# Analysis and Design of Flat Asymmetrical A-Sandwich Radomes

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***Abstract*—The purpose of this paper is to present analytical formulas for designing flat A-sandwich radomes in general case where the material types and the thicknesses of the skins are different. The transmission and reflection coefficients of flat A-sandwich radomes are presented for the general asymmetrical case. Based on the obtained formulas, the conditions of zero transmission loss are derived for various special cases under the assumption that the radome materials are lossless. The maximum of transmission loss is also presented for ultra-wideband applications of A-sandwich radomes. For verification, two radomes are designed by using the equations obtained in this paper, and the results are compared with those obtained by using commercial software HFSS and CST.**

***Index Terms*—A-Sandwich; Design; Radome; Transmission Loss.**

## I. INTRODUCTION

The radome is an essential part of communication and radar systems which have outdoor antennas. It is a cover which is transparent to radio waves and protects the antenna from the environmental phenomena such as wind, rain, snow, and dust [1]. Deployed on an aircraft or a missile, the radome must also withstand highly aerodynamic and thermal stresses [2].

Radomes can be classified into various types, classes, and styles according to their operating frequency, reliability, and dielectric wall construction, respectively [3]. Among the defined radome styles, A-sandwich radome is widely used because of its wideband capabilities as well as providing good strength-to-weight ratio [4]. A-sandwich radome is constructed of three layers: two skins and a core material. The dielectric constant of the skin materials is higher than the dielectric constant of the core material [3].

The presence of the radome in front of the antenna affects the system radiation characteristics. To observe the effect of the radome, the radiation patterns of the antenna are compared with those obtained in the presence of the radome. To characterise the electrical performance of the radome, the effect of the radome is expressed in terms of some parameters such as transmission loss, bore-sight error (BSE), and bore-sight error slope (BSES) [1]. To estimate these parameters, it is usually assumed that the radome is planar. This is a good approximation for the radomes with large curvature radius with respect to the operating wavelength. Hence, it is assumed that the radome is planar throughout this paper.

A planar A-sandwich radome is a flat multilayer structure whose analysis has been well established in the literature [1,5-8]. Traditionally, the transfer matrices of successive layers and the in-between interfaces are used to obtain the transmission and reflection coefficients of the radome. Also, some analytical formulas have been presented for the design of the symmetrical A-sandwich radomes [9,10].

In some cases, the design of an asymmetrical A-sandwich radome with two different skins may be desirable due to the different weather conditions existing inside and outside of the radome. Also, it has been shown that the asymmetrical A-sandwich radomes have the capability of more broadband performance [11]. Hence, this paper aims to extend the previous works to more general cases where the materials and the thicknesses of the skins are different. The design formulas presented in this paper include the oblique incidence and can also be used for the design of A-sandwich variable thickness radomes [12].

This paper is organised as follows. The transmission and reflection coefficients of A-sandwich radome are obtained using the transfer matrices of layers. Then, the conditions of zero transmission loss are presented for various cases. The maximum value of transmission loss is also obtained for A-sandwich radomes. Finally, two A-sandwich radomes are designed based on the formulas obtained in this paper, and their associated transmission loss is presented.

## II. CALCULATION OF TRANSMISSION AND REFLECTION COEFFICIENTS

### *A. Geometry*

The structure of a three-layer radome lying in free space is shown in Figure 1. It is assumed that the layers are flat and spread infinitely parallel to the *xy* plane. The thickness and the relative electric permittivity of the layer *i* (= 1, 2, 3) are, respectively, $t_i$ and $\varepsilon_{r,i}$ while the relative magnetic permeability of all layers are unity. Hence, the characteristic impedance of layer *i* ($\eta_i$) is

$$\eta_i = \sqrt{\frac{\mu_0}{\varepsilon_0 \varepsilon_{r,i}}} = \frac{\eta_0}{n_i} \tag{1}$$

