# Yaw Rate and Sideslip Control using PID Controller for Double Lane Changing

Zainab Zainal[1], Wan Rahiman[1] and M N R Baharom[2]
[1] *School of Electrical and Electronic Engineering, Universiti Sains Malaysia, 14300 Nibong Tebal, Penang, Malaysia*
[2] *Faculty of Electrical and Electronic Engineering, Universiti Tun Hussein Onn Malaysia, 86400 Batu Pahat, Johor, Malaysia*
*zz15_eee078@student.usm.my*

***Abstract*— Vehicle behavior and stability can be observed through the computer simulation using mathematical modeling approach consist the configuration of vehicle multibody system. This paper presents the estimation and control of the yaw rate and sideslip through vehicle lateral dynamic model with a PID controller. The stability analysis is conducted to a single-input-two-outputs (SITO) plant of front-steering-only vehicle model to observe the system response during double lane changing maneuver with constant speeds of 40 *km/h* and 90 *km/h*. Results show that a single-input-single-output (SISO) PID controller able to enhance the cornering limit by reducing the sideslip and optimized the yaw rate, especially for slow speed vehicle in order to maintain its stability states.**

***Index Terms*—Lateral vehicle dynamic; PID controller; Sideslip; Yaw rate.**

## I. INTRODUCTION

Road safety had been globally main concern and discussed issues. Focus for most of automotive industry especially in research and development sector is to produce the vehicle equipped with stability control systems such as auto-breaking system (ABS), electronic system control (ESC) and traction control (TC). Many countries had made mandatory for all new vehicle to have ESC. Hence, the yaw rate, $\dot{\psi}$ and sideslip, $\beta$ measurement are key parameters that contribute to the design of ESC in such vehicles. Vehicle instability occur when sideslip and yaw rate increases more than its allowable threshold, depend on the vehicle specification and tire model [1]. This is when the ESC will be activated for interaction with the vehicle system to make it stable. The value of yaw rate and sideslip also used for autonomous vehicle control that manipulated the input of steering wheel angle (SWA) while maneuvering [2].

Practically, the yaw rate sensor measures a vehicle's angular velocity around its vertical axis. It is related to the sideslip angle which is the angle between vehicle's heading and vehicle actual movement direction. Both can be estimated using mathematical model of lateral vehicle dynamic model [3]. Vast research on this topic had been conducted to replace the yaw rate sensor (gyroscopic device) to virtual sensor for more cost effective in vehicle manufacturing [4]. The aim of this study is to investigate the behavior of a linearized vehicle lateral dynamic model simulation with a PID controller for double lane changing maneuver. Single-input-single-output (SISO) controller applied to the single-input-two-outputs (SITO) plant as shown in Figure 1 to obtain insight into effects of PID controller interaction upon design trade-offs. The basic control objectives of applied PID controller is to meet desired trajectory path.

The remaining paper is organized as follows. In Section 2, vehicle dynamic model is described, and the state equations of linear vehicle system is given with small angle approximation, considered as the system plant. In Section 3, the controller design method are applied to observe the vehicle stability and trajectory for double lane changing. Simulation study on the performance of vehicle dynamic model stability is conducted. Results of the simulation outputs are discussed in Section 4. Finally, the conclusion and future works is made in Section 5.

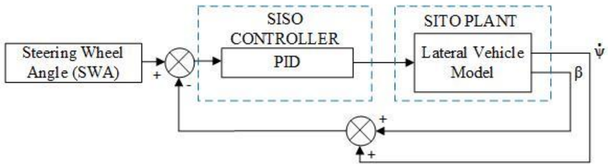



Figure 1: Block Diagram of Overall System

## II. MODEL OF LATERAL VEHICLE DYNAMIC

The philosophy of the simulation work is always to use simple models, in order to understand more aspects possible about the physical system. Mathematical models are available to predict the vehicle stability and handling. The major response of the vehicle can be explained based on a linear vehicle model [5].

The vehicle model used in this work is a bicycle model, using an augmented version of two-degree-of-freedom (2DOF) elementary automobile [6] as shown in Figure 2. This model is linearized for simplicity and has consistently been proven in various studies [7-9]. The terms $XY$ and $xy$, refers to the global and local coordinate frame, respectively. The nomenclatures used are presented in Table 1.

