A Model-free Approach to Optimal Chiller Loading Problem Using Global Simultaneous Perturbation Stochastic Approximation

Mohd Ashraf Ahmad, Raja Mohd Taufika Raja Ismail, Mohd Helmi Suid, Mohd Falfazli Mat Jusof, Zulkifli Musa, Nik Mohd Zaitul Akmal Mustapha, and Mohd Riduwan Ghazali

Faculty of Electrical and Electronics Engineering, Universiti Malaysia Pahang, 26600, Pekan, Pahang, Malaysia

mashraf@ump.edu.my

***Abstract*—This paper presents an investigation of a model-free approach using global simultaneous perturbation stochastic approximation (GSPSA) to optimal chiller loading problem. The GSPSA based method is employed to optimize the partial load ratio (PLR) of each chiller such that the total power consumption of multi-chiller systems is minimized. The main advantage of GSPSA is that it can produce fast design parameter without information of plant model by measuring the input-output data of the system. Our model-free design is validated using an experimental data from a well-known multi-chiller system of semiconductor factory in Hsinchu Scientific Garden, Taiwan. In addition, the performance of the GSPSA based method is compared to the other stochastic optimization based approaches. Simulation results illustrate that the GSPSA based method has a potential in minimizing power consumption of multi-chiller systems with less number of evaluated cost functions.**

***Index Terms*—Model-Free; Stochastic Approximation; Optimal Chiller Loading Problem; Energy Consumption.**

I. INTRODUCTION

Multi-chiller systems, which normally require huge electric power consumption, are mainly utilized to produce cooling energy for industrial and domestic facilities [1]. Recently, investigation on optimizing the partial load ratios (PLRs) to reduce the multi-chiller power consumption, which is called optimal chiller loading (OCL) problem, has become a popular research topic among control engineering researchers. Nevertheless, the difficulty in obtaining accurate models of the multi-chiller plants will make this problem more complex. Even though if it is possible, one requires a lot of time and effort to obtain such models. Hence, it is useful to use a model-free method for this kind of problem.

So far, a large number of tuning algorithms have been extensively presented to solve OCL problem. There are nature inspired algorithms such as particle swarm optimization [2], [3] and [4], genetic algorithms (GAs) [5] and [6], firefly algorithm [7], differential evolution [8], differential search [9], simulated annealing [10], gradient method [11], cuckoo search algorithm [12], bee swarm optimization [13], teaching learning based optimization algorithm [14], adaptive neuro-fuzzy inference systems and GA [15], artificial cooperative search algorithm [16], and firefly with opposition based learning [17].

However, most of the above approaches, which are based on population-based optimization, need large computation time during the tuning process. This is because the number of evaluated cost functions per iteration is proportional to the population number. Therefore, the design of an algorithm that produces less computation is important to solve this issue. Besides that, a global simultaneous perturbation stochastic approximation (GSPSA) [18] is one of the potential model-free approaches from this point of view. This is because the GSPSA method is based on trajectory-based optimization method that just use a single agent to update its design parameter. Moreover, this method, which is mainly derived from the conventional SPSA method [19], is known to be practical for various optimization problems even for high-dimensional design parameter [20], [21], [22], [23]. Also, it has been shown that the GSPSA method can achieve better local optimal values compared to the standard SPSA [21]. However, it is not clear whether it works for optimal chiller loading problem since the reports on the utilization of the GSPSA to the OCL problem are very few.

In this study, we explore the potential of the model-free based GSPSA approach for minimizing the power consumption of multi-chiller systems. As an initial study, our model-free design is tested using the experimental data of the real multi-chiller system in Hsinchu Scientific Garden in Taiwan [24], [25]. Next, we analyze the convergence speed and the minimum total power consumption of multi-chiller system to evaluate the performance of the model-free based GSPSA. Here, the convergence speed is recorded based on the number of evaluated cost functions. Finally, performance comparison between the GSPSA based method and other existing optimization algorithms, which are simulated annealing (SA) based method [10] and random search (RS) based method [26] is shown.

The organization of this paper is as follows. In Section 2, the problem of minimizing the power consumption of multi-chiller systems is formulated. Section 3 summarizes the GSPSA algorithm and its application in model-free design. In Section 4, the proposed method is validated to a given multi-chiller system. The analysis and comparative assessment between the GSPSA based method and SA and RS based methods are also discussed in this section. Finally, conclusions of our findings are presented in Section 5.