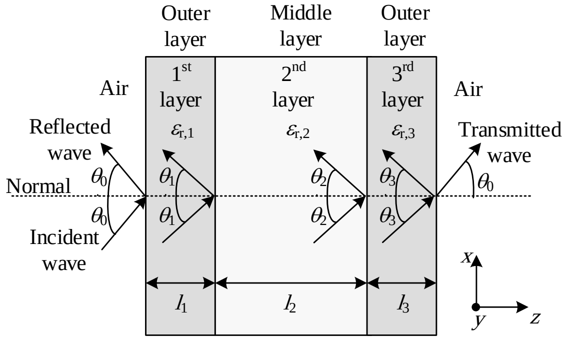


Figure 1: Structure of a flat A-sandwich radome.

where $\eta_0$ is the free space characteristic impedance, and $n_i$ is the refraction index of $i^{\text{th}}$ layer, i.e.

$$n_i = \sqrt{\varepsilon_{\mathrm{r},i}} \tag{2}$$

A uniform plane wave is an incident on the first layer of the radome propagating at an angle $\theta_0$ concerning the $z$-axis (Figure 1). The wave is refracted as it passes through different layers of the radome. According to the Snell's law, the angle between the direction of wave propagation in layer $i$ and the $z$-axis ($\theta_i$) is calculated as follows

$$\theta_i = \sin^{-1}\left(\frac{\sin(\theta_0)}{n_i}\right) \tag{3}$$

Also, the propagation constant of the wave along the $z$-axis in layer $i$ ($\gamma_{z,i}$) is as follows

$$\gamma_{z,i} = \mathrm{j}k_0 n_i \cos\theta_i = \mathrm{j}k_0\sqrt{n_i^2 - \sin^2\theta_0} \tag{4}$$

where $k_0$ is the wave number in free space and j denotes imaginary unit (i.e. $\mathrm{j} = \sqrt{-1}$ ).

### B. Analysis Formulations

Using the transfer matrix of successive layers, the radome solution takes the following form [1]

$$\begin{bmatrix} E_{\mathrm{T,i}} \\ E_{\mathrm{T,r}} \end{bmatrix} = \mathbf{A}\begin{bmatrix} E_{\mathrm{T,t}} \\ 0 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}\begin{bmatrix} E_{\mathrm{T,t}} \\ 0 \end{bmatrix} \tag{5}$$

$$\begin{aligned} \mathbf{A} = &\left(\prod_{i=1}^{i=3}\frac{1}{1+\rho_{i-1,i}}\begin{bmatrix} \mathrm{e}^{\mathrm{j}\delta_i} & \rho_{i-1,i}\mathrm{e}^{-\mathrm{j}\delta_i} \\ \rho_{i-1,i}\mathrm{e}^{\mathrm{j}\delta_i} & \mathrm{e}^{-\mathrm{j}\delta_i} \end{bmatrix}\right)\times \\ &\frac{1}{1+\rho_{3,0}}\begin{bmatrix} 1 & \rho_{3,0} \\ \rho_{3,0} & 1 \end{bmatrix} \end{aligned} \tag{6}$$

where $E_{\mathrm{T,i}}$, $E_{\mathrm{T,r}}$, and $E_{\mathrm{T,t}}$ are the transverse electric fields of the incident, reflected, and transmitted waves, respectively. Also, $\delta_i$ is the electrical thickness of the $i^{\text{th}}$ layer along the normal direction, i.e.:

$$\delta_i = -\mathrm{j}\gamma_{z,i}t_i = k_0 t_i\sqrt{n_i^2 - \sin^2\theta_0}, \quad i = 1,2,3 \tag{7}$$

In Equation (6), $\rho_{p,q}$ is the transverse reflection coefficient at the discontinuity between the successive layers $p$ and $q$ and is expressed regarding the corresponding transverse characteristic impedances as follows:

$$\rho_{p,q} = \frac{\eta_{\mathrm{T},q} - \eta_{\mathrm{T},p}}{\eta_{\mathrm{T},q} + \eta_{\mathrm{T},p}}, \quad p,q \in \{0,1,2,3\} \tag{8}$$

