Synthesis of Memristive Systems

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***Abstract*—A method of the synthesis of memristive systems with prescribed pinched hysteresis loops is described. Since the loop has a connection to the Fourier components of the system response to sinusoidal excitation, the method can be used for designing nonlinear applications of memristive systems that work on the principle of spectrum enrichment.**

***Index Terms*—Memristor; Pinched Hysteresis Loop; Spectrum; Synthesis.**

I. Introduction

The development of a nano-device in Hewlett-Packard Labs in 2008 [1] whose behavior resembles that of the memristor, i.e. the ideal circuit element introduced into the circuit theory in 1971 by L. Chua [2], initiated intensive research in the field of memristive systems [3], particularly towards nonvolatile memories for computer industry [4]. Since the memristor is, in fact, an analog circuit element, a number of papers have also been published about the potential applications of memristive systems for analog signal processing [5].

Current advances in nanotechnologies aim at creating conditions that enable designing and implementing memristive systems of prescribed characteristics. For example, some works describe an "HP memristor" with a special geometry which will provide a nonlinear relationship between memristance and the integral of the terminal voltage [6], [7]. However, papers dealing with a systematic design of memristive systems with prescribed behavior for concrete applications are not currently available.

The $v - i$ (voltage-current) pinched hysteresis loop as the well-known fingerprint of the memristive system is observable under its periodical excitation [8]. Since this loop is a Lissajous curve generated by periodical voltage and current waveforms, it raises the possibility of analyzing the connection between the type or the shape of the loop and the spectral components of the waveforms. The advantage of such an approach consists in coming to know the Fourier components of the response (for example the current) to the excitation (for example the voltage). This spectrum can be modified via tweaking the loop such that it will be optimal for a given application (for example with the aim to generate a dominant higher harmonic component for a frequency multiplier).

Since memristive systems introduce nonlinear distortion, particularly in the hard switching operation, some works deal with the analysis of this distortion, especially in the frequency domain. It is shown in [9] that, under certain conditions, dominant spectral components can appear in the response of the memristive system to sinusoidal excitation. This phenomenon can be made use of for designing analog systems that work on the principle of spectrum enrichment via the nonlinear operation and subsequent frequency-domain filtering. The paper [10] studies the generation of the second and higher harmonic components in the response of a passive memristor to sinusoidal excitation in a simple circuit with one memristor and also with four memristors in a bridge. It is proved that, in comparison with conventional diode-based circuits, memristive networks can provide considerably higher ratios of powers of the higher harmonics to the power of the driving signal.

This paper deals with the synthesis of memristive systems that can generate pinched hysteresis loops with the so-called higher-order touching points at the v-i origin. The synthesis is based on the knowledge that such a type of touching is possible only under a specific spectrum of the memristor current. The methodology described is also useful for various modifications of the task, and here it can serve as an introductory study of the synthesis of memristive systems with prescribed types of behavior.

II. Starting Points of the Synthesis

Consider a voltage-controlled first-order memristive system and its port and state equations

$$i(t) = g(x, v)v(t)\,, \tag{1}$$

$$\frac{dx}{dt} = f(x, v)\cdot \tag{2}$$

where, i, v, and x are the voltage, current, and internal state variable, g is the memductance, and f is a piece-wise-continuous function. The conclusions of this paper can be also used for the dual case of current-controlled memristive systems.

With the exception of special cases such as the ideal memristor in the sense of its axiomatic definition [2], the region of the values of the state variable x is limited to a certain subset $\mathbb{R}_x$ of real numbers. This is given by the principle of operation of the concrete mem-system. For example, the state variable of the well-known TiO_2 memristor [1] is the normalized width of the doped region, which can swing within the interval [0,1].

Consider a memristive system with its initial state $x_0 \in \mathbb{R}_x$. At time $t = 0$ we start its excitation by a sinusoidal voltage

$$v(t) = V_{max}\sin(\omega t), \tag{3}$$

where V_{max} is the amplitude, $\omega = 2\pi/T$ is the angular frequency, and T is the repeating period. Let us analyze only such systems (1), (2) that can provide transitions to periodical steady state, modeled by a periodical function $x(t) \in \mathbb{R}_X$, whose fundamental harmonic component will have the repeating frequency ω. Then the current i in (1) will be a periodic function of time which can be expanded into the Fourier series

$$i(t) = I_0 + \sum_{k=1}^{\infty}\left[I_k^c \cos(k\omega t) + I_k^s \sin(k\omega t)\right]. \tag{4}$$

The $v-i$ pinched hysteresis loop is a Lissajous curve drawn via a sinusoidal voltage v and periodical current i according to (3), (4).

