A Variable Sampling Interval Multivariate Exponentially Weighted Moving Average Control Chart Based on Median Time-to-Signal

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***Abstract*—In this study, the median time-to-signal (MTS) is used as an alternative measure to the average time-to-signal (ATS) in evaluating the performance of the variable sampling interval (VSI) multivariate exponentially weighted moving average (MEWMA) chart. Although the ATS is one of the most commonly used performance measures when the sampling interval is varied, it is not an accurate representation of the entire time-to-signal distribution of the VSI charts. Therefore, the percentage points (percentiles) of the time-to-signal distribution are provided for a more comprehensive study of the VSI MEWMA chart. A Monte Carlo simulation is used to calculate the MTS values for various magnitudes of shifts in the process mean vector. The optimal design strategy is to find the charting parameters having the minimum out-of-control MTS (MTS_1). A comparison study shows that the VSI MEWMA chart is more effective than the standard MEWMA chart with fixed sampling interval, in detecting shifts in the process mean vector in terms of the MTS.**

***Index Terms*—Median Time-To-Signal; Multivariate Exponentially Weighted Moving Average Chart; Simulation; Variable Sampling Interval.**

I. INTRODUCTION

One of the most popular statistical tools for continuous process monitoring is the control charts. In most practical situations, the quality of a product may depend on two or more characteristics which need to be monitored simultaneously to control the quality of the whole process and consequently, gives rise to the development of the multivariate control charts. For example, the multivariate exponentially weighted moving average (MEWMA) chart introduced by Lowry et al. [1] was found to be more effective than the Hotelling's T^2 chart in detecting small changes in the process mean vector. For recent literature on the MEWMA chart, see Chen et al. [2], Nishimura et al. [3], Park and Jun [4], Cheng et al. [5], Kim et al. [6] and Saleh and Mahmoud [7].

The usual practice of using a control chart for process monitoring is to take samples of fixed size from the process at a fixed sampling interval. The variable sampling interval (VSI) is one of the adaptive features of interest in control chart applications. The VSI charts work by varying the sampling interval according to the plotted statistics of the process data. Numerous findings showed that the VSI charts are able to detect shifts in the process faster than their corresponding standard charts. For the detailed discussion on recent development of the VSI charts, interested reader may refer to Chew et al. [8], Ershadi et al. [9], Guo and Wang [10], Patil and Shirke [11], Zhang et al. [12] and Amdouni et al. [13].

The statistical performance of the control charts is usually evaluated in terms of their run-length, which is the number of plotted chart statistic until an out-of-control signal is detected. For example, the average run-length (ARL) is commonly used as a performance measure for control charts. However, the time-to-signal is not a constant multiple of the ARL when the sampling interval is varied. Hence, the average time-to-signal (ATS), which is defined as the average time from the beginning of a process until the chart triggers an out-of-control signal is often used in the VSI charts [14]. In practice, the steady-state case is usually considered, where the process is initially in-control and then shifted out-of-control at some random time in the future suggesting the use of the steady-state average time-to-signal (SSATS) [15].

It is sensible to note that the ATS and SSATS do not represent the entire time-to-signal distribution of the VSI charts. The performance measure with respect to the percentage points (percentiles) provides a meaningful overview of the time-to-signal distribution [16]. This study complements the work of Lee and Khoo [17] such that the median time-to-signal (MTS) is employed as a new performance measure for the VSI MEWMA chart for the zero-state and steady-state cases. To the best of the authors' knowledge, papers dealing with the use of MTS in studying the performance of the multivariate control charts are not yet available in the literature. Hence, the objective of this study is to present a VSI MEWMA chart based on the MTS.

The rest of this article is structured as follows: Section II provides a review of the VSI MEWMA chart. Section III discusses the optimal statistical design of the VSI MEWMA chart based on the MTS. Section IV illustrates the time-to-signal distribution for a better understanding of the VSI MEWMA chart. Section V provides a comparison between the performances of the VSI MEWMA chart and the standard MEWMA chart with fixed sampling interval. Finally, Section VI concludes the paper.

