

# Four-Dimensional Homogeneous Systolic Pyramid Automata

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**Abstract**—Cellular automaton is famous as a kind of the parallel automaton. Cellular automata were investigated not only in the viewpoint of formal language theory, but also in the viewpoint of pattern recognition. Cellular automata can be classified into some types. A systolic pyramid automata is also one parallel model of various cellular automata. A homogeneous systolic pyramid automaton with four-dimensional layers (4-HSPA) is a pyramid stack of four-dimensional arrays of cells in which the bottom four-dimensional layer (level 0) has size  $a$  ( $a \geq 1$ ), the next lowest  $4(a-1)$ , and so forth, the  $(a-1)$ st four-dimensional layer (level  $(a-1)$ ) consisting of a single cell, called the root. Each cell means an identical finite-state machine. The input is accepted if and only if the root cell ever enters an accepting state. A 4-HSPA is said to be a real-time 4-HSPA if for every four-dimensional tape of size  $4a$  ( $a \geq 1$ ), it accepts the four-dimensional tape in time  $a-1$ . Moreover, a 1-way four-dimensional cellular automaton (1-4CA) can be considered as a natural extension of the 1-way two-dimensional cellular automaton to four-dimension. The initial configuration is accepted if the last special cell reaches a final state. A 1-4CA is said to be a real-time 1-4CA if when started with four-dimensional array of cells in nonquiescent state, the special cell reaches a final state. In this paper, we proposed a homogeneous systolic automaton with four-dimensional layers (4-HSPA), and investigated some properties of real-time 4-HSPA. Specifically, we first investigated the relationship between the accepting powers of real-time 4-HSPA's and real-time 1-4CA's. We next showed the recognizability of four-dimensional connected tapes by real-time 4-HSPA's.

**Index Terms**—Cellular Automaton; Diameter; Finite Automaton; Four-Dimension; Parallelism; Pattern Recognition; Real Time.

## I. INTRODUCTION

In recent years, due to the advances in many application areas such as dynamic image processing, computer animation, VR (virtual reality), AR (augmented reality), and so on, the study of four-dimensional pattern processing has been of crucial importance. And the question of whether processing four-dimensional digital patterns is much more difficult than three-dimensional ones is of great interest from the theoretical and practical standpoints. Thus, the study of four-dimensional automata as a computational model of four-dimensional pattern processing has been meaningful [4-23]. On the other hand, cellular automata has been investigated not only in the viewpoint of formal language theory, but also in the viewpoint of pattern recognition. Cellular automata can

be classified into some types [2]. A systolic pyramid automaton is also one parallel model of various cellular automata. In this paper, we proposed a homogeneous systolic pyramid automaton with four-dimensional layers (4-HSPA) as a new four-dimensional parallel computational model, and investigated some properties of real-time 4-HSPA.

Let  $\Sigma$  be a finite set of symbols. A four-dimensional tape over  $\Sigma$  is a four-dimensional rectangular array of elements of  $\Sigma$ . The set of all four-dimensional tapes over  $\Sigma$  is denoted by  $\Sigma^{(4)}$ . Given a tape  $x \in \Sigma^{(4)}$ , for each integer  $j$  ( $1 \leq j \leq 4$ ), we let  $l_j(x)$  be the length of  $x$  along the  $j$ th axis. The set of all  $x \in \Sigma^{(4)}$  with  $l_1(x) = n_1$ ,  $l_2(x) = n_2$ ,  $l_3(x) = n_3$ , and  $l_4(x) = n_4$  is denoted by  $\Sigma^{(n_1, n_2, n_3, n_4)}$ . When  $1 \leq i_j \leq l_j(x)$  for each  $j$  ( $1 \leq j \leq 4$ ), let  $x(i_1, i_2, i_3, i_4)$  denotes the symbol in  $x$  with coordinates  $(i_1, i_2, i_3, i_4)$ . Furthermore, we define:

$$x[(i_1, i_2, i_3, i_4), (i'_1, i'_2, i'_3, i'_4)],$$

when  $1 \leq i_j \leq i'_j \leq l_j(x)$  for each integer  $j$  ( $1 \leq j \leq 4$ ), as the four-dimensional input tape  $y$  satisfying the following conditions :