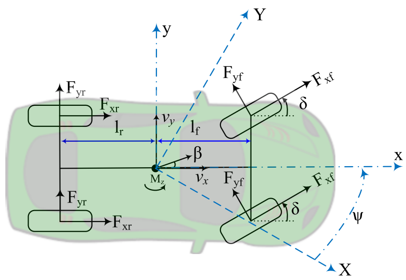



Figure 2: Planar Vehicle Dynamic

Table 1
Nomenclatures of Vehicle Parameters

| Parameter | Nomenclature | Unit |
|---|---|---|
| $v_x$ | Longitudinal velocity | $m/s$ |
| $F_{yf}$ | Front lateral tire force | $kg.m/s^2$ |
| $F_{yr}$ | Rear lateral tire force | $kg.m/s^2$ |
| $l_f$ | Distance from center of gravity to front axle | $m$ |
| $l_r$ | Distance from rear axle to center of gravity | $m$ |
| $m$ | Vehicle mass | $kg$ |
| $\delta$ | Steering wheel angle (SWA) | $rad$ |
| $\beta$ | Vehicle sideslip angle | $rad$ |
| $I_z$ | Yaw moment of inertia | $kg.m^2$ |
| $\dot{\psi}$ | Yaw rate of vehicle | $rad/s$ |
| $\dot{y}$ | Lateral velocity at center of gravity of vehicle | $m/s$ |
| $C_{\alpha f}$ | Cornering stiffness of front tire | $kN/rad$ |
| $C_{\alpha r}$ | Cornering stiffness of rear tire | $kN/rad$ |

### A. Dynamic Modelling

Using the Newton's second law of motion, the equation for lateral translational motion of the vehicle can be obtained as;

$$m(\ddot{y}+v_x\dot{\psi}) = F_{yf} + F_{yr} \tag{1}$$

Yaw dynamic is described when moment balance about the $z$ axis is considered as;

$$I_z\ddot{\psi} = l_f F_{yf} - l_r F_{yr} \tag{2}$$

The lateral tire force contributes to the impact of the vehicle heading angle due to many reason such as the road condition, tire characteristics, wind impact and etc. [10]. The equation of the front and rear tire lateral force can be simplified by assuming cornering stiffness of front and rear tire, $C_{\alpha f}$ and $C_{\alpha r}$, left and right tire acts in the same manner. Using small tire slip angles, lateral forces can be linearly approximated as follow:

$$F_{yf} = 2C_{\alpha f}\left(\delta - \frac{\dot{y}+l_f\dot{\psi}}{v_x}\right) \tag{3}$$

$$F_{yr} = 2C_{\alpha r}\left(-\frac{\dot{y}-l_r\dot{\psi}}{v_x}\right) \tag{4}$$

By substituting Equation (3) and (4) in Equation (1) and (2), the dynamic equations of the front-steering-only model can be written as:

$$m(\ddot{y}+v_x\dot{\psi}) = 2C_{\alpha f}\left(\delta - \frac{\dot{y}+l_f\dot{\psi}}{v_x}\right) + 2C_{\alpha r}\left(-\frac{\dot{y}-l_r\dot{\psi}}{v_x}\right) \tag{5}$$

$$I_z\ddot{\psi} = l_f\left(2C_{\alpha f}\left(\delta - \frac{\dot{y}+l_f\dot{\psi}}{v_x}\right)\right) - l_r\left(2C_{\alpha r}\left(-\frac{\dot{y}-l_r\dot{\psi}}{v_x}\right)\right) \tag{6}$$

When assuming longitudinal velocity $v_x$ to be constant;

$$\ddot{y} = \dot{\beta}v_x \tag{7}$$

the relationship between sideslip and yaw rate can be simplified as;

$$\dot{\beta} = -\left(\frac{2C_{\alpha f}+2C_{\alpha r}}{mv_x}\right)\beta + \left(-\frac{2C_{\alpha f}l_f - 2C_{\alpha r}l_r}{m{v_x}^2} - 1\right)\dot{\psi} + \frac{2C_{\alpha f}}{mv_x}\delta \tag{8}$$

$$\ddot{\psi} = \left(\frac{2C_{\alpha r}l_r - 2C_{\alpha f}l_f}{I_z}\right)\beta - \left(\frac{2C_{\alpha f}{l_f}^2 + 2C_{\alpha r}{l_r}^2}{I_z v_x}\right)\dot{\psi} + \frac{2C_{\alpha f}l_f}{I_z}\delta \tag{9}$$

