A Fault Detection Algorithm using Multiple Residual Generation Filters

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***Abstract*—This paper proposes a fault detection algorithm based on multiple residual generation filters for discrete-time systems. Residuals are generated from estimation errors between the reference filter and multiple residual generation filters. These filters utilize only finite observation on the most recent window. The reference filter gives optimal state estimates based on all sensors. One the other hand, one of multiple residual generation filters can give the sub-optimal state estimates which can be independent of faulty sensor. Then, the fault detection rule is developed to indicate presence of fault by checking the agreement of multiple residuals. Multiple test variables for the detection rule are defined using the chi-squared distribution with one degree of freedom. Via numerical simulations for the aircraft engine system, the proposed algorithm is verified.**

***Index Terms*—Fault Detection; Residual Generation; Estimation Filter; Kalman Filter.**

I. Introduction

A fault detection is an important and challenging problem in many disciplines such as chemical engineering, nuclear engineering, aerospace engineering, and automotive systems [1-3]. The essential step for the fault detection is to generate a set of variables known as residuals by using one or more residual generation filters. These residuals should ideally be zero (or zero mean) under no-fault conditions. In practical applications, the residuals are corrupted by the presence of noise, unknown disturbances, and uncertainties in the system model. Hence, in order to be useful in practical applications, they should be insensitive to noise, disturbances, and model uncertainties while maximally sensitive to faults.

As the residual generation filter, the Kalman filter has been adopted in the stochastic case where noises have to be considered [4-9]. Due to the compact representation and the efficient manner, the Kalman filter has been applied successfully for various areas including a fault detection. However, the Kalman filter has an infinite memory structure that utilizes all observations accomplished by equaling weighting and has a recursive formulation. Thus, the Kalman filter tends to accumulate the filtering error as time goes and can show even divergence phenomenon for temporary modeling uncertainties and round-off errors [10-15]. In addition, actually, long past measurements are not useful for detection of faults with unknown times of occurrence. Moreover, it is also known that the increase of the number of measurements for a detection decision will increase detection latency in a system for detecting a signal with unknown time of occurrence.

Therefore, this paper proposes a new fault detection algorithm using multiple residual generation filters for discrete-time systems with multiple sensors. The proposed fault detection algorithm is an alternative simple form of the early developed algorithm [16] with the iterative form. Residuals are generated from estimation errors between the reference filter and multiple residual generation filters. These filters utilize only finite observation on the most recent window. The reference filter gives optimal state estimates based on all sensors. One the other hand, one of multiple residual generation filters can give the sub-optimal state estimates which can be independent of faulty sensor. Then, the fault detection rule is developed to indicate presence of fault by testing the consistency of multiple residuals. Multiple test variables for the detection rule are defined using the chi-squared distribution with one degree of freedom. The proposed algorithm is verified via numerical simulations.

II. A Fault Detection Algorithm

A discrete time system with sensor faults can be modeled by state-space model as follows:

$$\begin{aligned} x(i+1) &= Ax(i) + Bu(i) + Gw(i), \\ z(i) &= Cx(t) + Ff(t) + v(i), \end{aligned} \tag{1}$$

where $z(i) \in R^m$ is the sensor observation vector with m sensors, $f(i) \in R^m$ is the unknown fault vector, and thus the observation matrix C has m rows as follows:

$$z(i) \equiv \begin{bmatrix} z_1(i) \\ z_2(i) \\ \vdots \\ z_m(i) \end{bmatrix}, \; f(i) \equiv \begin{bmatrix} f_1(i) \\ f_2(i) \\ \vdots \\ f_m(i) \end{bmatrix}, \; C \equiv \begin{bmatrix} C_1 \\ C_2 \\ \vdots \\ C_m \end{bmatrix} \tag{2}$$

and $x(i) \in R^n$ is the state vector, $u(i) \in R^l$ is the input vector. The system noise $w(i) \in R^p$ and the observation noise $v(i) \in R^q$ are zero-mean white Gaussian with covariance Q and R, respectively.