1. for each  $j$  ( $1 \leq j \leq 4$ ),  $l_j(y) = i_j - i'_j + 1$ ;
2. for each  $r_1, r_2, r_3, r_4$  ( $1 \leq r_1 \leq l_1(y)$ ,  $1 \leq r_2 \leq l_2(y)$ ,  $1 \leq r_3 \leq l_3(y)$ ,  $1 \leq r_4 \leq l_4(y)$ ),  $y(r_1, r_2, r_3, r_4) = x(r_1 + i'_1 - 1, r_2 + i'_2 - 1, r_3 + i'_3 - 1, r_4 + i'_4 - 1)$ . (We call  $x[(i_1, i_2, i_3, i_4), (i'_1, i'_2, i'_3, i'_4)]$  the  $[(i_1, i_2, i_3, i_4), (i'_1, i'_2, i'_3, i'_4)]$ -segment of  $x$ .)

For each  $x \in \Sigma^{(n_1, n_2, n_3, n_4)}$  and for each  $1 \leq i_1 \leq n_1$ ,  $1 \leq i_2 \leq n_2$ ,  $1 \leq i_3 \leq n_3$ ,  $1 \leq i_4 \leq n_4$ ,  $x[(i_1, 1, 1, 1)$ ,  $(i_1, n_2, n_3, n_4)$ ,  $x[(1, i_2, 1, 1)$ ,  $(n_1, n_2, n_3, n_4)$ ], and  $x[(1, 1, i_3, 1)$ ,  $(n_1, n_2, n_3, n_4)$ ] are called the  $i_1$ th (2-3) plane, the  $i_2$ th (1-3) plane, and the  $i_3$ th (1-2) plane of each time of  $x$ , and are denoted by  $x[i_1, *, *, *]$ ,  $x[* , i_2, *, *]$ , and  $x[* , *, i_3, *]$ , respectively.  $x[* , *, *, i_4]$  also has analogous meaning.

A four-dimensional homogeneous systolic pyramid automaton (4-HSPA) is a pyramidal stack of four-dimensional arrays of cells in which the bottom four-dimensional layer (level 0) has size  $4a$  ( $a \geq 1$ ), the next lowest  $4(a-1)$ , and so forth, the  $(a-1)$ st four-dimensional layer (level  $(a-1)$ ) consisting of a single cell, called the root. Each cell means an identical finite-state machine,  $M = (Q, \Sigma, \delta, \#, F)$ , where  $Q$  is a finite set of states,  $\Sigma \subseteq Q$  is a finite set of input states,  $\# \in Q - \Sigma$  is the *quiescent state*,  $F \subseteq Q$  is the set

of accepting states, and  $\delta: Q^{17} \rightarrow Q$  is the state transition function, mapping the current states of  $M$  and its 16 son cells in a  $2 \times 2 \times 2 \times 2$  block on the four-dimensional layer below into  $M$ 's next state. The input is accepted if and only if the root cell ever enters an accepting state. A 4-HSPA is said to be a real-time 4-HSPA if for every four-dimensional tape of size  $4a$  ( $a \geq 1$ ) it accepts the four-dimensional tape in time  $a - 1$ . By  $\mathcal{E}^R[4\text{-HSPA}]$  we denote the class of the sets of all the four-dimensional tapes accepted by a real-time 4-HSPA[1].

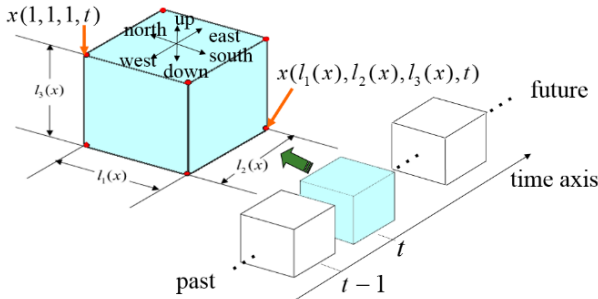


Figure 1: Four-dimensional input tape

A 1-way four-dimensional cellular automaton (1-4CA) can be considered as a natural extension of the 1-way two-dimensional cellular automaton to four dimensions [3]. The initial configuration of the cellular automaton is taken to be an  $l_1(x) \times l_2(x) \times l_3(x) \times l_4(x)$  array of cells in the nonquiescent state. The initial configuration is accepted if the last special cell reaches a final state.