Notation: The set of real numbers and the set of positive real numbers are defined by $\mathbb{R}$ and $\mathbb{R}_+$, respectively. The symbol $|S|$ is defined as the cardinality of a set *S*. For $\delta_{\max}, \delta_{\min} \in \mathbb{R}_+$, $\mathrm{sat}_\delta : \mathbb{R}^n \to \mathbb{R}^n$ represents the saturation function whose *i*-th element presented as follows:

The ith element of $\mathrm{sat}_{\delta}(\boldsymbol{x}) = \begin{cases} \delta_{\max} & \text{if} \quad \delta_{\max} < x_i, \\ x_i & \text{if} \quad \delta_{\min} \le x_i \le \delta_{\max}, \\ \delta_{\min} & \text{if} \quad x_i < \delta_{\min}, \end{cases}$

where $x_i \in \mathbb{R}$ is the i-th element of $\boldsymbol{x} \in \mathbb{R}^n$.

II. Problem Formulation

Consider the multi-chiller system consists of n chillers. Let Q_i (i = 1, 2, …,n) be the partial load ratio (PLR) of chiller i and P_i (Q_i) (i = 1, 2, …, n) be the power consumption of chiller i. In particular, Q_i is defined as the ratio of the chiller cooling load to the chiller power consumption [24], [25]. Note that the relation between Q_i and P_i is unknown due to the difficulty in developing a precise model of the plant. Nevertheless, we assume that the total power consumption of the multi-chiller system is measurable, which is given by

$$\overline{P}(Q_1, Q_2, \ldots, Q_n) = \sum_{i=1}^{n} P_i(Q_i) \cdot \tag{1}$$

Simultaneously, the designed partial load ratio Q_i also need to fulfil the given demanded cooling load, which is formulated by

$$CL = \sum_{i=1}^{n} Q_i R_i \ , \tag{2}$$

where R_i is the capacity of the i-th chiller. Then, the problem setting is stated as follows.

Problem 2.1 Let the explicit forms of the total power consumption P_i (I = 1, 2, …,n) are not known. Then, obtain partial load ratio Q_i (I = 1, 2, …,n) that minimizing $\overline{P}(Q_1, Q_2, \ldots, Q_n)$ subject to (2) and 0.3 <Q_i< 1, (I = 1, 2, …, n).

Remark 2.1 The designed partial load ratio Q_i must be greater than 0.3 to avoid the power surge in the chiller system [10].

III. Design of Model-free Approach Based on GSPSA

The mechanism to solve the Problem 2.1 is discussed in this section. Firstly, the summary of GSPSA algorithm [18] is explained. Secondly, we present the procedure to implement GSPSA algorithm in model-free design specifically for reducing the power consumption of the multi-chiller system.

A. Review on GSPSA

GSPSA is a stochastic gradient approximation algorithm to find the design parameters such that a given cost function is minimized. Consider a general optimization problem represented by

$$\min_{\boldsymbol{z} \in \mathbb{R}^n} h(\boldsymbol{z}), \tag{3}$$

where $h : \mathbb{R}^n \to \mathbb{R}$ is the cost function and $\boldsymbol{z} \in \mathbb{R}^n$ is the design parameter.

Then, the GSPSA algorithm [18] iteratively updates the design parameter to search a local optimal solution $\boldsymbol{z}^* \in \mathbb{R}^n$ of (3). The updated law is given by

$$\boldsymbol{z}(k+1) = \boldsymbol{z}(k) - a(k) g(\boldsymbol{z}(k), \Delta_1(k)) + b(k)\Delta_2(k) \tag{4}$$

for$k$ = 0, 1, …, where $a(k)$ and $b(k)$ are the gain sequences and $\Delta_2(k) \in \mathbb{R}^n$ are random perturbation vectors that are produced independently and $g(\boldsymbol{z}(k), \Delta_1(k))$ is the gradient approximation at the iterate $\boldsymbol{z}(k)$, which is expressed by

$$g(\boldsymbol{z}(k), \Delta_1(k)) = \begin{bmatrix} \dfrac{h(\boldsymbol{z}(k) + c(k)\Delta_1(k)) - h(\boldsymbol{z}(k) - c(k)\Delta_1(k))}{2c(k)\Delta_{11}(k)} \\ \vdots \\ \dfrac{h(\boldsymbol{z}(k) + c(k)\Delta_1(k)) - h(\boldsymbol{z}(k) - c(k)\Delta_1(k))}{2c(k)\Delta_{1n}(k)} \end{bmatrix} \tag{5}$$

