Analysis of the Multimode Fiber at Low-Frequency Passband Region

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***Abstract*—Multimode fibers have been used in communication systems for more than 40 years. The characteristics of such fibers have been investigated and it has shown that not just the 3-dB modal bandwidth of multimode fibers, but the high-frequency passbands can also be used to carry the signal; resulting in an increase of the data rate transmitted by a multimode fiber. However, the low-frequency passbands of multimode fibers have not been studied intensively. In this work, the characteristics of low-frequency passbands of multimode fibers are considered. The peak frequency, the amplitude, and the bandwidth of each possible passband are studied. From the simulation results, it is found that these parameters can be estimated. The approximation formulas for these three important parameters are given. Using the results found, the low-frequency passbands of multimode fibers can be utilized; thus, in comparison to the data rate obtained from the 3-dB modal bandwidth, a higher data rate for transmitting a signal over a multimode fiber can be increased at least 3 folds.**

***Index Terms*—Multimode Fibers; Passbands; Peak Frequency; Peak Amplitude, Bandwidth.**

I. Introduction

In optical communications, the signal is transformed into light and transmitted over an optical fiber. The wavelength to be used in optical communications is ranged between 800 to 1,600 nm and the available bandwidth is up to 10 THz [1]. Optical fibers can be categorized into 2 types: single-mode and multimode fibers. The main difference between these two is the number of guided modes carrying the optical signal; resulting in a limitation on the transmission bandwidth. In multimode fibers, the most common known bandwidth is the 3-dB modal bandwidth, which is limited to 300 to 500 MHz-km. To increase this limitation, a technique called subcarrier multiplexing (SCM) technique [2] has been done since at this high-frequency range, the frequency response of multimode fibers are frequency-selective channels; thus, there are many available passbands.

There were also some studies focusing on the characteristics of multimode fibers in the high-frequency range, and it was found that the signal can be transmitted over the high-frequency range of multimode fibers. Further studies on multimode fibers in terms of their stochastic model were done assuming that the mode distributions are Rayleigh distribution [3] and Gaussian distribution [4, 5]. It was found that the average amplitude of these passbands depends on the number of guided modes and the average bandwidth of these passbands depends on the standard deviation of the mode delays. Using SCM, the available bandwidth and the distance of the fiber were also considered [6]. Many studies and experiments [7–16] for transmitting a high data-rate signal over these passbands using SCM have been done.

Considering the research done in analyzing the frequency response of multimode fibers, it is viewed that only the 3-dB modal bandwidth and the high-frequency passband region with frequency higher than 1.5 GHz were done. The characteristics of the low-frequency passband region between 3-dB modal bandwidth and 1.5 GHz have not been studied. This frequency region can be viewed as potential channels in transmitting signal and will be analyzed in this paper. The peak frequency, peak amplitude and the available bandwidth of each possible passband in this region will be studied and some mathematical formula in estimating these parameters will be given.

The organization of this paper is done as follows. The frequency response of the multimode fiber is given in Section II. The characteristics of multimode fiber at low-frequency passband region are studied in Section III. In Section IV, the mathematical formulas for estimating the related parameters are presented. Finally in Section V, the conclusion is given.

II. Frequency Response of The Multimode Fiber

The impulse response [2, 3] of the multimode fiber with N_{mode} guided modes is shown in Equation (1).

$$h_{\text{fiber}}(t) = \sum_{n=1}^{N_{\text{mode}}} \delta\left(t - t_{d,n}\right) \tag{1}$$

where: $h_{\text{fiber}}(t)$ is the impulse response of the multimode fiber
N_{mode} is the number of guided modes
$t_{d,n}$ is the delay of the n^{th} mode.

Taking Fourier transform to (1), the frequency response of the complex envelope of the multimode fiber can be obtained, as shown in Equation (2).

$$H_{\text{fiber}}(f) = \sum_{n=1}^{N_{\text{mode}}} e^{-j2\pi f t_{d,n}} \tag{2}$$

From Equation (1) and (2), it is seen that these responses depend on two parameters: N_{mode} and $t_{d,n}$. Normally, the number of guided modes is in the range of 50 to 200. In this paper, the number of guided modes of 100 is used. Then, considering $t_{d,n}$, it is seen that this parameter can be described using its statistical property, that is, the probability density function, $f_{t_{d,n}}(t_{d,n})$. Assuming that $t_{d,n}$ is uniformly distributed about the average delay, $t_{d,avg}$, with the maximum deviation of $t_{d,dev}$; $f_{t_{d,n}}(t_{d,n})$ is given by Equation (3).