A 1-4CA is said to be a real-time 1-4CA if when started with an  $l_1(x) \times l_2(x) \times l_3(x) \times l_4(x)$  array of cells in the nonquiescent state, the special cell reaches a final state in time  $l_1(x) + l_2(x) + l_3(x) \times l_4(x) - 1$ . By  $\mathcal{E}^R[1\text{-4CA}]$  we denote the class of the sets of all the four-dimensional tapes accepted by a real-time 1-4CA [3].

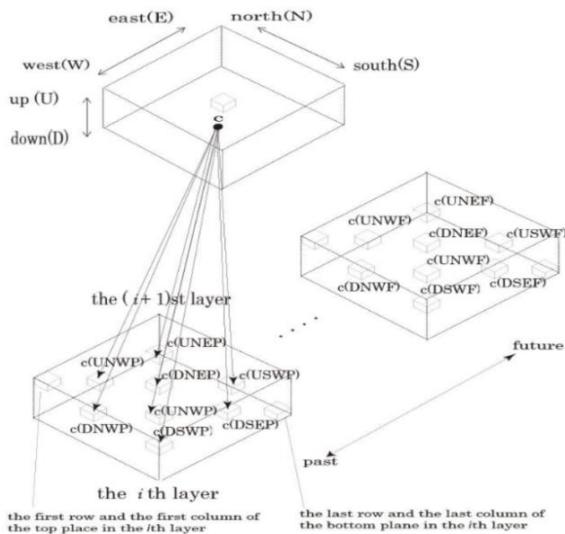


Figure 2: Four-dimensional homogeneous systolic pyramid automaton

## II. MAIN RESULTS

We mainly investigated a relationship between the accepting powers of real-time 4-HSPA's and real-time 1-4CA's. The following theorem implies that real-time 4-HSPA's are less powerful than real-time 1-4CA's.

**Theorem 2.1.**  $\mathcal{E}^R[4\text{-HSPA}] \subseteq \mathcal{E}^R[1\text{-4CA}]$ .

**Proof :** Let  $V = \{ x \in \{0,1\}^{(4)} \mid l_1(x) = l_2(x) = l_3(x) = l_4(x) \& [\forall i_1, \forall i_2, \forall i_3, \forall i_4 (1 \leq i_1 \leq l_1(x), 1 \leq i_2 \leq l_2(x), 1 \leq i_3 \leq l_3(x), 1 \leq i_4 \leq l_4(x)) [x(i_1, i_2, i_3, 1) = x(i_1, i_2, i_3, l_4(x))]] \}$ .

It is easily shown that  $V \in \mathcal{E}^R[1\text{-4CA}]$ . Below, we show that  $V \notin \mathcal{E}^R[4\text{-HSPA}]$ . Suppose that there exists a real-time 4-HSPA accepting  $V$ . For each  $t \geq 4$ , let

$$W(n) = \{ x \in \{0,1\}^{(4)} \mid l_1(x) = l_2(x) = l_3(x) = l_4(x) \& x[(1, 1, 1, 2), (t, t, t, t-1)] \in \{0\}^{(4)} \}.$$