It is noted that in this notation, the subscript 0 denotes the medium in which the radome has been inserted (i.e. air or free space). The transverse characteristic impedance for parallel and perpendicular polarisation is defined as follows:

$$\eta_{\mathrm{T},i} = \begin{cases} \dfrac{\eta_i}{\cos(\theta_i)}, & \text{perpendicular pol.} \\ \eta_i\cos(\theta_i), & \text{parallel pol.} \end{cases} \quad i = 0,1,2,3 \tag{9}$$

Using Equations (5) and (6), the transmission and reflection coefficients of the radome are obtained as follows:

$$\begin{aligned} T = \frac{E_\mathrm{t}}{E_\mathrm{i}} = \frac{E_{\mathrm{T,t}}}{E_{\mathrm{T,i}}} = \frac{1}{A_{11}} = \frac{\mathrm{e}^{\mathrm{j}(\delta_1+\delta_2+\delta_3)}}{\mathrm{e}^{2\mathrm{j}\delta_2}}\times \\ \frac{(\rho_{0,1}+1)(\rho_{1,2}+1)(\rho_{2,3}+1)(\rho_{3,0}+1)}{(\rho_{0,1}\rho_{1,2}+\mathrm{e}^{2\mathrm{j}\delta_1})(\rho_{2,3}\rho_{3,0}+\mathrm{e}^{2\mathrm{j}\delta_3})+(\rho_{0,1}+\mathrm{e}^{2\mathrm{j}\delta_1}\rho_{1,2})(\rho_{3,0}+\mathrm{e}^{2\mathrm{j}\delta_3}\rho_{2,3})} \end{aligned} \tag{10}$$

$$\begin{aligned} \Gamma = \frac{E_\mathrm{r}}{E_\mathrm{i}} = \frac{E_{\mathrm{T,r}}}{E_{\mathrm{T,i}}} = \frac{A_{21}}{A_{11}} = \frac{\mathrm{e}^{2\mathrm{j}\delta_2}}{\mathrm{e}^{2\mathrm{j}\delta_2}}\times \\ \frac{(\rho_{1,2}+\mathrm{e}^{2\mathrm{j}\delta_1}\rho_{0,1})(\rho_{2,3}\rho_{3,0}+\mathrm{e}^{2\mathrm{j}\delta_3})+(1+\mathrm{e}^{2\mathrm{j}\delta_1}\rho_{0,1}\rho_{1,2})(\rho_{3,0}+\mathrm{e}^{2\mathrm{j}\delta_3}\rho_{2,3})}{(\rho_{0,1}\rho_{1,2}+\mathrm{e}^{2\mathrm{j}\delta_1})(\rho_{2,3}\rho_{3,0}+\mathrm{e}^{2\mathrm{j}\delta_3})+(\rho_{0,1}+\mathrm{e}^{2\mathrm{j}\delta_1}\rho_{1,2})(\rho_{3,0}+\mathrm{e}^{2\mathrm{j}\delta_3}\rho_{2,3})} \end{aligned} \tag{11}$$

where $E_\mathrm{i}$, $E_\mathrm{r}$, and $E_\mathrm{t}$ are the electric fields of the incident, reflected, and transmitted waves, respectively.