### B. State Space Equation

The state equation of vehicle model for linear SITO system of single input variable, steering wheel angle, $\delta$; two output variables, sideslip angle, $\beta$ and yaw rate, $\dot{\psi}$ is obtained as the following:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \tag{10}$$

Where, $x = [\beta \quad \dot{\psi}]^T$ is the state vector; $u = \delta$ is the input; $y = [\beta \quad \dot{\psi}]$ is the output vector. A is the state matrix, B is input matrix and C is the output matrix as:

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} -\frac{2C_{\alpha f}+2C_{\alpha r}}{mv_x} & -\frac{2C_{\alpha f}l_f - 2C_{\alpha r}l_r}{m{v_x}^2} - 1 \\ -\frac{2C_{\alpha f}l_f - 2C_{\alpha r}l_r}{I_z} & -\frac{2C_{\alpha f}{l_f}^2 + 2C_{\alpha r}{l_r}^2}{I_z v_x} \end{bmatrix};$$

$$B = \begin{bmatrix} b_{11} \\ b_{21} \end{bmatrix} = \begin{bmatrix} \frac{2C_{\alpha f}}{mv_x} \\ \frac{2C_{\alpha f}l_f}{I_z} \end{bmatrix}; \qquad C = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{11}$$

## III. Design of Control System

To observe the system behavior, the configuration of the closed-loop system for SITO plant is considered as in Figure 3. The closed-loop system output is determined before adding the controller to the system. Using mixed values of estimated yaw rate, $\dot{\psi}$ and sideslip, $\beta$, the error signal is obtained for PID tuning purposed.

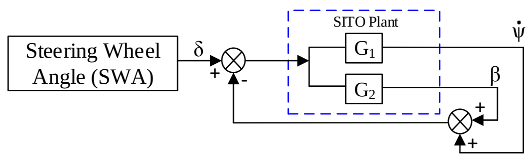



Figure 3: Block Diagram of Closed-Loop System

### A. Plant Transfer Function

The system transfer function for SITO model is defined as;

$$G_{sys} = G_1 + G_2 \tag{12}$$

where $G_1$ is the transfer function for yaw rate and $G_2$ is the transfer function of the sideslip from plant internal block. In the aforementioned section, the mathematical model is obtained when the longitudinal velocity, $v_x$, assumed to be constant all the time. Therefore, two sets of closed loop transfer function are used as in Equation (13) and (14) for vehicle speed of $40km/h$ and $90km/h$, respectively.

$$G_{sysA} = \begin{cases} G_{1A} = \dfrac{28s+352.8}{s^2+20s+117} \\ G_{2A} = \dfrac{3.2s+16.5}{s^2+20s+117} \end{cases} = \frac{31.2s+369.3}{s^2+20s+117} \quad (13)$$

$$G_{sysB} = \begin{cases} G_{1B} = \dfrac{28s+156.8}{s^2+8.9s+45.6} \\ G_{2B} = \dfrac{1.4s-19.2}{s^2+8.9s+45.6} \end{cases} = \frac{29.4s+137.6}{s^2+8.9s+45.6} \quad (14)$$

### B. PID Controller

PID control is simple and practical compare to other control method [11]. The PID controller developed in this study is designed to force the vehicle to follow a desired yaw rate, $\dot{\psi}$ and reduced the sideslip, $\beta$ as minimised as possible. Equation (15) refer to PID control law, which consist of proportional gain, $K_p$, integral gain, $K_i$ and derivative gain, $K_d$; summed to form the controller transfer function block, $G_c$.

$$G_c = K_p + \frac{K_i}{s} + K_d s \quad (15)$$

The parameters value of $K_p$, $K_i$ and $K_d$ are obtained using MATLAB SIMULINK built-in automated tuning tool as in Table 2. To observe the impact and objective of control method during vehicle maneuvering, the different control parameters are set for both test speeds so that the system response will be optimised.

Table 2
PID Tuning Parameters

| Parameter | Speed Slow $40km/h$ | Speed High $90km/h$ |
|---|---|---|
| $K_p$ | 2.07 | 0.57 |
| $K_i$ | 37.75 | 7 |
| $K_d$ | 0 | 0.01 |

## IV. Simulation System Configuration

In this section, the configuration of vehicle model and output validation method is discussed. Simulation works are conducted using MATLAB/SIMULINK.

