

A Quarter Car ARX Model Identification Based on Real Car Test Data

Dirman Hanafi¹, Mohammad Saiful Huq¹, Mohd Syafiq Suid¹, Mohd Fua'ad Rahmat²

¹*Advanced Mechatronic Research Group (AdMiRe), Faculty of Electrical and Electronic Engineering, Universiti Tun Hussein Onn Malaysia, 86400 Parit Raja, Batu Pahat, Johor, Malaysia.*

²*Faculty of Electrical Engineering, Universiti Teknologi Malaysia, 81310 Skudai, Johor, Malaysia.*
dirman@uthm.edu.my

Abstract—This paper presents a system identification of a quarter car passive suspension system dynamic model based on real-time running test car data. The input-output data of a car were recorded by test-driving the car on a road surface. The input variable is the vertical acceleration of the car shaft, while the output variable is the vertical acceleration of the body of the car. Two acceleration sensors were installed on the front right corner of the car: One on top of the suspension and another on the car shaft at the bottom of the suspension. The acquired data were used to identify the mathematical model of a quarter car passive suspension system dynamics. A quarter car passive suspension system was assumed to have an ARX model structure, hence qualifies to be a candidate model for system identification. The system identification algorithm used in this work was based on linear least-square estimation. The results showed that the best ARX model of the car passive suspension system model is produced with the best fit of 90.65%, Akaike's FPE is 5.315×10^{-6} . The output order of the model was found to be four, the input order is two and the time delay was one. The fit rate greater than 90% and along with a very small value for the FPE means that the system identification requirements are fulfilled and the identified model is acceptable.

Index Terms—Quarter Car Passive Suspension System Dynamics; Speed; Running Test; Artificial Road Surface; ARX Model.

I. INTRODUCTION

The main purposes of a car suspension system are to support the weight of vehicle, keep the wheels on the ground, minimize the transient force to the body, maintain good ride comfort, and enhance the handling performance [1]. It is impossible for the passive suspensions to achieve simultaneously the best performance of ride comfort and handling quality under all driving conditions [2,3].

Passive suspension systems consist of spring and damper. These parameters, i.e. the spring constant and the damping constant are fixed from the design stage itself; hence they are fixed and cannot be controlled [1,4]. Passive suspension systems with no controllable standard characteristics are the most widespread usage in vehicle production. Their popularity stems primarily from their simple design, high reliability and the passive nature of the operation (without any power supply) [5]. One of the major problems of passive suspension is that it transfers a lot of road input or causes unevenness of on-the-road car driving if it is heavily damped or the suspension is too hard [6].

The performance of such passive suspension systems can be improved by designing and analyzing the suspension system controller, leading to high fidelity mathematical

model that makes it necessary to capture accurate dynamics of the car suspension system [7].

An efficient and reliable way to determine the high fidelity mathematical model of a system is using system identification. System identification is an art of modeling. Its basic philosophy is to form a good mathematical model to fit input-output data of a system instead of building a model by its physical laws, although the physical structures provide some guidance in choosing model structures and experiment designs [8]. One of the well-known system identification algorithms is the least-squares estimation method [9,10] and it is applied in this paper.

A quarter-car model is an easy way to understand and represent the vehicle suspension system dynamics. A quarter car model describes the vibration transmission from the road surface to the body of the vehicle and eventually to the passenger [4]. Therefore, a quarter car model offers a quite reasonable representation of the actual suspension system dynamic [11].

In this paper, the quarter car is assumed to have an Autoregressive Exogenous (ARX) model structure and used to be the candidate model in system identification. ARX models constitute the simplest way of representing a dynamic system [12,13]. The ARX model is assumed as a single-input and single-output (SISO) model.

One way to collect a car input-output data can be done using test rig method [14], but in this case, the effect of environment, driving behavior, and other factors that affect a car suspension system are not accounted for. Therefore, the data produced are not maximally informative. To have an informative data in this work, input-output data of the car suspension system are collected by driving a test car on the artificial road surface. This road surface is the real road surface imitation with known dimensions [7].

II. SYSTEM IDENTIFICATION

Constructing models from observed data is one of the fundamental practices in science and engineering. Several methodologies and nomenclatures have been developed in different application areas relating to modeling. Within the control system area, the techniques are known under the term System Identification [15,16].

In system identification, the parameters of the model are estimated using estimation algorithm. One of the well-known estimation algorithms is the least square. The method of least squares is about estimating parameters by minimizing the squared discrepancies between observed data and their

expected values [9,16]. The system identification block diagram is as shown below:

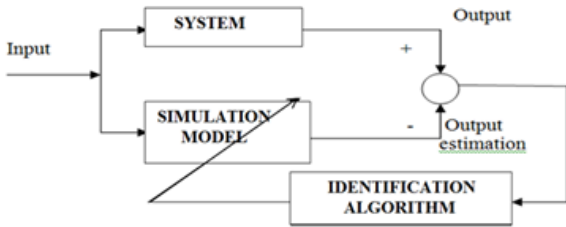


Figure 1: System Identification Block Diagram

The determination of the model of a dynamical system from the observed input-output data using system identification procedure entails the following three steps:

1. The input-output data collection through an experimental procedure.
2. A set of candidate models (the model structure).
3. A criterion to select a particular model in the set, based on the information in the data.