In Equation (5), $c(k)$ is another gain and $\Delta_{1i}(k)$ is the i-th element of a random vector $\Delta_1(k) \in \mathbb{R}^n$. The detail of the GSPSA algorithm and also the procedure to choose $a(k)$, $b(k)$, $c(k)$ and the random vectors, and $\Delta_2(k)$ is reported in [18]. The termination criterion used in this algorithm is given by

$$\left| h(\boldsymbol{z}(k+1)) - h(\boldsymbol{z}(k)) \right| < \varepsilon, \tag{6}$$

where ε be a small number. Then, the algorithm stops with the optimal solution $\boldsymbol{z}^* := \arg \min_{\boldsymbol{z} \in \{\boldsymbol{z}(0), \boldsymbol{z}(1), \ldots, \boldsymbol{z}(k+1)\}} h(\boldsymbol{z}) \cdot$

Remark 3.1 In Equation (4), the term $b(k)\Delta_2(k)$ is added to the standard update rule of SPSA. This term plays an important role in the global convergence of GSPSA. In particular, this term gives additional effort to the updated rule in avoiding local minima problem in the standard SPSA algorithm.

B. Design of Model-free Method

With the GSPSA algorithm in the previous sub-section, the procedure of model-free based GSPSA design for optimal chiller problem is given by:

Step 1: Determine the GSPSA gain sequences $a(k)$, $b(k)$ and $c(k)$. Determine ε and $\boldsymbol{z}(0)$. Set k = 1.

Step 2: Let the cost function to solve Problem 2.1 is given by

$$J(Q_1, Q_2, \ldots, Q_n) = \overline{P}(Q_1, Q_2, \ldots, Q_n) + w\left| CL - \sum_{i=1}^{n} Q_i R_i \right|, \tag{7}$$

where w is weight. Then perform GSPSA algorithm, i.e., by regarding J and Q_i (I = 1, 2, …,n) as h and z_i (I = 1, 2, …, n), respectively.

Step 3: After $\left| h(\boldsymbol{z}(k+1)) - h(\boldsymbol{z}(k)) \right| < \varepsilon$, the convergence speed of tuning process, the optimal partial load ratio $Q_i^* = z_i^*$ (I = 1, 2, …, n), and the total power consumption $\overline{P}(Q_1^*, Q_2^*, \ldots, Q_n^*)$ are analyzed.

In order to solve constraint 0.3 < Q_i < 1, (I = 1, 2, …,n), we apply a saturation function $\mathrm{sat}_\delta(\cdot)$ in (4) and the modified updated law is expressed by

$$\mathbf{z}(k+1) = \mathrm{sat}_\delta(\mathbf{z}(k) - a(k)g(\mathbf{z}(k), \Delta_1(k)) + b(k)\Delta_2(k)) \quad (8)$$

where the value of $\delta_{\max} = 1$ and $\delta_{\min} = 0.3$. Here, the modified updated law in (8) is used in the model-free based GSPSA method throughout this paper.

IV. Simulation Results

The performance of the GSPSA based method is evaluated in this section. Here, a multi-chiller model is used to realize our model-free scheme. Firstly, we summarize the multi-chiller system in [24], [25]. Secondly, the GSPSA based algorithm is implemented to the system.

A. Multi-chiller Model

A multiple chiller system normally consists more than two chillers connected by parallel or series piping to a distributed system. This system often applies to air-conditioning systems since they provide operational flexibility, standby capacity and less disruption maintenance. In a multiple chiller system with all-electric cooling, the power consumption of each centrifugal chiller is a function of its PLR in a given wet-bulb temperature. That is,

$$P_i = v_i + y_i Q_i + x_i Q_i^2, \quad (9)$$

where v_i, y_i and x_i are the coefficients of interpolation for P_i-Q_i curve, which is discussed in detail in [6], [11], [24] and [25].