A. Reference and Residual Generation Filters

Firstly, a reference filter and multiple residual generation filters are developed for residual generation in the proposed fault detection algorithm. They adopts the well-known finite memory structure estimation filter with a matrix form[11]. If there is no fault, $f(i)=0$, the finite memory structure estimation filter provides the state estimate $\hat{x}(i)$ of the system state $x(i)$ using only the most recent finite sensor observations on the window $[i-M, i]$. M is called the window length. That is, past sensor observations outside the window are discarded. When the system of (1) is completely observable, the finite memory structure estimation filter is represented by following simple matrix form [11].

$$\hat{x}(i) = H\left[Z(i) - \Lambda U(i)\right] \qquad (3)$$

with the filter gain matrix H,

$$H \equiv \left[\left(\Gamma^T \Pi^{-1} \Gamma\right)^{-1} \Gamma^T \Pi^{-1}\right] \qquad (4)$$

Matrices Λ, Γ, Π, Ξ in (3) and (4) are as follows:

$$\Lambda \equiv -\begin{bmatrix} CA^{-1}B & CA^{-2}B & \cdots & CA^{-M+1}B & CA^{-M}B \\ 0 & CA^{-1}B & \cdots & CA^{-M+2}B & CA^{-M+1}B \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & CA^{-1}B \end{bmatrix},$$

$$\Gamma \equiv \begin{bmatrix} CA^{-M} \\ CA^{-M+1} \\ \vdots \\ CA^{-1} \end{bmatrix},$$

$$\Pi \equiv \Xi\left[diag\left(\overbrace{Q \quad Q \quad \cdots \quad Q}^{M}\right)\right]\Xi^T + \left[diag\left(\overbrace{R \quad R \quad \cdots \quad R}^{M}\right)\right],$$

$$\Xi \equiv \begin{bmatrix} CA^{-1}G & CA^{-2}G & \cdots & CA^{-M+1}G & CA^{-M}G \\ 0 & CA^{-1}G & \cdots & CA^{-M+2}G & CA^{-M+1}G \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & CA^{-1}G \end{bmatrix},$$

The most recent finite sensor observations and inputs on the window $[i-M, i]$ are represented by:

$$Z(i) \equiv \left[z^T(i-M) \quad z^T(i-M+1) \quad \cdots \quad z^T(i-1)\right]^T,$$
$$U(i) \equiv \left[u^T(i-M) \quad u^T(i-M+1) \quad \cdots \quad u^T(i-1)\right]^T.$$

From the deadbeat property [11], the finite memory structure estimation filter can show the fast convergence and thus provide the fast tracking performance.

For the linear discrete time-invariant system (1) with multiple sensors $z(i)$ and sensor fault $f(i)$, two kinds of finite memory structure filter are defined using only the most recent finite sensor observations on the window $[i-M, i]$. The first one is the reference filter which utilizes observations of all sensor. The second one is the residual generation filter which reflects observations of a part of sensors. The estimation difference between the reference filter and the residual generation filter is used for generating residual.

The reference filter $\hat{x}(i)$ is given by (3) and processes the most recent finite observations $z(i)$ for all sensors. That is, the reference filter $\hat{x}(i)$ must be affected by the fault $f(i)$ of (2). On the other hand, the residual generation filter $\hat{x}_p(i)$ is obtained from:

$$\hat{x}_p(i) = H_p Z_p(i) - H\Lambda U(i),$$

when $1 \le p \le m$. H_p and $Z_p(i)$ can be obtained from H and $z(i)$ for the pth residual generation filter. In $Z_p(i)$, the pth observation part for the pth sensor is excluded from $z(i)$ as follows:

$$Z_p(i) \equiv \left[\bar{z}_p^T(i-M) \quad \bar{z}_p^T(i-M+1) \quad \cdots \quad \bar{z}_p^T(i-1)\right]^T.$$