The generic form of the ARX model is written as [12,13,16]:

$$y(kT) = -a_1y(k-1)T - \dots - a_{n_a}y(k-n_a)T + b_1u(k-1)T + \dots + b_{n_b}u(k-d-n_b)T \quad (1)$$

where, n_a and n_b are output variable and input variable orders respectively and d is the time delay.

The best parameter estimation using the least square estimation is given by:

$$\hat{\theta}_N = \left[\frac{1}{N} \sum_{t=1}^N \varphi(t)\varphi^T(t) \right]^{-1} \left[\frac{1}{N} \sum_{t=1}^N \varphi(t)y^T(t) \right] \quad (2)$$

where:

$$\varphi(t) = [-y(k-1)T \dots -y(k-n_a)T \quad u(k-1)T + \dots + u(k-d-n_b)T]$$

$$\theta^T = [a_1 \dots a_{n_a} \quad b_1 \dots b_{n_b}]$$

III. INPUT OUTPUT DATA COLLECTION

Input-output data are a crucial factor in determining the accuracy of the model parameters produced in system identification. The data should be maximally informative, including all factors that influence system dynamics. Therefore, in this work the input-output data of car suspension system were collected through experiments or real test system. A test car is derived on a special road event that is named artificial road surface [7,17]. Car sprung mass vertical acceleration and unsprung mass vertical acceleration are the output variable and input variable respectively, which were measured using two accelerometers. Installation of each of the accelerometers and the data collection processes are illustrated in Figure 2 until Figure 4.



Figure 2: Accelerometer Sensor Installation for Vertical Body Acceleration

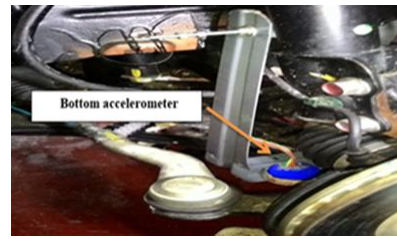


Figure 3: Accelerometer Sensor Installation for Vertical Shaft Acceleration



Figure 4: Running Test Car on Artificial Road Surface

Artificial road surfaces are made of plywood and beam woods as shown in Figure 4. Beam woods with various dimensions were placed on the plywood at a random distance. To ensure that both the right and left sides of car suspensions experience the same vibrations, the artificial road surfaces were made identical on both sides.

IV. ARX MODEL OF A QUARTER CAR PASSIVE SUSPENSION SYSTEM

A simple quarter car passive suspension system model consists of one-fourth of the body mass and the suspension components [17,18] as shown in Figure 5, where M_s is the sprung mass, Z_s is sprung mass position, K_s is the suspension spring element, C_s is the suspension damping element and Z_o is the unsprung mass position. Applying Newton's Second Law, the equation of simple a quarter car dynamics is:

$$M_s \ddot{Z}_s = C_s (\dot{Z}_o - \dot{Z}_s) + K_s (Z_o - Z_s) \quad (3)$$

The input and output variables are sprung mass vertical acceleration and unsprung mass vertical acceleration, respectively. By differentiating Equation (1) twice and assuming that the output variable is $y = \frac{d^2 Z_s}{dt^2}$ and the input variable is $u = \frac{d^2 Z_o}{dt^2}$, the following equation is obtained:

$$M_s \ddot{y} + C_s \dot{y} + K_s y = C_s \dot{u} + K_s u \quad (4)$$

The discrete form of Equation (2) is as below:

$$y(kT) = -a_1y(k-1)T - a_2y(k-2) + b_1u(k-1)T \quad (5)$$

Equation (3) has the similar structure with the ARX model.

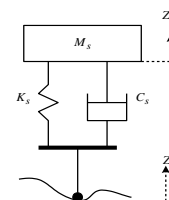


Figure 5: A Quarter Car Dynamic Model

V. SYSTEM IDENTIFICATION AND VALIDATION

The input-output data of the quarter car suspension system were collected from the experimental test as previously mentioned. From a set of data, the best ARX model of a quarter car passive suspension system was identified using the least square estimation algorithm.

The data of sprung mass vertical acceleration as the output variable and the unsprung mass vertical acceleration as input variable are shown in Figure 6(a) and (b), respectively. The data was divided into two parts, in which the first half was used for estimating the model parameter and the second half was for model validation purpose.