B. Six Chillers Example

This section presents the evaluation of the GSPSA based method performance for the six chillers system using the multi-chiller model in the previous sub-section. Also, the performances of the GSPSA based method is assessed to several demanded cooling loads of multi-chiller systems. The coefficients of this six chillers system, which is based on the experimental data in a semiconductor factory located in Hsinchu Scientific Garden in Taiwan, are given in Table 1.

Table 1
Values of v_i, y_i and x_i coefficients and R_i

Chiller i	v_i	y_i	x_i	R_i
1	399.345	-122.12	770.46	1280
2	287.116	80.04	700.48	1280
3	-120.505	1525.99	-502.14	1280
4	-19.121	898.76	-98.15	1280
5	-95.029	1202.39	-352.16	1250
6	191.750	224.86	524.04	1250

For the parameters of GSPSA, we choose the gain sequences of the GSPSA based method $a(k) = 0.0002/(k + 101)^{0.8}$, $b(k) = 0.005/(((k + 1)^{0.8})\ln((k + 1)^{0.2} + 500))^{0.5}$ and $c(k) = 0.001/(k + 1)^{1/6}$. Furthermore, we also provide the parameters of other stochastic optimization parameters, which are the SA [10] and RS [26] based methods, for comparative assessment evaluation. In particular, the SA based approach with initial temperature T_0 =120, final temperature T_f = 0.0001 and cooling rate ρ = 0.98 are chosen. However, for the RS based algorithm, we do not require any parameters to be selected. See [10] and [26] for the detail of both algorithms. The small number $\varepsilon = 0.001$, the weight w = 1 and the initial conditions Q_i (0) = 1, (i = 1, 2, …, 6) are used for both GSPSA and SA based approaches. In order to see the stochastic effect, we execute 500 runs for the GSPSA, SA and RS based methods. Then, after each algorithm is terminated, we evaluate the performance of each method based on the statistical analysis of the cost function $J(Q_1, Q_2, \ldots, Q_6)$, total power consumption $\overline{P}(Q_1, Q_2, \ldots, Q_n)$, the error between the demanded cooling load and produced cooling load $\left|CL - \sum_{i=1}^{6} Q_i R_i\right|$ and convergence speed. In particular, the values of mean, best, worst and standard deviation (Std.) of them are recorded after 500 runs. In this study, the performance of the convergence speed is obtained based on the number of evaluated cost function N_f after the termination criterion in (6) is achieved.

Table 2 shows the statistical analysis of the cost function $J(Q_1, Q_2, \ldots, Q_6)$, total power consumption $\overline{P}(Q_1, Q_2, \ldots, Q_n)$, the error between the demanded cooling load and produced cooling load $\left|CL - \sum_{i=1}^{6} Q_i R_i\right|$ and the convergence speed N_f. The best optimal partial load ratios of the GSPSA based method in comparison with the SA and RS based methods are shown in Table 3. It shows that, in terms of the cost function, total power consumption and error between the demanded cooling load and produced cooling load, the GSPSA based method obtains the lowest mean, worst and standard deviation values than the SA and RS based methods for every demanded cooling load. The GSPSA based method also obtains the lowest best value in most of the performance criteria. Furthermore, the GSPSA based method produces a better average number of evaluated cost function than SA and RS based methods in all cases. Hence, this fact means that the GSPSA based method can reduce the power consumption of multi-chiller systems with high probability and practical convergence speed.

V. Conclusion

In this paper, a study on a model-free approach based on global simultaneous perturbation stochastic approximation (GSPSA) for optimal chiller loading has been performed. The GSPSA based method is validated on a multi-chiller system in [24], [25], which is based on real experimental data. The numerical results show that the GSPSA based method outperforms the SA and RS based approaches from the viewpoints of minimum power consumption and convergence speed for the various demanded cooling load.

Table 2
Performance comparison of the GSPSA, SA [10] and RS [26] based approaches from 500 runs.