$$f_{t_{d,n}}\left(t_{d,n}\right)=\begin{cases}\dfrac{1}{2t_{d,dev}}; & t_{d,avg}-t_{d,dev}\le t_{d,n}\le t_{d,avg}+t_{d,dev}\\ 0\ ; & \text{otherwise}\end{cases} \tag{3}$$

Applying N_{mode} of 100, $t_{d,avg}$ of 5 μs, and $t_{d,dev}$ of 2.50 ns to Equation (3), the frequency response of the multimode fiber from (2) can be determined. The magnitude responses from three different sets of delays are shown in Figure 1.

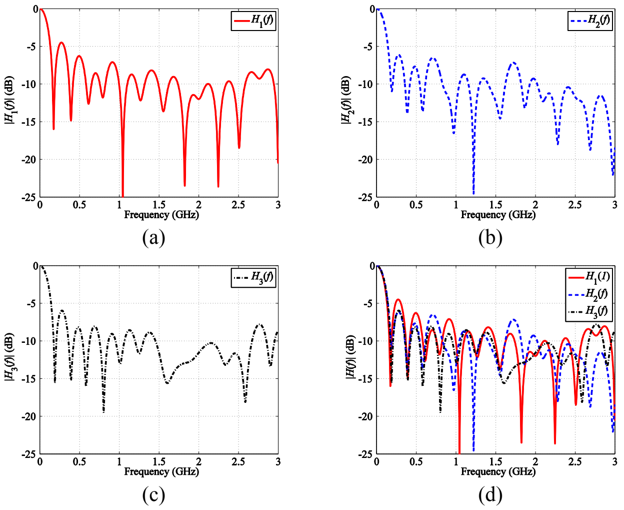

Figure 1: Magnitude response of the complex envelope of the multimode fiber modeled by (2) with N_{mode} = 100, $t_{d,avg}$ = 5 μs, and $t_{d,dev}$ = 2.50 ns. Three responses correspond to three different sets of delays

Figure 1 shows that in all the three sub-figures ((a) to (c)), the 3-dB modal bandwidth is approximately 100 MHz. Further, there are passbands available for the frequency higher than 0.2 GHz. However, these passbands are not located at the same frequencies as the set of the delays changes. Considering particularly in the frequency range between 0.2 – 1.5 GHz, it can be seen from Figure 1(d) that the passbands from all the three responses are mostly located at the same frequencies with a variation of the magnitude. At this point, it is seen that if these low-frequency passbands can be used in signal transmission, a higher data rate signal can be transmitted compared to that from only the 3-dB modal bandwidth.

III. Characteristics of The Multimode Fiber at Low-Frequency Passband Region

In this section, three important parameters that is, peak frequency, peak amplitude, and bandwidth of the available passbands in the low-frequency region of the multimode fiber are studied. These parameters are described in Figure 2.

From Figure 2, the peak frequency of a passband is the frequency where the amplitude of such passband is at the highest level, namely at the peak amplitude. The bandwidth of a passband is determined from the 3-dB bandwidth of the passband. For example, in this figure, the peak frequency is at 0.29 GHz; the peak amplitude is at -4.5 dB; and the bandwidth is approximately 200 MHz.

Applying N_{mode} of 100, $t_{d,avg}$ of 5 μs, and $t_{d,dev}$ 2.50 ns to (3), 1,000 sets of delays are generated and 1,000 frequency responses can be simulated. The average peak frequency, bandwidth, and peak amplitude can be determined from the simulations. Changing $t_{d,dev}$ from 2.50 ns to 5.00 and 7.50 ns, 1,000 sets of delays and frequency responses for each $t_{d,dev}$ can be obtained; thus, three considered parameters are determined.

