). This case is currently under further investigation and may be the subject of another publication. On the other hand, the second case, i.e., reduced sidelobes, can be implemented directly in practice with the available attenuators and phase shifters as a new tapering method for sidelobe reduction.

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