Performance criteria	*CL*		Model-free approaches GSPSA	SA [10]	RS [26]
$J(Q_1, Q_2, \ldots, Q_6)$ ($\times 10^3$)	6858	Mean	**4.7465**	4.7667	4.7847
		Best	4.7388	**4.7386**	4.7387
		Worst	**4.8757**	6.0841	6.2580
		Std.	**0.0142**	0.1265	0.1770
	6477	Mean	**4.4301**	4.4762	4.4587
		Best	**4.4218**	4.4219	4.4218
		Worst	**4.5110**	6.2746	6.6390
		Std.	**0.0130**	0.1734	0.2124
	6096	Mean	**4.1675**	4.2564	4.2380
		Best	4.1441	4.1442	**4.1440**
		Worst	**4.3508**	6.9344	7.0200
		Std.	**0.0309**	0.3465	0.4220
$\overline{P}(Q_1, Q_2, \ldots, Q_n)$ ($\times 10^3$)	6858	Mean	**4.7327**	4.7522	4.7609
		Best	**4.6498**	4.6979	4.7294
		Worst	**4.8130**	5.4258	5.4960
		Std.	**0.0154**	0.0710	0.1218
	6477	Mean	**4.4211**	4.4632	4.4442
		Best	**4.3894**	4.4089	4.4140
		Worst	**4.4915**	5.3525	5.4960
		Std.	**0.0106**	0.0941	0.1071
	6096	Mean	**4.1607**	4.2237	4.2005
		Best	**4.1249**	4.1382	4.1389
		Worst	**4.3237**	5.4575	5.4960
		Std.	**0.0266**	0.1662	0.1996
$\left\lvert CL - \sum_{i=1}^{6} Q_i R_i \right\rvert$	6858	Mean	**13.7914**	16.8690	23.8577
		Best	**0.0061**	0.0143	0.0098
		Worst	**175.5118**	658.3284	762.0000
		Std.	**19.0308**	59.6908	120.8715
	6477	Mean	**9.0144**	15.8328	14.7250
		Best	0.0760	**0.0156**	0.0324
		Worst	**54.0658**	922.0811	1143.0000
		Std.	**10.4549**	84.2637	107.9464
	6096	Mean	**6.8530**	36.1287	37.5326
		Best	**0.0057**	0.0084	0.0181
		Worst	**118.6105**	1476.9119	1524.0000
		Std.	**12.0426**	182.9972	224.1947
N_f	6858	Mean	**1756.42**	2841.30	2721.95
		Best	42.00	70.00	**1.00**
		Worst	**5170.00**	6010.00	14963.00
		Std.	1112.04	**595.67**	2115.57
	6477	Mean	**2322.60**	2921.40	2826.85
		Best	198.00	280.00	**1.00**
		Worst	5934.00	**5400.00**	11262.00
		Std.	1263.80	**614.18**	1992.77
	6096	Mean	**3060.30**	3068.00	3148.50
		Best	76.00	40.00	**1.00**
		Worst	9482.00	**5390.00**	12211.00
		Std.	1869.10	**761.70**	2186.12

Table 3
The best optimal PLR and its total power consumption of the GSPSA, SA [10] and RS [26] based methods from 500 runs.

CL	Optimal values	GSPSA	SA [10]	RS [26]
6858	Q_1^*	0.8058	0.8127	0.8052
	Q_2^*	0.7606	0.7422	0.7545
	Q_3^*	1.0000	1.0000	1.0000
	Q_4^*	1.0000	1.0000	1.0000
	Q_5^*	1.0000	1.0000	1.0000
	Q_6^*	0.8341	0.8461	0.8412
	$\overline{P}(Q_1^*, Q_2^*, ..., Q_6^*)$	4737.80	4739.13	4738.10
6477	Q_1^*	0.7319	0.7107	0.7251
	Q_2^*	0.6482	0.6662	0.6567
	Q_3^*	1.0000	1.0000	1.0000
	Q_4^*	1.0000	1.0000	1.0000
	Q_5^*	0.9999	1.0000	1.0000
	Q_6^*	0.7204	0.7237	0.7185
	$\overline{P}(Q_1^*, Q_2^*, ..., Q_6^*)$	4421.47	4424.77	4421.36
6096	Q_1^*	0.6485	0.6365	0.6398
	Q_2^*	0.5620	0.5660	0.5717
	Q_3^*	1.0000	0.9981	1.0000
	Q_4^*	0.9996	1.0000	1.0000
	Q_5^*	0.9995	1.0000	1.0000
	Q_6^*	0.6902	0.5994	0.5882
	$\overline{P}(Q_1^*, Q_2^*, ..., Q_6^*)$	4144.25	4146.27	4143.86

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