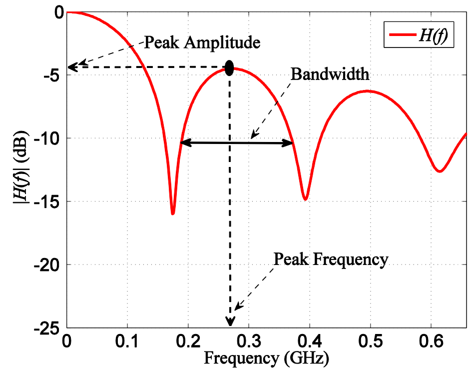

Figure 2: Three important passband parameters to be studied

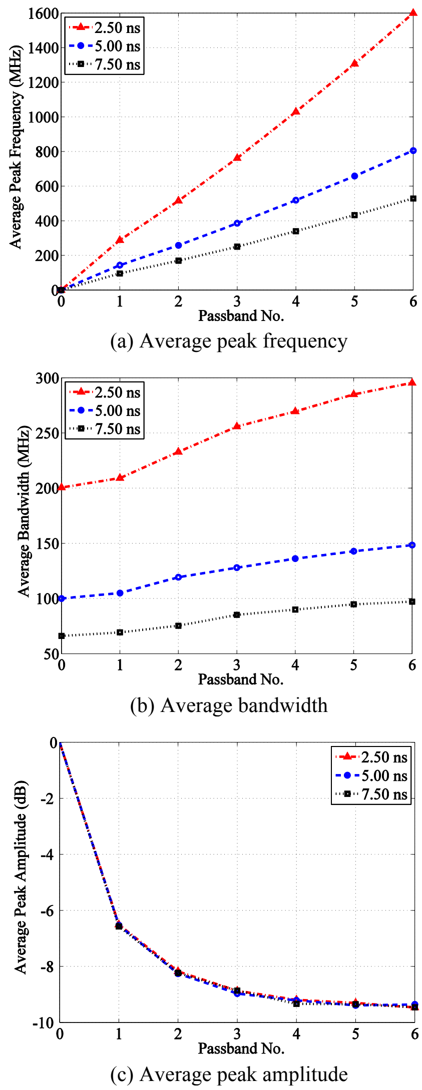

Figure 3: (a) Average peak frequency, (b) average bandwidth, and (c) average peak amplitude; obtained from 1,000 sets of frequency responses for 3 values of maximum delay deviation

From Figure 3, it is seen that only the first six passbands are considered. Three parameters of these passbands are studied as shown in Figure 3(a), (b), and (c). The average peak frequency of the passbands is shown in Figure 3(a). It is seen that for $t_{d,dev}$ of 2.50 ns, the peak frequency of the passband increases almost linearly as the function of the passband sequence. For example, at the passband no. 2, 3, 4, and 5, the average peak frequencies are 288, 515, 762, and 1,029 MHz, respectively. Similar results are also obtained for the case of $t_{d,dev}$ of 5.00 and 7.50 ns. The difference among these three cases is that for the peak frequency of a particular passband no., the highest peak frequency is from the case of $t_{d,dev}$ of 2.50 ns, while the lowest peak frequency is from the case of $t_{d,dev}$ of 7.50 ns. Based on all the three cases, the zeroth frequency is for the 3-dB modal band.

The average bandwidth of the passbands is shown in Figure 3(b). For $t_{d,dev}$ of 2.50 ns, the bandwidth slightly increases from 200 MHz to 300 MHz as the passband no. increases. Similar results are obtained for the other two values of $t_{d,dev}$. Specifically, for $t_{d,dev}$ of 5.00 ns, the bandwidth increases from 100 MHz to 150 MHz, and for $t_{d,dev}$ of 7.50 ns, the bandwidth increases from 66 MHz to 97 MHz.

The average peak amplitude is shown inFigure 3(c). It is clearly seen that for all the three values of $t_{d,dev}$, the results are almost identical, that is, as the average peak amplitude reduces as the passband no. increases. However, after the passband no. 4, the average peak amplitude is kept almost constant; that is, about -9.5 dB. From this figure, it can be seen that the average peak amplitude depends only on the passband no. The value of $t_{d,dev}$ does not affect the average peak amplitude of the passbands.

From Figure 3(a) and (b), it can be clearly seen that two parameters that affect the average peak frequency and the average bandwidth of the passbands are the passband no. and the value of $t_{d,dev}$. Further, from Figure 3(c), the only parameter affecting the value of the average peak amplitude of the passbands is the passband no. Considering these relationships, it can be seen that the average peak frequency, bandwidth, and peak amplitude of the passbands in the low-frequency passband region of the multimode fiber can be estimated if the passband no. and $t_{d,dev}$ are known. The formulas for approximating these parameters are analyzed in the next section.