Sixteen sons of the root cell  $A(t-1,1,1,1)$  of  $M A(t-2,1,1,2,1)$ ,  $A(t-2,1,2,2,1)$ ,  $A(t-2,2,1,2,1)$ ,  $A(t-2,2,2,2,1)$ ,  $A(t-2,1,1,3,1)$ ,  $A(t-2,1,2,3,1)$ ,  $A(t-2,2,1,3,1)$ ,  $A(t-2,2,2,3,1)$ ,  $A(t-2,1,1,2,2)$ ,  $A(t-2,1,2,2,2)$ ,  $A(t-2,2,1,2,2)$ ,  $A(t-2,2,2,2,2)$ ,  $A(t-2,1,1,3,2)$ ,  $A(t-2,1,2,3,2)$ ,  $A(t-2,2,1,3,2)$ ,  $A(t-2,2,2,3,2)$  are denoted by  $C^{(UNWP)}$ ,  $C^{(USWP)}$ ,  $C^{(USEP)}$ ,  $C^{(UNEP)}$ ,  $C^{(DNWP)}$ ,  $C^{(DSWP)}$ ,  $C^{(DSEP)}$ ,  $C^{(DNEP)}$ ,  $C^{(UNWF)}$ ,  $C^{(USWF)}$ ,  $C^{(USEF)}$ ,  $C^{(UNEF)}$ ,  $C^{(DNWF)}$ ,  $C^{(DSWF)}$ ,  $C^{(DSEF)}$ ,  $C^{(DNEF)}$ , respectively. For each  $x$  in  $W(n)$ ,  $x(UNWP)$ ,  $x(USWP)$ ,  $x(USEP)$ ,  $x(UNEP)$ ,  $x(DNWP)$ ,  $x(USWP)$ ,  $x(USEP)$ ,  $x(UNEP)$ ,  $x(UNWF)$ ,  $x(USWF)$ ,  $x(USEF)$ ,  $x(UNEF)$ ,  $x(DNWF)$ ,  $x(USWF)$ ,  $x(USEF)$ ,  $x(UNEF)$  are the states of  $C^{(UNWP)}$ ,  $C^{(USWP)}$ ,  $C^{(USEP)}$ ,  $C^{(UNEP)}$ ,  $C^{(DNWP)}$ ,  $C^{(DSWP)}$ ,  $C^{(DSEP)}$ ,  $C^{(DNEP)}$ ,  $C^{(UNWF)}$ ,  $C^{(USWF)}$ ,  $C^{(USEF)}$ ,  $C^{(UNEF)}$ ,  $C^{(DNWF)}$ ,  $C^{(DSWF)}$ ,  $C^{(DSEF)}$ ,  $C^{(DNEF)}$ , at time  $t-2$ , respectively. Let  $\sigma(x) = (x(UNWP), x(USWP), x(USEP), x(UNEP), x(DNWP), x(USWP), x(USEP), x(UNEP)), \gamma(x) = (x(UNWF), x(USWF), x(USEF), x(UNEF), x(DNWF), x(USWF), x(USEF), x(UNEF))$ , and  $\rho(x) = (x(UNWP), x(USWP), x(USEP), x(UNEP), x(DNWP), x(USWP), x(USEP), x(UNEP), x(UNWF), x(USWF), x(USEF), x(UNEF), x(DNWF), x(USWF), x(USEF), x(UNEF))$ . Then, the following two propositions must hold.

**Proposition 2.1.** (i) For any two tapes  $x, y \in W(n)$  whose 1st cubes are same,  $\sigma(x) = \sigma(y)$ . (ii) For any two tapes  $x, y \in W(n)$  whose  $n$ th cubes are same,  $\gamma(x) = \gamma(y)$ .

**Proof:** From the mechanism of each cell, it is easily seen that the states of  $C^{(UNWP)}$ ,  $C^{(USWP)}$ ,  $C^{(USEP)}$ ,  $C^{(UNEP)}$ ,  $C^{(DNWP)}$ ,  $C^{(DSWP)}$ ,  $C^{(DSEP)}$ ,  $C^{(DNEP)}$  are not influenced by the information of 1st cube. From this fact, we have (i). The proof of (ii) is the same as that of (i).

**Proposition 2.2.** For any two tapes  $x, y \in W(t)$  whose 1st cube are different,  $\sigma(x) \neq \sigma(y)$ .

**Proof:** Suppose to the contrary that  $\sigma(x) = \sigma(y)$ . We consider two tapes  $x', y' \in W(t)$  satisfying the following :

- 1st cube of  $x$  and  $n$ th cube of  $x$  are equal to 1st cube of  $x'$ , respectively
- 1st cube of  $y'$  is equal to 1st cube of  $y$ , and  $n$ th cube of  $y'$  is equal to 1st cube of  $x$ .