In case $n_1 = n_3$ and $t_1 = t_3$, Equations (10) and (11) are reduced as follows

$$T = \frac{\mathrm{e}^{\mathrm{j}(2\delta_1+\delta_2)}(\rho_{0,1}^2-1)(\rho_{1,2}^2-1)}{\mathrm{e}^{2\mathrm{j}\delta_2}(\mathrm{e}^{2\mathrm{j}\delta_1}+\rho_{0,1}\rho_{1,2})^2-(\rho_{0,1}+\mathrm{e}^{2\mathrm{j}\delta_1}\rho_{1,2})^2} \tag{12}$$

$$\begin{aligned} \Gamma = &\frac{\mathrm{e}^{2\mathrm{j}\delta_2}(\rho_{1,2}+\mathrm{e}^{2\mathrm{j}\delta_1}\rho_{0,1})(\rho_{1,2}\rho_{0,1}+\mathrm{e}^{2\mathrm{j}\delta_1})}{\mathrm{e}^{2\mathrm{j}\delta_2}(\mathrm{e}^{2\mathrm{j}\delta_1}+\rho_{0,1}\rho_{1,2})^2-(\rho_{0,1}+\mathrm{e}^{2\mathrm{j}\delta_1}\rho_{1,2})^2} - \\ &\frac{(1+\mathrm{e}^{2\mathrm{j}\delta_1}\rho_{0,1}\rho_{1,2})(\rho_{0,1}+\mathrm{e}^{2\mathrm{j}\delta_1}\rho_{1,2})}{\mathrm{e}^{2\mathrm{j}\delta_2}(\mathrm{e}^{2\mathrm{j}\delta_1}+\rho_{0,1}\rho_{1,2})^2-(\rho_{0,1}+\mathrm{e}^{2\mathrm{j}\delta_1}\rho_{1,2})^2} \end{aligned} \tag{13}$$

It is noted that the transmission loss of the radome is due to two factors: *a*) dielectric loss of the radome layers and *b*) reflection from the radome. If it is assumed that the dielectric loss is zero, the reflection is the only factor that causes the transmission loss, and the following relation holds

$$|T|^2 = 1 - |\Gamma|^2 \tag{14}$$

## III. CONDITIONS OF ZERO TRANSMISSION LOSS

Optimal design of a radome aims to maximise the magnitude of the radome transmission coefficient ($|T|$) within the desired frequency range. In other words, it is ideal to have a radome with $|T| = 1$. It is clear that $|T| < 1$ if the layers of the radome are made of lossy materials. Hence, the first condition for having $|T| = 1$ is that all three layers should be lossless (i.e. $n_i \in \text{Reals}, \ i = 1,2,3$).

For a radome with lossless materials, the condition of having $|T| = 1$ is equivalent to the condition of $\Gamma = 0$. To avoid the magnitude calculation of $T$, the equivalent condition of having zero reflection (i.e. $\Gamma = 0$) is derived. According to Equation (11), the following equation should be satisfied in order to have $\Gamma = 0$

$$\begin{aligned} \mathrm{e}^{2\mathrm{j}\delta_2}(\rho_{1,2}+\mathrm{e}^{2\mathrm{j}\delta_1}\rho_{0,1})(\rho_{2,3}\rho_{3,0}+\mathrm{e}^{2\mathrm{j}\delta_3}) = \\ -(1+\mathrm{e}^{2\mathrm{j}\delta_1}\rho_{0,1}\rho_{1,2})(\rho_{3,0}+\mathrm{e}^{2\mathrm{j}\delta_3}\rho_{2,3}) \end{aligned} \tag{15}$$

Assuming that $\rho_{1,2}+\mathrm{e}^{2\mathrm{j}\delta_1}\rho_{0,1}\neq 0$ Equation (15) can be rewritten as follows:

$$\mathrm{e}^{2\mathrm{j}\delta_2}=-\frac{\left(1+\mathrm{e}^{2\mathrm{j}\delta_1}\rho_{0,1}\rho_{1,2}\right)\left(\rho_{3,0}+\mathrm{e}^{2\mathrm{j}\delta_3}\rho_{2,3}\right)}{\left(\rho_{1,2}+\mathrm{e}^{2\mathrm{j}\delta_1}\rho_{0,1}\right)\left(\rho_{2,3}\rho_{3,0}+\mathrm{e}^{2\mathrm{j}\delta_3}\right)} \tag{16}$$