### A. Vehicle Parameters

The parameters value of vehicle model for this work are as listed in Table 3. The car model parameters are obtained from [12] is type of passenger cars defined in ISO3833 [13] so that it is applicable to the test track.

Table 3
Vehicle Parameters

| Parameter | Value | Parameter | Value |
|---|---|---|---|
| $m$ | $2000kg$ | $I_z$ | $3500kg.m^2$ |
| $l_f$ | $1.4m$ | $C_{\alpha f}$ | $35kN/rad$ |
| $l_r$ | $1.4m$ | $C_{\alpha r}$ | $70kN/rad$ |

### B. ISO 3888-2 Double Lane Change Test

The vehicle stability is observed by assuming a double lane change test track as desired vehicle trajectory. Figure 4 shows the International Standard ISO 3888-2 (obstacle avoidance test) track used to evaluate the handling performance of a vehicle [14].

This test is also part of the vehicle design procedures and vehicles assessment. In this paper, the test track is used to validate the output of the simulation of vehicle motion along the planar plane.

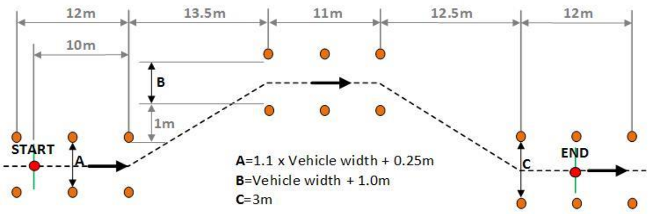


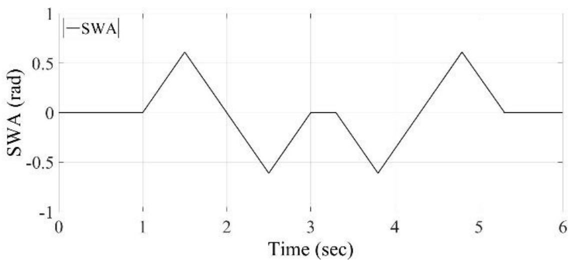

Figure 4: ISO 3888-2 Double Lane Change Test Track

## V. Simulation Results And Analysis

The results of open-loop (OL), closed-loop (CL) and added PID controller vehicle dynamics using two different lateral speeds at $40km/h$ and $90km/h$ are presented in this section.

### A. Yaw Rate and Sideslip

Figure 5 and Figure 6 shows the estimation of the yaw rate, $\dot{\psi}$ and sideslip, $\beta$ of the simulated vehicle model at both test speeds, slow and high. The performance of the proposed controller design is compared to the OL and CL vehicle system. The SWA input is given differently for both speed test so that the vehicle follows the desired trajectory path. The turning radius increased when vehicle is at slow speed. This gives the idea of turning ratio for the vehicle model specification. When in high speed, the turning will be sharper and faster compared to slow speed vehicle maneuvering. The SWA increased almost double for vehicle at constant speed of $90km/h$.




(a)

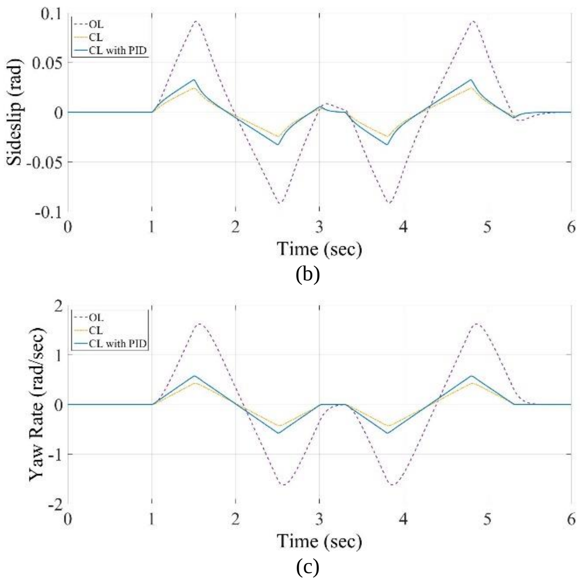



Figure 5: Simulation Outputs of Vehicle (A) Input SWA (B) Yaw Rate, $\dot{\psi}$ And (C) Sideslip, $\beta$ With Constant Speed at $40km/h$

As predicted, the system without any control method will produced high yaw rate and sideslip. Using a PID controller, the yaw rate can be corrected to desired output. The sideslip also reduced for the system with PID. Optimum stability can be achieved when the yaw rate value almost imitate the input SWA and sideslip is close to zero.