As is easily seen,  $x' \in V$  and so  $x'$  is accepted by  $M$ . On the other hand, from Proposition 2.1(ii),  $\gamma(x') = \gamma(y')$ . From Proposition 2.1(i),  $\sigma(x) = \sigma(x')$ ,  $\sigma(y) = \sigma(y')$ . It follows that  $y'$  must be also accepted by  $M$ . This contradicts the fact that  $y'$  is not in  $V$ .  $\square$

**Proof of Theorem 2.1 (continued):** Let  $p(t)$  be the number of tapes in  $W(t)$  whose 1st cubes are different, and let  $Q(t) = \{ \sigma(x) \mid x \in W(t) \}$ , where  $k$  is the number of states of each cell

of  $M$ . Then,  $p(t) = 2^{t^2}$ , and  $Q(t) \leq k^4$ . It follows that  $p(n) > Q(t)$  for large  $t$ . Therefore, it follows that for large  $t$ , there must be two tapes  $x, y$  in  $W(t)$  such that their 1st cubes are different and  $\sigma(x) = \sigma(y)$ . This contradicts Proposition 2.2, so we can conclude that  $V \notin \mathcal{L}^R[4\text{-HSPA}]$ . In the case of four-dimension, we can show that  $V \notin \mathcal{L}^R[4\text{-HSPA}]$  by using the same technique. This completes the proof of Theorem 2.1.

We next show the recognizability of four-dimensional connected tapes by real-time 4-HSPA's by using the same technique of Ref.[3]. Let  $x$  in  $\{0,1\}^{(4)}$ . A maximal subset  $P$  of  $N^4$  satisfying the following conditions is called a 1-component of  $x$ .

1. For any  $(i_1, i_2, i_3, i_4) \in P$ , we have  $1 \leq i_1 \leq l_1(x)$ ,  $1 \leq i_2 \leq l_2(x)$ ,  $1 \leq i_3 \leq l_3(x)$ ,  $1 \leq i_4 \leq l_4(x)$ , and  $x(i_1, i_2, i_3, i_4) = 1$ .
2. For any  $(i_1, i_2, i_3, i_4), (i_1', i_2', i_3', i_4') \in P$ , there exists a sequence  $(i_1, 0, i_2, 0, i_3, 0, i_4, 0), (i_1, 1, i_2, 1, i_3, 1, i_4, 1), \dots, (i_1, 4, i_2, 4, i_3, 4, i_4, 4)$  of elements in  $P$  such that  $(i_1, 0, i_2, 0, i_3, 0, i_4, 0) = (i_1, i_2, i_3, i_4)$ ,  $(i_1, 4, i_2, 4, i_3, 4, i_4, 4) = (i_1', i_2', i_3', i_4')$ , and  $|i_1, j - i_1, j - 1| + |i_2, j - i_2, j - 1| + |i_3, j - i_3, j - 1| + |i_4, j - i_4, j - 1| \leq 1 (1 \leq j \leq 4)$ . A tape  $x \in \{0, 1\}^{(4)}$  is called *connected* if there exists exactly one 1-component of  $x$ .

Let  $Tc$  be the set of all the four-dimensional connected tapes. Then, we have:

**Theorem 2.2.**  $Tc \notin \mathcal{L}^R[4\text{-HSPA}]$ .

### III. CONCLUSION

The technique of AR (augmented reality) or VR (virtual reality) progresses like the Pokemon GO and the Virtual Cinema in the world. The virtual technique will spread steadily among our societies in future. Thus, we think that the study of four-dimensional automata is very meaningful as a computational model of four-dimensional pattern processing. In this paper, we proposed a homogeneous systolic pyramid automaton with four-dimensional layers, and investigated a relationship between the accepting powers of homogeneous systolic pyramid automaton with four-dimensional layers (4-HSPA) and one-way four-dimensional cellular automata (1-4CA) in real time, and showed that real-time 4-HSPA's are less powerful than real time 1-4CA's.

It will be interesting to investigate an alternating version or synchronized alternating version of homogeneous systolic pyramid automaton with four-dimensional layers. Moreover, we think that there are many interesting open problems for digital geometry. Among them, the problem of connectedness, especially topological component is one of the most interesting topics [17].

Finally, we hope that some unsolved problems concerning this paper will be explicated in the near future.

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