This equation can be satisfied only if the magnitude of the right side is unity, i.e.:

$$\left|\frac{\left(1+\mathrm{e}^{2\mathrm{j}\delta_1}\rho_{0,1}\rho_{1,2}\right)\left(\rho_{3,0}+\mathrm{e}^{2\mathrm{j}\delta_3}\rho_{2,3}\right)}{\left(\rho_{1,2}+\mathrm{e}^{2\mathrm{j}\delta_1}\rho_{0,1}\right)\left(\rho_{2,3}\rho_{3,0}+\mathrm{e}^{2\mathrm{j}\delta_3}\right)}\right|=1 \tag{17}$$

The condition of Equation (17) can be rewritten as follows:

$$\begin{aligned}&\frac{1+\rho_{0,1}^2\rho_{1,2}^2+2\rho_{0,1}\rho_{1,2}\cos\left(2\delta_1\right)}{\rho_{0,1}^2+\rho_{1,2}^2+2\rho_{0,1}\rho_{1,2}\cos\left(2\delta_1\right)}=\\&\qquad\frac{1+\rho_{2,3}^2\rho_{3,0}^2+2\rho_{2,3}\rho_{3,0}\cos\left(2\delta_3\right)}{\rho_{2,3}^2+\rho_{3,0}^2+2\rho_{2,3}\rho_{3,0}\cos\left(2\delta_3\right)}\end{aligned} \tag{18}$$

This equation shows that for a given set of $\{n_1, n_2, n_3, t_1\}$ one may find some values of $t_3$ that can lead to the zero-transmission loss. After determining $t_3$, $t_2$ can be found by using Equation (16).

In case $n_1 = n_3$, then $\rho_{3,0}=-\rho_{0,1},\rho_{2,3}=-\rho_{1,2}$ and Equations (15) and (18) are reduced to the following equations, respectively:

$$\tan^2\left(\delta_1\right)=\tan^2\left(\delta_3\right) \tag{19}$$

$$\begin{aligned}&\left(\rho_{0,1}+\mathrm{e}^{-2\mathrm{j}\delta_1}\rho_{1,2}\right)\left(1+\mathrm{e}^{-2\mathrm{j}\delta_1}\rho_{0,1}\rho_{1,2}\right)\mathrm{e}^{2\mathrm{j}\delta_2+4\mathrm{j}\delta_1}=\\&\qquad\left(\rho_{0,1}+\mathrm{e}^{2\mathrm{j}\delta_1}\rho_{1,2}\right)\left(1+\mathrm{e}^{2\mathrm{j}\delta_1}\rho_{0,1}\rho_{1,2}\right)\end{aligned} \tag{20}$$

It is noted that both of Equations (19) and (20) should be satisfied in order to have $|T| = 1$. Also, it is reminded that $\delta_i$ ($i$ = 1, 2, 3) has been defined in Equation (7) and $\tan(\delta_i)$ should not be confused with loss tangent. According to the Equation (19), the electrical thickness of the skin layers should satisfy:

$$\delta_1-\delta_3=\nu\pi \tag{21}$$

where $\nu$ is an integer number.

In addition to the assumption of the same material type for the skins, if the core layer is also assumed to be filled with air (i.e. $n_2 = 1$), then $\rho_{1,2}=-\rho_{0,1}$ and Equation (20) is reduced to the following equations:

$$\mathrm{e}^{\mathrm{j}2\delta_1}=1 \tag{22}$$

$$\mathrm{e}^{2\mathrm{j}\delta_2}=-\mathrm{e}^{-2\mathrm{j}\delta_1}\frac{1-\mathrm{e}^{2\mathrm{j}\delta_1}\rho_{0,1}^2}{1-\mathrm{e}^{-2\mathrm{j}\delta_1}\rho_{0,1}^2} \tag{23}$$

In other words, the condition expressed in Equation (20) is met if either of the Equations (22) or (23) is satisfied.