The starting condition for a correct operation of an arbitrary memristive system is zero current for zero voltage, or the passage of the loop through the $v-i$ origin. For the excitation (3) and the response (4), the consequent condition for the cosine components of the current is

$$I_0 + \sum_{k=1}^{\infty} I_k^c = 0. \tag{5}$$

According to the character of the loop about the $v-i$ origin, the loops can be either of type-I (crossing arms of the loop) or of type-II (non-crossing touching) [11]. The ideal memristor according to Chua [2] can exhibit only type-I loops, which must be odd-symmetric [12]. The order of touching, n, is defined in [13]. It is shown in [14] that every loop must fulfill the following conditions for the sine and cosine components of the Fourier series (4):

$$\sum_{k=0}^{\infty} I_{2k+1}^c = 0,$$

$$\sum_{k=1}^{\infty} 2k I_{2k}^s = 0, \tag{6a}$$

$$\sum_{k=0}^{\infty} (2k+1)^2 I_{2k+1}^c = 0,$$

…

The continuation depends on the loop type:
Type-I (n even):

$$\sum_{k=1}^{\infty} (2k)^{n-1} I_{2k}^s = 0,$$

$$\sum_{k=0}^{\infty} (2k+1)^n I_{2k+1}^c = 0, \tag{6b}$$

$$\sum_{k=1}^{\infty} (2k)^{n+1} I_{2k}^s \neq 0.$$

Type-II (n odd):

$$\sum_{k=0}^{\infty} (2k+1)^{n-1} I_{2k+1}^c = 0,$$

$$\sum_{k=1}^{\infty} (2k)^n I_{2k}^s = 0, \tag{6c}$$

$$\sum_{k=0}^{\infty} (2k+1)^{n+1} I_{2k+1}^c \neq 0.$$

Comparing the first Equation in 6(a) and the condition (5), it is obvious that the sum of even-order cosine components and the DC component must also be zero:

$$I_0 + \sum_{k=1}^{\infty} I_{2k}^c = 0. \tag{7}$$

Equations (6), (5), and possibly also (7) represent theoretical starting points of the synthesis of memristive systems.

III. General Procedure of the Synthesis

Equation (6) for even or odd order of touching n represents a set of n+1 equations with an infinite number of unknowns, with the limiting condition in the form of the inequality (6b) or (6c). This set provides an infinite number of solutions, each modeling a memristive system with the given spectrum of the current for the sinusoidal excitation (3). The task is to find the model of a system whose loop has the n^{th}-order touching. This system will be selected from the above set on the basis of some other limiting conditions. Examples of these conditions are as follows:

i. The system must be an ideal memristor (but only if it is possible in principle, thus only for an even n).
ii. The frequency bandwidth of the current must be as low as possible.
iii. Conditions (i) and (ii) must be fulfilled simultaneously, if possible.

Regardless of the order of touching, the synthesis of the corresponding memristive system can be done as follows:

a) We select a set of sine and cosine components of the current that conforms to Equation (6) for a given order of touching n.
b) We write formally the memductance g as a ratio

$$g = \frac{i}{v} = \frac{\sum_{k=1}^{\infty}\left[I_k^c \cos(k\omega t) + I_k^s \sin(k\omega t)\right]}{V_{\max}\sin(\omega t)}. \tag{8}$$

c) We express the cosines and sines of the multiple angles $k\omega t$ via the sines and cosines of the simple angle ωt. We can use the well-known identities such as [15]

$$\cos(k\omega t) = T_k(\cos(\omega t)), \tag{9}$$

$$\sin(k\omega t) = \sin(\omega t)\, U_{k-1}(\cos(\omega t)). \tag{10}$$

where T_k and U_k are the Chebyshev polynomials of the first and the second kind. Then the memductance will be a nonlinear function of the sine and cosine of the angle ωt:

$$g = \frac{\sum_{k=1}^{\infty}\left[I_k^c T_k(\cos(\omega t)) + \sin(\omega t) I_k^s U_{k-1}(\cos(\omega t))\right]}{V_{\max}\sin(\omega t)}. \tag{11}$$

It is obvious that the numerator of (11) is a polynomial of the sines and cosines of the angle ωt, thus it can be modified by the familiar rules such as $\sin^2(\alpha) = 1-\cos^2(\alpha)$. That is why a number of different forms of Equation (11) can enter the synthesis, each generating a different model of the memristive system. However, all of them will provide identical behavior under concrete conditions (type of the excitation and the initial state).