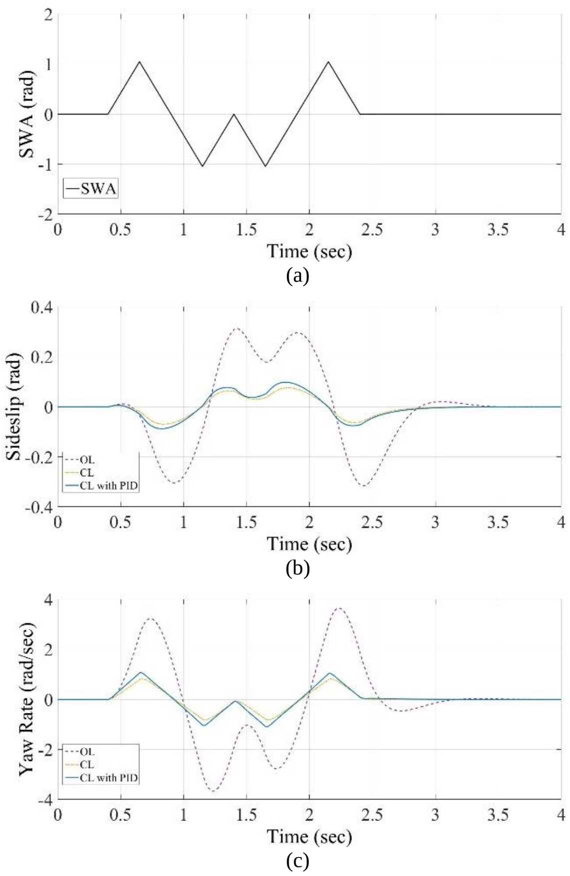



Figure 6: Simulation Outputs of Vehicle (A) Input SWA (B) Yaw Rate, $\dot{\psi}$ And (C) Sideslip, $\beta$ With Constant Speed at $90km/h$

## B. Vehicle Trajectory

The trajectory of the vehicle system for double lane change simulation are shown in Figure 7 and Figure 8. The vehicle stability of the slow speed, $40km/h$ and high speed, $90km/h$ can be seen more clearly so that it is easier to understand. Using the same vehicle dynamic model, the maneuvering through double lane change test track is in better stability for low speed.

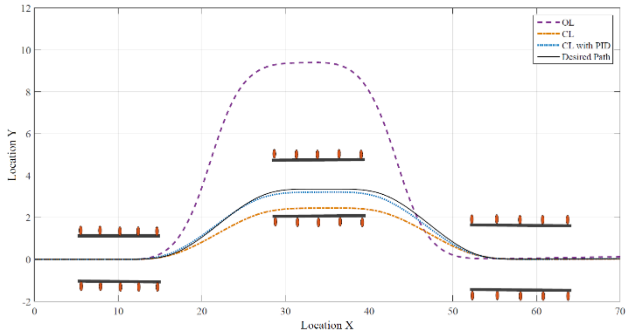



Figure 7: Vehicle Trajectory for Double Lane Change at Constant Speed, $40km/h$

When at $90km/h$ speed, the result shows that the vehicle failed to maneuver double lane change test track. Constrain of the SWA input make the vehicle slip through the next lane slower than expected. This problem can be solve using adaptive auto tuning PID control method.