## IV. MAXIMUM OF TRANSMISSION LOSS

In ultra-wideband applications, it is desirable to estimate the minimum possible value of the radome transmission coefficient. In this section, the contribution of the dielectric loss of the radome to transmission loss is ignored, and the loss of the layers is assumed to be zero. This assumption is a good approximation for radomes with thin and low loss layers.

In A-sandwich radomes, the materials are usually chosen such that $n_1$, $n_3 > n_2 \geq 1$. Therefore,

$$\begin{aligned}&\rho_{1,2},\rho_{3,0}>0\\&\rho_{0,1},\rho_{2,3}<0\end{aligned} \tag{24}$$

It is noted that Equation (24) holds for all incident angles in perpendicular polarisation whereas it only holds for the low incident angles (incidence near the normal direction) for parallel polarisation.

For a given A-sandwich radome, the minimum value of $|T|$ occurs when the magnitude of the denominator in Equation (10) becomes maximum. Concerning Equation (24), the denominator is maximised when

$$\mathrm{e}^{\mathrm{j}\delta_1}=\mathrm{e}^{\mathrm{j}\delta_2}=\mathrm{e}^{\mathrm{j}\delta_3}=-1 \tag{25}$$

Therefore, the minimum possible value of the transmission coefficient is obtained as follows:

$$\begin{aligned}|T|_{\min}&=\frac{\left(\rho_{0,1}+1\right)\left(\rho_{1,2}+1\right)\left(\rho_{2,3}+1\right)\left(\rho_{3,0}+1\right)}{\left(\rho_{0,1}\rho_{1,2}-1\right)\left(\rho_{2,3}\rho_{3,0}-1\right)-\left(\rho_{0,1}-\rho_{1,2}\right)\left(\rho_{3,0}-\rho_{2,3}\right)}\\&=\frac{2\eta_{\mathrm{T},0}\eta_{\mathrm{T},1}\eta_{\mathrm{T},2}\eta_{\mathrm{T},3}}{\eta_{\mathrm{T},0}^2\eta_{\mathrm{T},2}^2+\eta_{\mathrm{T},1}^2\eta_{\mathrm{T},3}^2}\end{aligned} \tag{26}$$

## V. SIMULATION RESULTS

Based on the equations derived in this paper, two separate computer codes were developed in MATLAB [13] for the design and the transmission loss calculations. In this section, the results of the codes are presented for the design of two radomes. In the first design, the effect of loss of radome materials on the design accuracy is considered. In the second case, an asymmetrical A-sandwich radome is designed for an oblique incidence.

### *A. Radome #1*

In the first problem, a radome is to be designed to have a zero-transmission loss at $f$ = 10 GHz for the normal incidence (i.e. $\theta_0 = 0°$ with respect to Figure 1). The skins have a dielectric constant of 4.4, and a loss tangent of 0.016 and their thickness is $t_1 = t_3$ = 1 mm. Also, the core has a dielectric constant of 1.1 and a loss tangent of 0.001.

Neglecting the loss of the radome materials, the reflection coefficients at the interfaces are $\rho_{0,1}=-\rho_{3,0}\approx -0.3543$ and $\rho_{1,2}=-\rho_{2,3}\approx 0.3333$. Also, the electrical thickness of the skins are $\delta_1=\delta_3\approx 0.4396$. To obtain the optimal thickness of the core, Equation (20) can be reformulated as follows:

$$\mathrm{e}^{2\mathrm{j}\delta_2}=\mathrm{e}^{-4\mathrm{j}\delta_1}\frac{\left(\rho_{0,1}+\mathrm{e}^{2\mathrm{j}\delta_1}\rho_{1,2}\right)\left(1+\mathrm{e}^{2\mathrm{j}\delta_1}\rho_{0,1}\rho_{1,2}\right)}{\left(\rho_{0,1}+\mathrm{e}^{-2\mathrm{j}\delta_1}\rho_{1,2}\right)\left(1+\mathrm{e}^{-2\mathrm{j}\delta_1}\rho_{0,1}\rho_{1,2}\right)} \tag{27}$$

According to Equation (27), $\delta_2 \approx 1.0980$ and the minimum optimal thickness of the core is obtained as $t_2 \approx 4.9949 mm$.