All the mathematical arrangements must be correct: They must start from identities that hold for an arbitrary range of the arguments of the functions. For example, the well-known identities

$$\sin(\alpha) = \sqrt{1-\cos^2(\alpha)}\,, \tag{12}$$

$$\cos(\alpha) = \sqrt{1-\sin^2(\alpha)}\,. \tag{13}$$

are not correct because the square root generates only nonnegative numbers. The correct versions of (12) and (13) are

$$\sin(\alpha) = \mathrm{sgn}(\sin(\alpha))\sqrt{1-\cos^2(\alpha)}\,, \tag{14}$$

$$\cos(\alpha) = \mathrm{sgn}(\cos(\alpha))\sqrt{1-\sin^2(\alpha)}\,. \tag{15}$$

where sgn is a sign function, which provides 1 for positive argument, -1 for negative argument, and 0 for zero argument.

Only a correctly modified formula (11) can generate the class of models of memristive systems with unified and correct behavior under the above conditions.

d) According to (3), we replace the function $\sin(\omega t)$ in the correctly modified formula (11) with a normalized voltage, v/V_{max}, and the function $\cos(\omega t)$ with the state variable x. After the corresponding arrangement, the memductance (11) will be in the form (1).

The state variable x is governed by the differential equation

$$\frac{dx}{dt} = -\frac{\omega}{V_{\max}}v\,,\ x \in [-1,1]\,, \tag{16}$$

where ω, V_{max} are parameters (therefore constants) of the driving signal that generates the hysteresis loop. A correct solution to this equation, i.e. the signal $\cos(\omega t)$, is obtained under the initial condition $x_0 = 1$.

Equation (16) together with the equation for the memductance, which is dependent on the state variable and the voltage, represents a complete mathematical model (1), (2) of the designed memristive system. In most cases, this model must be eventually optimized in order to be as simple as possible. The basic possibilities consist in the transformation of the state equation (16) (transfer to another state variable that simplifies the model), modification of the memductance formula (11), or an additional modification of the Fourier series of the current that would result in the simplification of the memductance formula (11).

IV. Example of the Synthesis

The procedure will be explained on the example of a memristive system that would be able, under the excitation (3), to generate a type-II pinched hysteresis loop with third-order touching. Then Equation (6) for the current spectral components are in the form:

$$I_1^c + I_3^c + I_5^c + I_7^c + I_9^c + \ldots = 0\,,$$

$$2I_2^s + 4I_4^s + 6I_6^s + 8I_8^s + 10I_{10}^s + \ldots = 0\,,$$

$$I_1^c + 9I_3^c + 25I_5^c + 49I_7^c + 81I_9^c + \ldots = 0\,, \tag{17}$$

$$8I_2^s + 64I_4^s + 216I_6^s + 512I_8^s + 1000I_{10}^s + \ldots = 0\,,$$

$$I_1^c + 81I_3^c + 625I_5^c + 2041I_7^c + 6561I_9^c + \ldots \neq 0\,. \tag{18}$$

Without the inequality (18), Equations (17) represent a set of four equations with an infinite number of unknowns. In fact, it can be divided into two individual sets, i.e. two equations for the cosine terms and two equations for the sine terms of the current:

$$I_1^c + I_3^c + I_5^c + I_7^c + I_9^c + \ldots = 0\,,$$

$$I_1^c + 9I_3^c + 25I_5^c + 49I_7^c + 81I_9^c + \ldots = 0\,, \tag{19}$$

$$I_2^s + 2I_4^s + 3I_6^s + 4I_8^s + 5I_{10}^s + \ldots = 0\,,$$

$$I_2^s + 8I_4^s + 27I_6^s + 64I_8^s + 125I_{10}^s + \ldots = 0\,. \tag{20}$$

Each non-trivial solution of the set (19) is represented by at least three non-zero cosine terms of the current. Let us select this triad as the harmonic components of orders 1, 3, and 5, assuming that all the higher-order cosine components are negligible. Then (19) provides the solution

$$I_1^c = I^c\,,\ I_3^c = -1.5I^c\,,\ I_5^c = 0.5I^c\,, \tag{21}$$

where I^c is a free parameter. This solution concurrently conforms to the inequality (18).