II. A REVIEW OF THE VSI MEWMA CHART

The standard MEWMA chart takes samples from a process, each of size n at a fixed sampling interval h for process monitoring. The VSI MEWMA chart is implemented by deciding on the time for the next sampling according to the current plotted statistic on the standard MEWMA chart. In addition to the control limit H, a warning limit w is introduced for the VSI MEWMA chart such that $0 < w < H$. The value of w is used to determine the change between the long sampling interval h_1 and the short sampling interval h_2, where $h_2 < h < h_1$. Following the implementation of the VSI feature in Castagliola et al. [18], the values of h_1 and h_2 are chosen to obtain an in-control average sampling interval $E(h)$, such that $E(h) = h$. The in-control average sampling interval of the control charts is defined as follows:

$$E(h) = \mathrm{ATS}_0 / \mathrm{ARL}_0 \tag{1}$$

where ATS_0 and ARL_0 are the in-control average time-to-signal and the in-control average run-length, respectively.

In this study, it is assumed that we have an independent and identical observation $\boldsymbol{X}_k^T = (X_1, X_2, \ldots, X_p)$ with p random variables following a multivariate normal distribution of $\boldsymbol{X}_k \sim N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma}_0)$ where $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}_0$ are the known process mean vector and the known in-control covariance matrix, respectively. The objective of this process monitoring is to detect a shift in the process mean vector $\boldsymbol{\mu}$. At sampling point t, the sample mean vector $\bar{X}_{kt}$ is given as $\bar{X}_{kt} = \frac{1}{n}\sum_{v=1}^{n} X_{vkt}$ for observation $v = 1, 2, \ldots, n$. Then, the corresponding standardized sample mean for quality characteristic k at sampling point t is given as:

$$Z_{kt} = \frac{\sqrt{n}\left(\bar{X}_{kt} - \mu_{0k}\right)}{\sigma_{0k}}, \; k = 1, 2, \ldots, p \tag{2}$$

where $\boldsymbol{\mu}_{0K}$ and $\boldsymbol{\sigma}_{0k}$ are the kth component of the in-control process mean vector $\boldsymbol{\mu}_0$ and the kth component of the in-control standard deviation vector $\boldsymbol{\sigma}_0$, respectively.

Lowry et al. [1] proposed the MEWMA vector as follows:

$$\mathbf{W}_t = r\mathbf{Z}_t + (1-r)\mathbf{W}_{t-1}, \text{ for t} = 1, 2, 3, \ldots, \tag{3}$$

where $\boldsymbol{W}_0$ is a zero vector and r is the smoothing constant, such that $0 < r \leq 1$. The plotted statistic on the MEWMA chart is defined as follows:

$$T_t^2 = \mathbf{W}_t^T \boldsymbol{\Sigma}_{\mathbf{W}}^{-1} \mathbf{W}_t, \text{ for } t = 1, 2, 3, \ldots, \tag{4}$$

where $\boldsymbol{\Sigma}_{\mathbf{W}}$ is the covariance matrix of $\mathbf{W}_t$. Here, the asymptotic covariance matrix $\boldsymbol{\Sigma}_{\mathbf{W}} = \left(\frac{r}{2-r}\right)\boldsymbol{\Sigma}_{\mathbf{Z}}$ is used, where $\boldsymbol{\Sigma}_{\mathbf{Z}}$ is the correlation matrix of Z_t. The MEWMA chart signals when the plotted chart statistic $T_t^2 > H$ indicating that the process is out-of-control.

Furthermore, Lowry et al. [1] have shown that the performance of a MEWMA chart depends only on the shift in the process mean vector through the non-centrality parameter. This shift is defined as the square root of the non-centrality parameter, i.e. $\delta = \sqrt{\mathbf{v}^T \boldsymbol{\Sigma}_Z^{-1} \mathbf{v}}$, where $\mathbf{v}$ is the standardized mean vector, such that the kth component of $\mathbf{v}$ is $v_k = (\mu_k - \mu_{0k})/\sigma_{0k}$, where μ_k is the kth component of the out-of-control process mean vector $\boldsymbol{\mu}_1$. The value of the performance measure is the same for any out-of-control process mean vectors that have the same distance from the in-control process mean vector because of the directional invariance property of the MEWMA chart, where $\boldsymbol{\Sigma}_{\boldsymbol{Z}}$ remains the same.

The VSI MEWMA chart consists of three regions which are the safety, warning and out-of-control regions. The two sampling intervals of the VSI MEWMA chart function is given as:

$$h = \begin{cases} h_1 & \text{if } T_t^2 \leq w \\ h_2 & \text{if } w < T_t^2 \leq H \end{cases}.$$

If a sample falls inside the safety region $(T_t^2 \leq w)$, then the next sample should be taken at a long sampling interval h_1. In contrast, if a sample falls inside the warning region $(w < T_t^2 \leq H)$, then the next sample should be taken at a short sampling interval h_2. If a sample falls in the action region $(T_t^2 > H)$, then the process is out-of-control and a corrective action is needed to identify and eliminate the assignable cause(s) to bring the process back into the in-control state. The first sampling interval when the process monitoring has just started can be chosen at random. Alternatively, we can use tightening control as recommended by most researchers, i.e. Costa [19], Chen et al. [20] and Guo and Wang [10], where the short sampling interval h_2 is used as it provides an additional protection against problems that may arise during start-up or restart-up.