The reflection coefficient of the designed radome is calculated for two cases of lossless and lossy materials, and the associated transmission loss is plotted in Figure 2 and Figure 3, respectively. In these figures, the results obtained from two well-known commercial software HFSS [14] and CST [15] also accompany the analytical results. These figures show that there is a good coincidence between the analytical results and those obtained from the commercial software. As seen in Figure 2, the transmission loss becomes zero at 10 GHz when loss of materials is neglected. If the loss of the materials is included in calculations, Figure 3 shows that the minimum of transmission loss occurs at 9.951 GHz with the level of 0.0304 dB. This implies that neglecting the loss of the materials in radome design has caused a shift of less than 0.5% in the design frequency.

To observe the behaviour of the radome at larger bandwidth, the transmission loss of the designed radome from 1 GHz up to 110 GHz is plotted in Figure 4 for two cases of neglecting and including the loss of radome materials. According to Equation (26), the transmission loss will be less than 6.9145 dB if the loss of materials is neglected. This limit is also plotted with a dotted line in Figure 4. As seen in this figure, the curve of transmission loss does not exceed the limit even at large frequencies of about 100 GHz. This is because the imaginary parts of $\delta_1$ and $\delta_2$, which are, respectively, 0.0088 and 0.0055, are much less than unity. It is noted that the transmission loss exceeds the limit (i.e. 6.9145 dB) at about 245 GHz due to the loss of the radome materials.

*B. Radome #2*

In the next problem, a radome is to be designed to have a zero-transmission loss at 10 GHz for a parallel polarised plane wave with the incidence angle of $\theta_0 = 30°$ (Figure 1). The layer #1 has a dielectric constant of $\varepsilon_{r,1} = 4.4$ and its thickness is $t_1 = 1$ mm. The layers #2 and #3 have dielectric constants of 1.1 and 2.1, respectively.

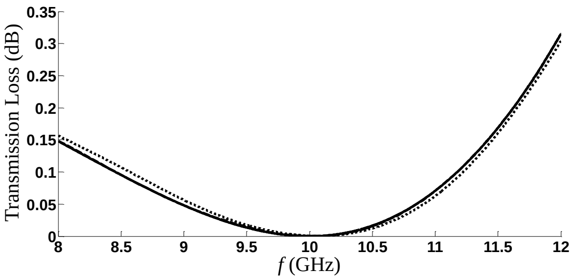


Figure 2: Transmission loss of the radome #1 in X-band if the loss of materials is neglected in simulations: Analytical results (solid line); HFSS (dashed line); CST (dotted line).

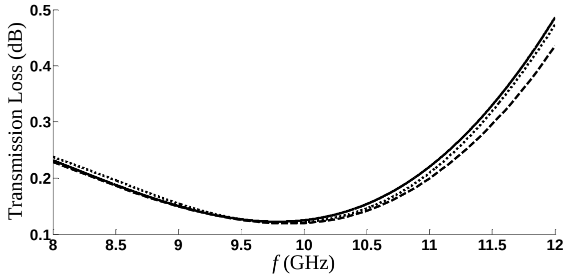


Figure 3: Transmission loss of the radome #1 in X-band if the loss of materials is included in simulations: Analytical results (solid line); HFSS (dashed line); CST (dotted line).

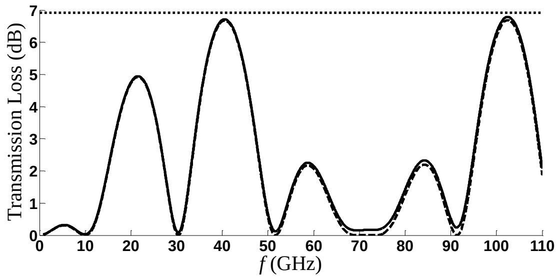


Figure 4: Transmission loss of the radome #1 from 1 GHz up to 110 GHz for two cases where the loss of materials is neglected (dashed line) and the loss of materials is included (solid line). The dotted line represents the maximum possible value.