For a maximum simplification of the synthesis in its starting phase, consider a trivial solution of the set (20), i.e. all the sine components of the current will be zero. Let us also null all the remaining spectral components that cannot affect the condition (21).

Substituting (21) in (8) yields for the memductance

$$g = \frac{I^c}{V_{\max}\sin(\omega t)}\left[\cos(\omega t) - 1.5\cos(3\omega t) + 0.5\cos(5\omega t)\right]. \tag{22}$$

Utilizing (11) and rearranging lead to

$$g = \frac{8I^c}{V_{\max}}\cos(\omega t)\sin^3(\omega t)\,. \tag{23}$$

There are several ways how to assign the state variable and the driving voltage to the sine and cosine functions in (23). One of them is as follows:

$$g(x,v)=\frac{8I^c}{V_{\max}}\cos(\omega t)\frac{(V_{\max}\sin(\omega t))^3}{V_{\max}^3}=\frac{8I^c}{V_{\max}^4}xv^3. \quad (24)$$

An analysis of (24) reveals that the memductance can be positive and also negative, and the corresponding pinched hysteresis loops would be drawn also outside the first and third quadrants of the *v-i* space. To design a passive memristive system, the numerator in (8) should be completed by a proper term which concurrently does not violate the conditions (17) and (18) of 3rd-order touching. For example, we can add the first sine-type harmonic component, and Equation (23) will be modified to

$$g=\frac{8I^c}{V_{\max}}\cos(\omega t)\sin^3(\omega t)+\frac{I_1^s}{V_{\max}}. \quad (25)$$

The selection

$$I_1^s>\frac{3\sqrt{3}}{2}I^c\approx 2.598\,I^c. \quad (26)$$

assures the passivity of the designed system. From the point of view of the model (24), the last term in (25) represents a positive fixed conductance which shifts the working region of the memductance to the space of positive numbers.

Analogously, it is possible to start the synthesis from the sine, not the cosine terms (Equation 20), and to get another set of memristive systems with equivalent behavior under the given conditions.

The designed memristive system (25) has been analyzed in SPICE. The results are summarized in the following Section.

V. Simulation

The SPICE model of the system (25) was used for the simulation, with the following parameters:

V_{max} = 1V, f = ω/(2π)=1 Hz, I^c=2 mA, I_1^s =7 mA.

The schematic for SPICE modeling is shown in Figure 1. The variable part of the memductance (25) is modeled by the controlled current source Gmem and the fixed part by the resistor R0.

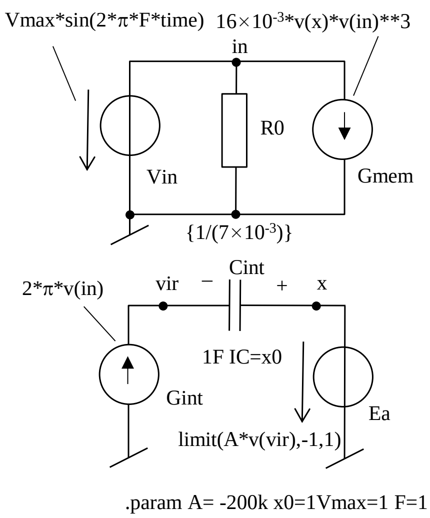


Figure 1: Schematic for SPICE model of memristive system described by Equation (16) and (25)