III. Optimal Design of the VSI MEWMA Chart

The optimal statistical design of the VSI MEWMA chart, based on the MTS involves the computation of five charting parameters, i.e. the smoothing constant r, the control limit H, the warning limit w, the long sampling interval h_1 and the short sampling interval h_2 which minimizes the out-of-control MTS (MTS_1) for both the zero-state and steady-state cases for the given values of the in-control median time-to-signal MTS_0, the sample size n, the in-control average sampling interval $E(h)$ and the number of quality characteristics p.

A Monte Carlo simulation is conducted using the Statistical Analysis System (SAS) to compute the MTS of the VSI MEWMA chart. Monte Carlo simulation has been used extensively to obtain the different performance measures of the control charts. For example, see Yew et al. [21] and Cheng et al. [5]. Following Graham et al. [22] and Huang et al. [23], 100 000 simulation runs are performed to obtain the MTS_1 of the VSI MEWMA chart.

Referring to Graham et al. [24], the in-control average run-length of $\mathrm{ARL}_0 \approx 500$ is used as the industry standard value and it corresponds to an in-control median run-length of $\mathrm{MRL}_0 \approx 350$ for the standard charts. Hence, the in-control median time-to-signal of $\mathrm{MTS}_0 = 350$ is chosen because the time-to-signal is simply the multiplication of the run-length and the fixed sampling interval for the standard charts, where we use $h = 1$. The control limits H are computed to obtain the specified MTS_0. On the other hand, the warning limits w are computed for each combination of (h_1, h_2) to obtain the in-control average sampling interval (see Equation 1), where the ATS_0 and ARL_0 are obtained from the SAS program.

Some constraints are considered for the optimal design of the VSI MEWMA chart based on the MTS. The smoothing constant is set with a step size of 0.005 for $0 < r \leq 0.10$ and a step size of 0.01 for $0.10 < r \leq 1.00$. For the VSI MEWMA chart, only two sampling intervals are used, i.e. h_1 and h_2 such that $E(h) = h$. Here, the short sampling interval is set to be $h_2 = 0.1$ and the long sampling interval is set to be $h_1 = h + 0.1$, $h + 0.2$, $h + 0.3$, …, 3.0, where $h_2 < h < h_1$ so that the $E(h) = h = 1.0$.

IV. Time-to-Signal Distribution of the VSI MEWMA Chart

The percentage points of the time-to-signal for the steady-state VSI MEWMA chart with $E(h) = 1$, $n = 3$, $p = 4$, $\delta_{opt} \in \{0.50, 1.00\}$ and $ATS_0 \approx 500$ are presented in Table 1 to give information regarding the performance of the VSI MEWMA chart. The 1st, 5th, 10th, 20th, 30th, 40th, 50th, 60th, 70th, 80th and 90th percentage points (or percentiles) of the time-to-signal distribution are computed using the SAS program. Note that the 50th percentile is the MTS. Table 1 shows that the MTS_0 value of the VSI MEWMA chart is less than the corresponding ATS_0 value. For example, for the VSI MEWMA chart with $(r, H, w, h_1, h_2) = (0.130, 15.587, 2.849, 2.2, 0.1)$, the MTS_0 is 335.8 while the ATS_0 is 487.41. Based on the entire time-to-signal distribution, the ATS_0 lies between the 60th and 70th percentiles of the time-to-signal distribution.

From Table 1, it is observed that the ATS_1 values are larger than the corresponding MTS_1 values for all the given δ_{opt} except for $\delta_{opt} = 1.50$. For example, when $\delta_{opt} = 0.50$, $E(h) = 1$, $n = 3$ and $p = 4$ are considered, $ATS_1 = 47.01$ but $MTS_1 = 33.2$ ($< ATS_1$) at $\delta = 0.25$ for the VSI MEWMA chart with the optimal charting parameters $(r, H, w, h_1, h_2) = (0.130, 15.587, 2.849, 2.2, 0.1)$. This example shows that practitioner who tends to equate the ATS and the MTS is incorrect and misleading.