H_p is the filter gain for the pth residual generation filter as follows:

$$H_p \equiv \left[\left(\Gamma_p^T \Pi_p^{-1} \Gamma_p\right)^{-1} \Gamma_p^T \Pi_p^{-1}\right]$$

where Γ_p and Π_p are obtained from Γ and Π using the matrix C of (2) without the pth row. Thus, if a fault occurs in one of sensors, the specific residual generation filter is not affected by a sensor fault $f(i)$ of (2).

B. *Residual Generation*

The reference filter $\hat{x}(i)$ and pth residual generation filter $\hat{x}_p(i)$ give their estimation errors $\tilde{x}(i)$, $\tilde{x}_p(i)$ and error covariances $\Sigma(i)$, $\Sigma_p(i)$ as follows:

$$\begin{aligned} \tilde{x}(i) &= x(i) - \hat{x}(i), \\ \Sigma(i) &= E\left[x(i) - \hat{x}(i)\right]\left[x(i) - \hat{x}(i)\right]^T \\ &= \left(\Gamma^T \Pi^{-1} \Gamma\right)^{-1} \equiv \Omega(M) \\ \tilde{x}_p(i) &= x(i) - \hat{x}_p(i), \\ \Sigma_p(i) &= E\left[x(i) - \hat{x}_p(i)\right]\left[x(i) - \hat{x}_p(i)\right]^T \\ &= \left(\Gamma_p^T \Pi_p^{-1} \Gamma_p\right)^{-1} \equiv \Omega_p(M) \end{aligned} \qquad (5)$$

A residual for the fault detection is defined by the following estimation disagreement:

$$\begin{aligned}\gamma_p(i) &= \hat{x}_p(i) - \hat{x}(i) \\ &= [x(i) - \hat{x}(i)] - [x(i) - \hat{x}_p(i)] \\ &= \tilde{x}(i) - \tilde{x}_p(i).\end{aligned} \tag{6}$$

Since reference and residual generation filters are linear, two kinds of estimates are unbiased for the fault-free system. Hence, residual $\gamma_p(i)$ becomes:

$$E[\gamma_p(i)] = E[\tilde{x}(i)] - E[\tilde{x}_p(i)] = 0.$$

The covariance $\Gamma_p(i)$ of residual $\gamma_p(i)$ is represented by:

$$\begin{aligned}\Gamma_p(i) &= E[\tilde{x}(i) - \tilde{x}_p(i)][\tilde{x}(i) - \tilde{x}_p(i)]^T \\ &= \Omega(M) - E\{\tilde{x}(i)[\tilde{x}_p(i)]^T\} \\ &\quad - E\{\tilde{x}_p(i)[\tilde{x}(i)]^T\} + \Omega_p(M).\end{aligned} \tag{7}$$

where:

$$\begin{aligned}&E\{\tilde{x}(i)[\tilde{x}_p(i)]^T\} \\ &= E[\tilde{x}(i)][\{x(i) - \hat{x}(i)\} + \{\hat{x}(i) - \hat{x}_p(i)\}]^T \\ &= \Omega(M) + E\{\tilde{x}(i)[\hat{x}(i) - \hat{x}_p(i)]^T\}\end{aligned} \tag{8}$$

So, the residual $\gamma_p(i)$ is in the zero-mean Gaussian distribution for the fault-free because the residual is the linear combination of $\tilde{x}(i)$ and $\tilde{x}_p(i)$.