Using Equations (8) and (9), the interface reflection coefficients are $\rho_{0,1} \approx -0.3033$, $\rho_{1,2} \approx 0.2883$, $\rho_{2,3} \approx -0.1282$, and $\rho_{3,0} \approx 0.1442$. Also, the electrical thickness of layer #1 is $\delta_1 \approx 0.4270$ Using Equation (18), the electrical thickness of layer #3 is calculated as $\delta_3 = 1.3195$ which leads to the thickness of $t_3 = 4.6288$ mm. Then, using Equation (16) the electrical thickness of layer #2 is calculated as $\delta_2 = 0.6936$ which in turn results in the thickness of $t_2 = 3.5893$ mm. It is noted that Equation (18) does not have a real value solution for $t_3$ if the incident angle is greater than about 35.2°.

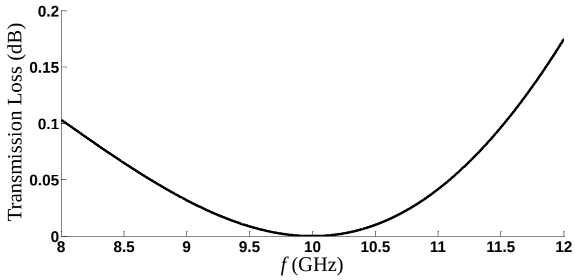


Figure 5: Transmission loss of the radome #2 in X-band.

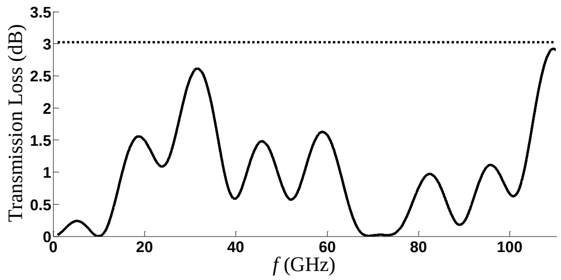


Figure 6: Transmission loss of the radome #2 at frequencies up to 110 GHz. The dotted line represents the maximum possible value.

The transmission loss of the designed radome is plotted in Figure 5. As shown in this figure, the transmission loss is zero at 10 GHz as expected. To observe the behaviour of the radome at larger bandwidth, the transmission loss of the designed radome is plotted in Figure 6 over the range from 1 GHz up to 110 GHz. According to Equation (26), the transmission loss will always be less than 3.0263 dB. This limit is also plotted with a dotted line in Figure 6. As seen in this figure, the curve of the transmission loss never crosses the limit (even at higher frequencies which have not been shown here) because the materials of this radome have been assumed to be lossless.

## VI. Conclusions

Using the transfer matrices of the successive radome layers, the transmission and reflection coefficients of flat A-sandwich radomes were obtained for the general case where the incidence angle is oblique, and the materials of the skins are different. Based on the obtained formulas, the conditions of zero transmission loss were derived for various general and special cases. The maximum possible value of transmission loss was also presented for ultra-wideband applications of A-sandwich radomes.

Two radomes with typical values of dielectric constants were designed by using the equations presented in this paper. In the first case, the incident wave was normal to the radome surface, and the radome materials were lossy. The analytical results of transmission loss had a good coincidence with those obtained from HFSS and CST confirming the validity of the derived analytical equations. The loss of the radome materials caused a shift of less than 0.5% in the optimal frequency. Also, the transmission loss of the radome was less than the presented limit at frequencies up to 110 GHz. In the second case, the radome was designed for an oblique incidence, and its associated transmission loss was presented in X-band and at frequencies up to 110 GHz. This work can be extended to the design of more complex structures such as C-sandwich radomes [3].

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