In addition, the smaller percentage points (1st to 10th percentiles) of the time-to-signal distribution in Table 1 when the process is in-control ($\delta = 0$) shows the early false alarm rates of the VSI MEWMA chart. Next, the higher percentage points (> 60th percentiles) when the process shifts by a certain magnitude show the probabilities that the VSI MEWMA chart would trigger an out-of-control signal. From these results, it can be concluded that the percentiles of the time-to-signal distribution provide a more meaningful interpretation for the in-control and out-of-control performances of the VSI MEWMA chart, which is not possible with the ATS alone.

Table 1
Percentage points of the time-to-signal distribution for the steady-state VSI MEWMA chart when $ATS_0 \approx 500$.

Optimal charting parameters	δ	ATS	Percentage points										
			1st	5th	10th	20th	30th	40th	50th	60th	70th	80th	90th
	0.00	487.41	0.6	18.6	45.5	104.0	169.8	246.0	335.8	444.8	586.1	789.2	1131.8
$\delta_{opt} = 0.50$	0.25	47.01	0.4	1.9	5.2	11.4	17.7	24.9	33.2	43.5	56.5	75.1	107.0
$(r, H, w, h_1, h_2) =$	0.50	7.70	0.3	0.6	0.9	1.8	3.2	4.8	6.0	7.8	9.8	12.3	16.6
(0.130, 15.587,	1.00	2.92	0.2	0.3	0.4	0.5	0.6	1.0	2.6	2.8	4.7	4.9	7.0
2.849, 2.2, 0.1)	1.50	1.98	0.1	0.2	0.2	0.3	0.4	0.5	2.3	2.4	2.5	4.5	4.7
	2.00	1.56	0.1	0.2	0.2	0.2	0.3	0.3	0.5	2.3	2.4	2.4	4.5
	0.00	495.99	2.5	23.9	51.1	109.3	174.7	251.1	342.8	453.3	598.6	799.0	1153.1
$\delta_{opt} = 1.00$	0.25	133.39	0.6	6.8	14.1	30.1	47.9	68.7	92.9	122.1	159.9	214.2	306.4
$(r, H, w, h_1, h_2) =$	0.50	17.67	0.2	0.9	2.5	4.7	6.9	9.7	12.7	16.5	21.2	27.9	39.3
(0.340, 16.630,	1.00	2.55	0.1	0.2	0.3	0.5	1.0	2.1	2.3	2.5	3.2	4.2	4.9
3.331, 1.9, 0.1)	1.50	1.51	0.1	0.1	0.2	0.3	0.3	0.5	2.0	2.1	2.1	2.2	2.6
	2.00	1.11	0.1	0.1	0.1	0.2	0.2	0.2	0.3	2.0	2.0	2.0	2.1

Note: Optimal charting parameters are obtained from Table 4 in [17].

V. Performance Comparison Between the VSI MEWMA Chart and the Standard MEWMA Chart Based on the Median Time-to-Signal

Table 2 shows the optimal charting parameters (r, H, w, h_1, h_2) of the VSI MEWMA chart for $p \in \{2, 5\}$, $n = 5$, $E(h) = 1$ and $MTS_0 = 350$ at the specific shifts $\delta \in \{0.25, 0.50, 1.00, 1.50, 2.00, 2.50\}$. The MTS_1 values for the VSI MEWMA chart are compared with the MTS_1 values for the standard MEWMA chart. The results in Table 2 shows that the MTS_1 values for the VSI MEWMA chart are smaller than the standard MEWMA chart for all the shifts in the process mean vector. This result shows that the performance of the VSI MEWMA chart is better than the standard MEWMA chart for both the zero-state and steady-state cases.

Table 2
Zero-state and steady-state optimal charting parameters and the corresponding MTS_1 values for the VSI MEWMA chart and the standard MEWMA chart when $MTS_0 = 350$

δ_{opt}	p	Zero-state				Steady-state			
		VSI MEWMA		Standard MEWMA		VSI MEWMA		Standard MEWMA	
		(r, H, w, h_1, h_2)	MTS_1	(r, H)	MTS_1	(r, H, w, h_1, h_2)	MTS_1	(r, H)	MTS_1
0.25	2	(0.095, 10.700, 0.720, 3.0, 0.1)	12.5	(0.11, 10.929)	26	(0.010, 5.999, 0.5378, 3.0, 0.1)	7.9	(0.10, 10.790)	25
	5	(0.090, 16.918, 2.994, 3.0, 0.1)	20.3	(0.05, 15.827)	34	(0.010, 11.070, 2.391, 3.0, 0.1)	6.3	(0.05, 15.827)	32
0.50	2	(0.280,11.985, 0.739, 3.0, 0.1)	3.3	(0.16, 11.419)	9	(0.010, 5.999, 0.538, 3.0, 0.1)	2.8	(0.19, 11.614)	9
	5	(0.250, 18.390, 3.121, 2.9, 0.1)	4.7	(0.17, 17.928)	12	(0.010, 11.070, 2.391, 3.0, 0.1)	2.4	(0.15, 17.766)	11