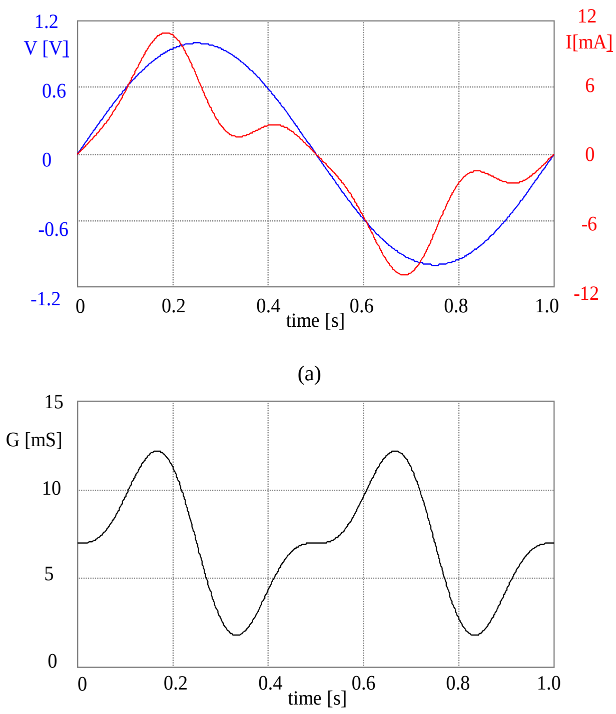
The bottom part of Figure 1 models the differential equation (16). The state variable *x* is represented by the voltage of node x. The controlled source Ea models the operational amplifier in inverting circuitry, which forms, together with the capacitor Cint, the converter of the source Gint current to a voltage which is equal to the time-domain integral of current. The amplifier limits its output voltage within the interval [-1, 1] V according to the limitation of the state variable in Equation (16).

The simulation results are shown in Figure 2. The hysteresis loop clearly shows its tangential type about the *v* – *i* origin. The Fourier analysis confirms the spectrum of current as input data of the synthesis. Note that the memductance exhibits two times higher repeating frequency than the frequency of the driving signal.

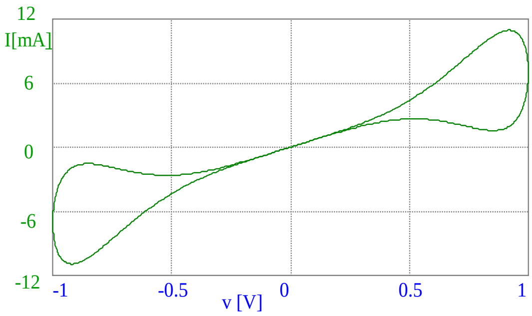
VI. Conclusions

The paper analyzes connections between the spectral components of the current flowing through a memristive system driven by a sinusoidal voltage, and the model of the system, i.e. its state equation and the dependence of memductance on state and terminal voltage. Although all the conclusions hold for voltage-controlled memristive systems, they can be also applied to the dual current-controlled systems driven by current sources.

The proposed procedure of the synthesis is based on several simplifications. As a consequence, not all the memristive systems that conform to the requirements can be found. The designed systems generate a current response to a sinusoidal voltage within the limited frequency band, which follows from the purpose of the synthesis, and the state variable is chosen such that it corresponds to the state variable of the ideal generic memristor [16]. Since the proposed method starts from general connections between the characteristics of memristive systems and the spectral terms of their terminal signals, it has a potential for overcoming the above limitations.



(b)



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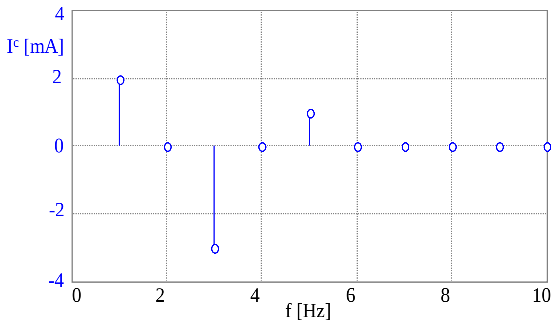

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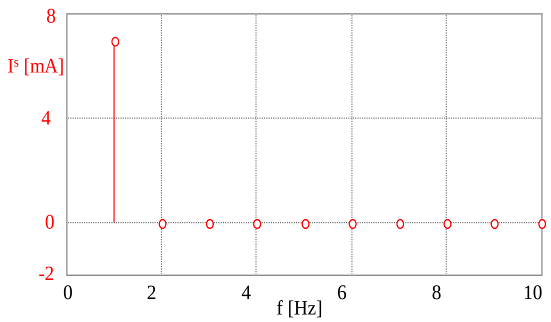

(e)

Figure 2: SPICE simulation of the model from Figure 1: (a) voltage and current waveforms, (b) memductance vs time, (c) type-II pinched hysteresis loop with 3rd-order touching at *v* – *i* origin, (d), (e) cosine and sine spectral terms of current

ACKNOWLEDGMENT

This work was supported by the Czech Science Foundation under grant No. 14-19865S, Czech Republic. The research is a part of the COST Action IC1401 and it is financially supported by the Czech Ministry of Education under grant No. LD15033. For the research, the infrastructure of K217 UD Brno was used.

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