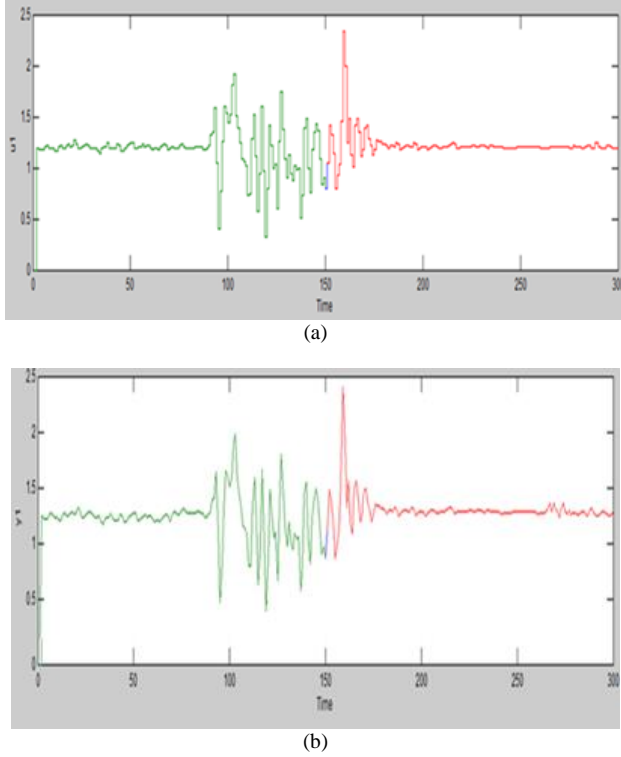


Figure 6: Input-Output Data (a) Sprung mass vertical acceleration signal as output variable (b) Unsprung mass vertical acceleration as input signal

The best ARX model of a quarter car passive suspension system dynamic produced by system identification technique for the data is shown in Figure 6 as Equation (6). The model has output order (n_a) = 4, the input order (n_b) = 2 and time delay (d) = 1. The model validation gives the best fit and Akaike's Final Prediction Error (FPE) are 90.65% and 5.315×10^{-6} respectively. On the other hand, the comparison between the measured signal and the model response is presented in Figure 7. The autocorrelation of the residual and the cross correlation between the residual and the input are described in Figures 8 and 9, respectively.

$$\begin{aligned}
 y(kT) = & 0.9669y((k-1)T) + 0.0134y((k-2)T) \\
 & -0.0004y((k-3)T) + 0.0057y((k-4)T) \\
 & +0.9973u((k-1)T) - 0.9728u((k-2)T) \\
 & +0.9973u((k-1)T) - 0.9728u((k-2)T)
 \end{aligned} \quad (6)$$

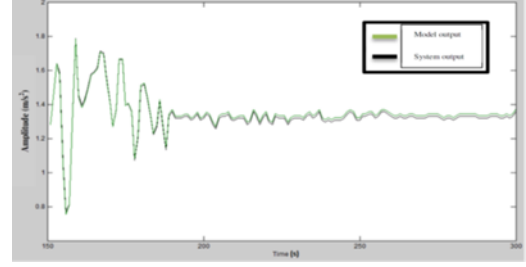


Figure 7: Comparison between Measured Signal and Model Response

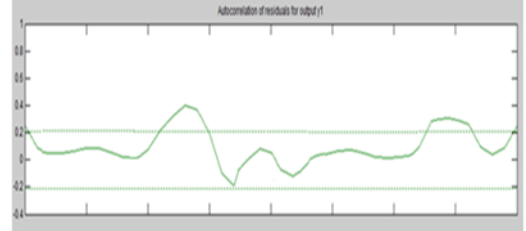


Figure 8: Auto Correlation of Residual

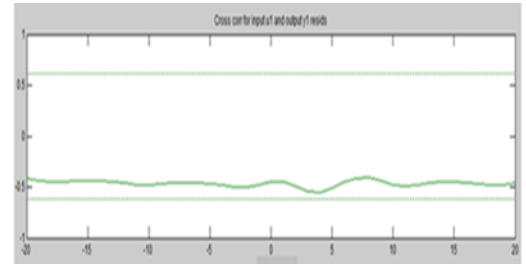


Figure 9: Cross Correlation of Residual and Input

VI. DISCUSSION

The best ARX model of a quarter car passive suspension system has been identified in this work using system identified technique. The input-output data used in system identification were collected by running a test car on a road surface specially tailored to create the desired vibration condition. The Least square estimation algorithm was achieved with a significantly high fit value and very small Akaike's FPE value. Additionally, the value for the residual autocorrelation and the residual cross-correlation with input were also found to be within limits. It means the system identification criteria are fulfilled, and the resulting ARX model is acceptable.

VII. CONCLUSION

The ARX models of a quarter car passive suspension system with body mass (sprung mass) vertical acceleration as the output variable and tire mass (unsprung mass) vertical acceleration as the input variable have been obtained through system identification technique. The best model has output order (n_a) = 4, input order (n_b) = 2, delay (d) = 1, best fit = 90.65% and Akaike's FPE = 5.315×10^{-6} . The residual autocorrelation and residual cross-correlation with input were also found to be well within limits. Therefore, it can be concluded that the criteria for a successful system identification are fulfilled and the resulting ARX model can be used for analyzing and designing the suspension controllers.

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