δ_{opt}	p	Zero-state				Steady-state			
		VSI MEWMA		Standard MEWMA		VSI MEWMA		Standard MEWMA	
		(r, H, w, h_1, h_2)	MTS_1	(r, H)	MTS_1	(r, H, w, h_1, h_2)	MTS_1	(r, H)	MTS_1
1.00	2	(0.480, 12.320, 2.057, 1.5, 0.1)	0.3	(0.50, 12.305)	3	(0.455, 12.300, 0.847, 2.7, 0.1)	0.4	(0.43, 12.245)	3
	5	(0.440, 18.793, 3.709, 2.3, 0.1)	0.4	(0.40, 18.680)	4	(0.305, 18.558, 3.056, 3.0, 0.1)	0.5	(0.30, 18.499)	4
1.50	2	(0.620,12.410, 4.587, 1.1, 0.1)	0.2	(0.30, 12.030)	2	(0.905, 12.450, 4.260, 1.1, 0.1)	0.2	(0.60, 12.355)	2
	5	(0.715, 18.905, 9.234, 1.1, 0.1)	0.2	(0.63, 18.829)	2	(0.705, 18.907, 3.849, 2.2, 0.1)	0.2	(0.62, 18.826)	2
2.00	2	(0.680, 12.426, 4.608, 1.1,0.1)	0.1	(0.70, 12.385)	1	(0.670, 12.425, 4.610, 1.1, 0.1)	0.1	(0.70, 12.385)	1
	5	(0.770, 18.914, 9.230, 1.1, 0.1)	0.1	(0.80, 18.857)	1	(0.745, 18.909, 9.251, 1.1, 0.1)	0.1	(0.70, 18.845)	1
2.50	2	(0.605, 12.397, 4.598, 1.1, 0.1)	0.1	(0.65, 12.370)	1	(0.595, 12.393, 4.620, 1.1, 0.1)	0.1	(0.60, 12.355)	1
	5	(0.660, 18.902, 9.232, 1.1, 0.1)	0.1	(0.65, 18.834)	1	(0.635, 18.892, 9.268, 1.1, 0.1)	0.1	(0.70, 18.845)	1

VI. CONCLUSIONS

The standard MEWMA chart employs fixed sampling interval. In this study, we demonstrated the improved performance of the VSI MEWMA chart based on the MTS for monitoring shift in the process mean vector. The Monte Carlo simulation is conducted to compute the MTS for the VSI MEWMA chart. A procedure for the optimal statistical design of the VSI MEWMA chart is presented to find the combination of the charting parameters by minimizing the MTS_1. The comparison study between the VSI MEWMA chart and the standard MEWMA chart shows that the VSI MEWMA chart is more effective in detecting shifts in the process mean vector, in terms of the MTS.

REFERENCES

[1] C. A. Lowry, W. H. Woodall, C. W. Champ, and S. E. Rigdon, "A multivariate exponentially weighted moving average control chart," *Technometrics*, vol. 34, no. 1, pp. 46-53, 1992.

[2] N. Chen, Z. Li, and Y. Ou, "Multivariate exponentially weighted moving-average chart for monitoring Poisson observations," *Journal of Quality Technology*, vol. 47, no. 3, pp. 252-263, 2015.

[3] K. Nishimura, S. Matsuura, and H. Suzuki (2015), "Multivariate EWMA control chart based on a variable selection using AIC for multivariate statistical process monitoring," *Statistics & Probability Letters*, vol. 104, pp. 7-13, 2015.

[4] J. Park and C. H. Jun, "A new multivariate EWMA control chart via multiple testing," *Journal of Process Control*, vol. 26, pp. 51-55, 2015.

[5] Y. Cheng, A. Mukherjee, and M. Xie, "Simultaneously monitoring frequency and magnitude of events based on bivariate gamma distribution," *Journal of Statistical Computation and Simulation*, vol. 87, no. 9, pp. 1723-1741, 2017.