IV. Formulas for Estimating Low-Frequency Passband Parameters

A. Average Peak Frequency ($f_{peak,avg}$)

From Figure 3(a), it can be seen that the average peak frequency increases linearly as the passband no. increases. Further, changing the value of $t_{d,dev}$ results in the average peak frequency to change. Applying the linear regression analysis, the mathematical formula for estimating the average peak frequency for a particular passband no. is given in Equation (4).

$$f_{peak,avg}(i) = \begin{cases} 0 & ; \quad i = 0 \\ \dfrac{1.305i + 0.0298}{2t_{d,dev}} & ; \quad i = 1,2,\ldots,6 \end{cases} \tag{4}$$

where i is the passband no.

From Equation (4), it is seen that a particular passband no., $f_{peak,avg}$ can be determined if $t_{d,dev}$ is known. For the case of i = 0, it means the 3-dB modal bandwidth; thus, the frequency to be used for this has to be zero. The plots of $f_{peak,avg}$ for different cases are shown next.

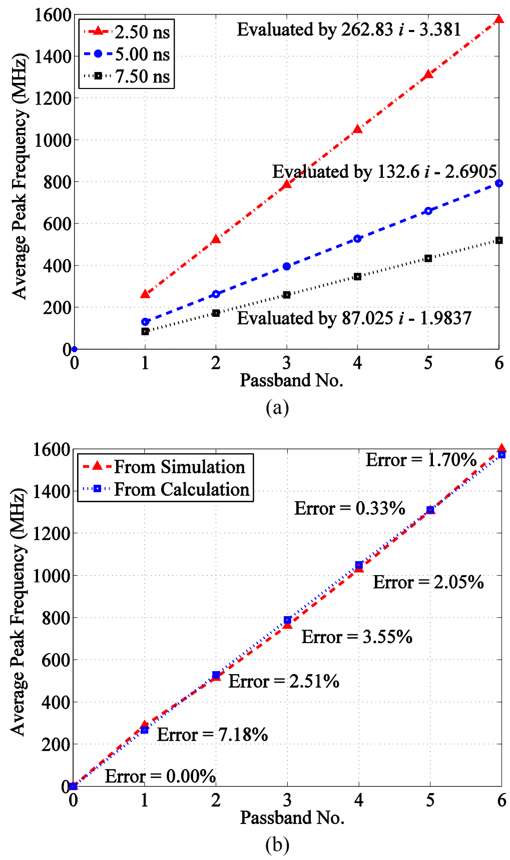

Figure 4: (a) The average peak frequency evaluated from Equation (4), and (b) The error between the simulation and calculation for the average peak frequency with $t_{d,dev}$ = 2.50 ns.

From Figure 4 (a), $f_{peak,avg}$ evaluated from Equation (4) for three values of $t_{d,dev}$ are plotted. It can be seen that $f_{peak,avg}$ grows linearly as the function of the passband no. The equation for evaluating $f_{peak,avg}$ for a particular value $t_{d,dev}$ is also shown. And, in Figure 4(b), by comparing between the simulation and calculation, the error is presented for the case of $t_{d,dev}$ = 2.50 ns. The errors for passband no. 1 to 6 are 7.18%, 2.51%, 3.55%, 2.05%, 0.33%, and 1.70%, respectively. It can be seen that the maximum error is 7.18 % at the Passband No. 1. Further, as the passband no. increases, all the errors are less than 4 %. This implies that there is a strong agreement between the simulation and calculation from Equation (4); thus, the given formula in Equation (4) can be certainly be used in estimating the average peak frequency of the low-frequency passbands of the multimode fiber.

B. Average Bandwidth (BW_{avg})

From Figure 3(b), it can be seen that for a particular $t_{d,dev}$, the average bandwidth slightly increases as the passband no. increases. Further, changing the value of $t_{d,dev}$, lead to the changes in the average bandwidth. From this figure, applying the polynomial regression analysis with the degree of 3, a formula for estimating the average bandwidth for a particular passband no. can be obtained and given in Equation (5).