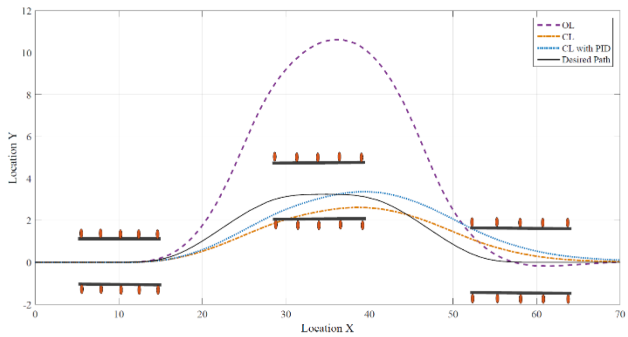



Figure 8: Vehicle Trajectory for Double Lane Change at Constant Speed, $90km/h$

## VI. CONCLUSION

In this paper, the prediction of better stability when using PID controller for double lane change vehicle maneuvering is achieved. The proposed design method is limited to slow speed vehicle due to SISO controller design; mixed system outputs as system feedback. Future work includes an advanced PID design and tuning techniques of TISO controller for SITO plant. With more sophisticated controller type the nonlinear system can also be considered. The results indicate that the relationship between yaw rate and sideslip estimation can be interesting solution to the auto steering wheel input estimation for autonomous setting of double lane changing test.

## ACKNOWLEDGMENT

This research was supported by Multi-Agent System for Data Link Interaction in SPALAN Grant (grant number: 1001/PELECT/814265).

## REFERENCES


[1] Q. Lu, P. Gentile, A. Tota, A. Sorniotti, P. Gruber, F. Costamagna, *et al.*, "Enhancing vehicle cornering limit through sideslip and yaw rate control," *Mechanical Systems and Signal Processing,* vol. 75, pp. 455-472, 2016.

[2] O. Töro, T. Bécsi, and S. Aradi, "Design of lane keeping algorithm of autonomous vehicle," *Periodica Polytechnica. Transportation Engineering,* vol. 44, p. 60, 2016.W.-K. Chen, *Linear Networks and Systems* (Book style). Belmont, CA: Wadsworth, 1993, pp. 123–135.

[3] R. Tchamna and I. Youn, "Yaw rate and side-slip control considering vehicle longitudinal dynamics," *International Journal of Automotive Technology,* vol. 14, p. 53, 2013.B. Smith, "An approach to graphs of linear forms (Unpublished work style)," unpublished.

[4] S. Sen, S. Chakraborty, and A. Sutradhar, "Estimation of vehicle yaw rate and lateral motion for dynamic stability control using unscented Kalman filtering (UKF) approach," 2015.

[5] J. Kong, M. Pfeiffer, G. Schildbach, and F. Borrelli, "Kinematic and dynamic vehicle models for autonomous driving control design," in *Intelligent Vehicles Symposium (IV), 2015 IEEE*, 2015, pp. 1094-1099.

[6] R. Rajamani, *Vehicle dynamics and control*: Springer Science & Business Media, 2011.

[7] M. T. Emirler, İ. M. C. Uygan, B. Aksun Güvenç, and L. Güvenç, "Robust PID steering control in Parameter Space for highly automated driving," *International Journal of Vehicular Technology,* vol. 2014, 2014.

[8] A. Sharmin and R. Wan, "An Autonomous Lane-Keeping Ground Vehicle Control System for Highway Drive," in *9th International Conference on Robotic, Vision, Signal Processing and Power Applications*, 2017, pp. 351-361.

[9] V. Cerone, M. Milanese, and D. Regruto, "Simulation results on combined automatic lane keeping and driver's maneuvers," in *Control Conference (ECC), 2007 European*, 2007, pp. 1241-1248.

[10] G. Erdogan, S. Hong, F. Borrelli, and K. Hedrick, "Tire sensors for the measurement of slip angle and friction coefficient and their use in stability control systems," *SAE International Journal of Passenger Cars-Mechanical Systems,* vol. 4, pp. 44-58, 2011.

[11] P. Zhao, J. Chen, Y. Song, X. Tao, T. Xu, and T. Mei, "Design of a control system for an autonomous vehicle based on adaptive-pid," *International Journal of Advanced Robotic Systems,* vol. 9, p. 44, 2012.

[12] P. J. T. Venhovens and K. Naab, "Vehicle dynamics estimation using Kalman filters," *Vehicle System Dynamics,* vol. 32, pp. 171-184, 1999.

[13] A. E. d. N. y. Certificación and O. I. p. a. l. Normalització, *ISO 3833: Road Vehicles : Types : Terms and Definitions*: ISO, 1977.

[14] I. Norm, "3888-2 November 2002," *Passenger cars–Test track for a severe lane-change manoeuvre–Part,* vol. 2.