[6] S. Kim, M. K. Jeong, and E. A. Elsayed, "Generalized smoothing parameters of a multivariate EWMA control chart," *IISE Transactions*, vol. 49, no. 1, pp. 58-69, 2017.

[7] N. A. Saleh and M. A. Mahmoud, "Accounting for phase I sampling variability in the performance of the MEWMA control chart with estimated parameters," *Communications in Statistics - Simulation and Computation*, vol. 46, no. 6, pp. 4333-4347, 2017.

[8] X. Y. Chew, M. B. C. Khoo, S. Y. The, and M. H. Lee, "The run sum Hotelling's X^2 control chart with variable sampling intervals," *Quality and Reliability Engineering International*, vol. 32, no. 7, pp. 2573-2590, 2016.

[9] M. J. Ershadi, R. Noorossana and S. T. A. Niaki, "Economic-statistical design of simple linear profiles with variable sampling interval," *Journal of Applied Statistics*, vol. 43, no. 8, pp. 1400-1418, 2016.

[10] B. Guo and B. X. Wang, "The variable sampling interval S^2 chart with known or unknown in-control variance," *International Journal of Production Research*, vol. 54, no. 11, pp. 3365-3379, 2016.

[11] S. H. Patil and D. T. Shirke, "Economic design of variable sampling interval sign control chart," *Journal of Industrial and Production Engineering*, vol. 34, no. 4, pp. 253-260, 2017.

[12] X. Zhang, J. Liu, and C. Tan, "Economic design based on a multivariate Bayesian VSI chart with a dual control limit," *Communications in Statistics - Theory and Methods*, vol. 46, no. 14, pp. 6808-6822, 2017.

[13] A. Amdouni, P. Castagliola, H. Taleb, and G. Celano, "A variable sampling interval Shewhart control chart for monitoring the coefficient of variation in short production runs," *International Journal of Production Research*, vol. 55, no. 19, pp. 5521-5536, 2017.

[14] Z. Li, C. Zou, Z. Gong, and Z. Wang, "The computation of average run length and average time to signal: an overview," *Journal of Statistical Computation and Simulation*, vol. 84, no. 8, pp. 1779-1802, 2014.

[15] Z. G. Stoumbos, J. Mittenthal, and G. C. Runger, "Steady-state-optimal adaptive control charts based on variable sampling intervals," *Stochastic Analysis and Applications*, vol. 19, no. 6, pp. 1025-1057, 2001.

[16] M. B. C. Khoo, "Performance measures for the Shewhart control chart," *Quality Engineering*, vol. 16, no. 4, pp. 585-590, 2004.

[17] M. H. Lee and M. B. C. Khoo, "Design of a multivariate exponentially weighted moving average control chart with variable sampling intervals," *Computational Statistics*, vol. 29, no. 1-2, pp. 189-214, 2014.

[18] P. Castagliola, G. Celano, and S. Fichera, "Evaluation of the statistical performance of a variable sampling interval R EWMA control chart," *Quality Technology & Quantitative Management*, vol. 3, no. 3, pp. 307-323, 2006.

[19] A. F. Costa, "$\bar{X}$ chart with variable sample size and sampling intervals." *Journal of Quality Technology*, vol. 29, no. 2, pp. 197-204, 1997.

[20] Y. K. Chen, H. C. Liao, and C. T. Chen, "Economic design of VSI GCCC charts for correlated samples from high-yield processes." *South African Journal of Industrial Engineering*, vol. 24, pp. 88-101, 2013.

[21] S. Y. Yew, M. B. C. Khoo, W. L. Teoh, S. Y. Teh, and W. C. Yeong, "A comparative study of the group runs and side sensitive group runs control charts." *Pertanika Journal of Science & Technology*, vol. 24, no. 1, pp. 177-189, 2016

[22] M. A. Graham, A. Mukherjee, and S. Chakraborti, "Distribution-free exponentially weighted moving average control charts for monitoring unknown location." *Computational Statistics & Data Analysis*, vol. 56, no. 8, pp. 2539-2561, 2012.

[23] W. Huang, L. Shu, J. Cao, and K. L. Tsui, "Probability distribution of CUSUM charting statistics." *IIE Transactions*, vol. 48, no. 4, pp. 324-332, 2016.

[24] M. A. Graham, S. Chakraborti, and A. Mukherjee, "Design and implementation of CUSUM exceedance control charts for unknown location." *International Journal of Production Research*, vol. 52, no. 18, pp. 5546-5564, 2014.