From Equation (5), it can be seen that for a particular passband no., BW_{avg} can be calculated if the passband no. and $t_{d,dev}$ are known. For the case of $i = 0$, it can be seen that the bandwidth is just the 3-dB modal bandwidth. This is in agreement with the results found in the previous research [6–7]. The plots of BW_{avg} for different cases are shown in Figure 5.

$$BW_{avg}(i) = \frac{1}{2t_{d,dev}}\begin{cases} 1 & ; \quad i = 0 \\ \frac{1}{1{,}000}\begin{pmatrix} 0.54i^3 - 14.35i^2 \\ +164i + 895 \end{pmatrix} & ; \quad i = 1,2,\ldots,6 \end{cases} \tag{5}$$

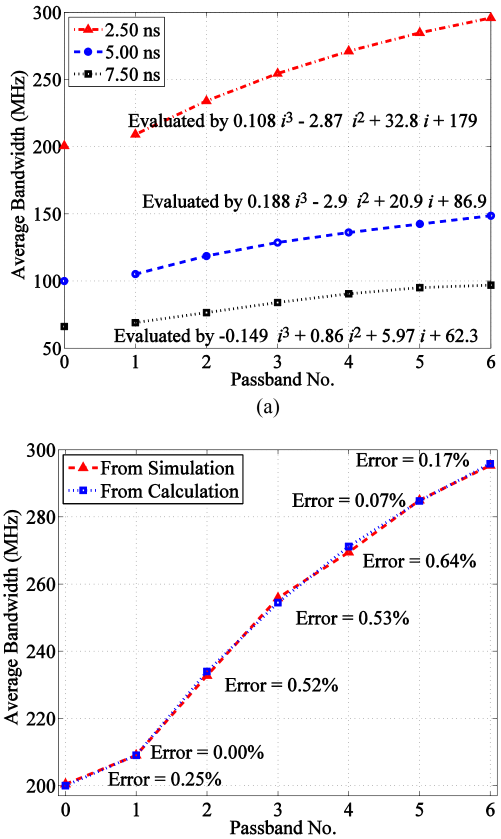

Figure 5: (a) The average bandwidth evaluated from Equation (5), and (b) The error between the simulation and calculation for the average bandwidth with $t_{d,dev}$ = 2.50 ns.

The plot of BW_{avg} evaluated from Equation (5) is shown in Figure 5(a). Three values of $t_{d,dev}$ are used resulting in three corresponding curves. Comparing these curves to Figure 3(b), it can be seen that they are almost identical; that is, as the passband no. increases, the average bandwidth also increases. Additionally, for a particular $t_{d,dev}$, it can be seen that the bandwidths of passband no. 1 to 6 are slightly larger than the 3-dB modal bandwidth. For example, for $t_{d,dev}$ of 2.50 ns, the 3-dB modal bandwidth is 200 MHz, while the bandwidth of passband no. 1 to 6 is ranged between 235 to 295 MHz.

To compare between the simulation and calculation, the case of $t_{d,dev}$ of 2.50 ns is depicted in Figure 5(b). The errors in percentage for the passband no. 1 to 6 are 0, 0.52, 0.53, 0.64, 0.07, and 0.17, respectively. The maximum error is less than 1 %. These results clearly indicate that the estimation formula given in Equation (5) can be used to determine the average bandwidth of the low-frequency passbands of the multimode fiber.

C. *Average Peak Amplitude ($A_{peak,avg}$)*

Considering Figure 3(c), the average peak amplitude of a low-frequency passband depends solely on the passband no. Using this information and the results from this figure, the average peak amplitude (in dB) can be computed using the following equation.

$$A_{peak,avg}(i) = \begin{cases} 0 & ; \quad i = 0 \\ \begin{pmatrix} 0.177i^2 - 1.76i \\ -5.09 \end{pmatrix} & ; \quad i = 1,2,\ldots,6 \end{cases} \tag{6}$$

where i is the passband no.

Applying the second degree of polynomial in the regression analysis, the mathematical formula for calculating $A_{peak,avg}$ for a particular passband no. can be obtained, as shown in (6). It can be seen that as the passband no. increases, $A_{peak,avg}$ becomes less. The results from the calculation and simulation are given in Figure 6.

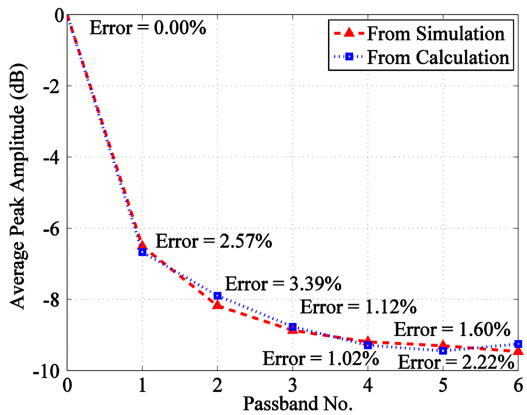

Figure 6: The average peak amplitude from simulation and calculation from (6).

The average peak amplitudes from the simulation and the calculation are shown in Figure 6. For both curves, the average peak amplitude decreases as the passband no. increases. A strong decrease happens at the low passband no. Further, the average peak amplitude becomes almost stable at the passband no. 4 to 6. These results are also consistent with the research done previously for the high-frequency passbands for the multimode fiber that the average peak amplitude of the passbands is a constant depending on the number of guided modes [6]. Considering the effectiveness of the formula in Equation (6) for estimating the average peak amplitude, the error is computed. The errors in percentage for passband no. 1 to 6 are 2.57, 3.39, 1.12, 1.02, 1.60, and 2.22, respectively. It is clearly seen that they strongly agree with the maximum error less than 3.5 %. This approximation formula is shown to be a useful one for determining the average peak amplitude for a particular low-frequency passband of the multimode fiber.

Considering the simulation results in last section and the approximation formulas shown in this section, it can be seen that if the maximum delay deviation is known, three important parameters for a low-frequency passband of the multimode fiber, namely the average peak frequency, the average bandwidth, and the average peak amplitude can be estimated. Normally, the maximum delay deviation of the guided modes depends on the type of multimode fiber and the length of the fiber. The smaller deviation is from the graded index multimode fiber. Further, as the fiber becomes longer, the delay spread of the guided modes becomes larger; thus, higher maximum delay deviation. Using the average peak frequencies with SCM, the suitable subcarrier frequencies are known. Knowing the average bandwidth, the data rate in a particular passband can be determined. The average peak amplitude of a particular passband can be used for setting the proper amplification needed at the end of the fiber in order to compensate the attenuation occurred in the fiber. If all of the six passbands were utilized, the bandwidth for transmitting a signal over a multimode fiber can be increased significantly. Applying $t_{d,dev}$ of 2.50 ns to (4) to (6), the three important parameters are shown in Table 1.

Table 1
Average peak frequency, average bandwidth, and average peak amplitude of low-frequency passbands from (4), (5), and (6) for $t_{d,dev}$ of 2.50 ns

	Passband No.						
	0	1	2	3	4	5	6
$f_{peak,avg}$ (MHz)	0	267	528	789	1,050	1,310	1,572
BW_{avg} (MHz)	200	209	234	254	271	285	296
$A_{peak,avg}$ (dB)	0	-6.7	-7.9	-8.8	-9.3	-9.5	-9.3

From Table 1, for $t_{d,dev}$ of 2.50 ns, three parameters for all low-frequency passbands are given. The peak frequencies are at 267, 528, 789, 1,050, 1,310, and 1,572 MHz. These frequencies can be used as the subcarrier frequencies in transmitting signals over SCM. The peak amplitudes of these passbands are also given in the table. These can be used in the equalization process at the end of the fiber in order to undo the loss happened in the passbands. Considering the average bandwidth, it is seen that the combined bandwidth from all of the passbands is 1,549 MHz, which is 7.7 times larger than the 3-dB modal bandwidth (i.e., 200 MHz). With this considerable increase of the available bandwidth, the combined data rate transmitted over these passbands can be up to 3.8 times larger than the data rate sent by the 3-dB modal band. From this consideration, it implies that utilizing these low-frequency passbands, the total data rate transmitted over the multimode fiber can be significantly increased.

V. CONCLUSIONS

The low-frequency passband region of the multimode fiber is studied and analyzed in this paper. It is found that there are some passbands in this region which can be used in signal transmission. From the simulation results, it is found that, for these passbands, three important parameters, namely the average peak frequency, the average bandwidth, and the average peak amplitude, can be predicted. Depending on the passband no. and the maximum delay deviation of the guided modes, the formulas for estimating these parameters are found. The estimation results strongly aligned with the results found in the simulation. Therefore, the characteristics of the low-frequency passbands of the multimode fiber can be precisely estimated. If these low-frequency passbands of the multimode fiber are utilized to transmit subcarrier signals with Subcarrier Multiplexing technique, the data rate can be increased by at least 3 folds in comparison to the data rate delivered only by the 3-dB modal bandwidth of the multimode fiber.

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