$\Gamma_p(i)$ in (7) requires the cross-covariance of two residuals because they are correlated for the system noise $w(i)$. Actually, the system noise is a common input to $\tilde{x}(i)$ and $\tilde{x}_p(i)$. These cross-covariances can be shown to be same as the estimation error covariance of the reference filter filter. Because the finite memory structure filter is satisfying the orthogonality:

$$\begin{aligned}&E[x(i) - \hat{x}_p(i)]\alpha^T(\xi) \text{ for all} \\ &\alpha(\xi) \in \bar{z}_p(\xi),\ i - M < \xi < i.\end{aligned} \tag{9}$$

Thus, $E\{\tilde{x}(i)[\tilde{x}_p(i)]^T\}$ in (7) satisfies the orthogonality because:

$$x(\xi) - \hat{x}_p(\xi) \in z(\xi). \tag{10}$$

As mentioned before, both the reference filter and residual generation filter utilize the most recent finite sensor observations, thus $E\{\tilde{x}(i)[\hat{x}(i) - \hat{x}_p(i)]^T\}$ in (8) is zero with applying (10) to all time. $E\{\tilde{x}_p(i)[\tilde{x}(i)]^T\}$ in (9) is also same result, so the following is satisfied:

$$E\{\tilde{x}(i)[\tilde{x}_p(i)]^T\} = E\{\tilde{x}_p(i)[\tilde{x}(i)]^T\} = \Omega(M) \tag{11}$$

And then, $\Gamma_p(i)$ can be represented by estimation error covariances (5) of the reference filter and residual generation filter as follows:

$$\Gamma_p(i) = \Omega_p(M) - \Omega(M).$$

Hence, the covariance $\Gamma_p(i)$ of residual $\gamma_p(i)$ can be obtained from the off-line computation because $\Omega(M)$ and $\Omega_p(M)$ in (5) require computation only on the interval $[0, M]$ once and are time-invariant for all windows.

C. Test Variables and Detection Rule

A test variable is formulated based on the residual $\gamma_p(i)$ of (6) and its covariance. Each component of $\gamma_p(i)$ is considered to enhance performance of a fault detection. The qth component of residual $\gamma_p(i)$ is defined by $\gamma_p^q(i)$, where $1 \le q \le m$. Using this residual component, the test variable $t_p(i)$ for the pth sensor can be defined as follows:

$$t_p(i) = \frac{[\gamma_p^q(i)]^2}{\sigma_p} \tag{12}$$

where σ_p is the covariance of $\gamma_p^q(i)|\{f(i) = 0\}$ as:

$$\begin{aligned}\sigma_p &= E\{[\gamma_p^q(i)]^2 \,|\, f(i) = 0\} \\ &= [\Omega_p(M)]_q - [\Omega(M)]_q\end{aligned}$$

where $[\Omega(M)]_q$ and $[\Omega_p(M)]_q$ are the qth diagonal component $\Omega(M)$ and $\Omega_p(M)$ of (5), respectively.

If there is a fault in the sensor, the corresponding test variable $t_p(i)$ might be highly affected and declare a fault first. To compare with the test variable, a threshold value is required. The threshold value is set relatively to the sensitivity of residuals to the sensor fault. That is, too low threshold value causes excessive false alarm rate, on the other hand, too high one brings about insensitive fault detection. Because the test variable (11) forms a chi-squared distribution, a threshold value can be obtained from the chi-squared distribution function under the consideration of rational probability false alarm (PFA). The relationship between the threshold value and the PFA is represented as one degree of freedom chi-squared distribution function. The pth sensor fault can be detected for the chosen threshold value using the following detection rule: faulty if

$t_p(i) \geq threshold\ value$ and non-faulty if $t_p(i) < threshold\ value.$

D. Numerical Simulations

The promptness of a fault detection must be considered as one of the important performance criteria. Therefore, numerical simulations are performed for the aircraft engine system in [7,16] in order to show the adjustment of detection latency. In the aircraft engine system, it is important to be able to detect and locate the faulty sensor because the sensor fault can cause rapid instability and loss of control. The aircraft engine system in [7,16] is modeled by the following 3rd order discrete-time system:

$$A = \begin{bmatrix} 0.9305 & 0 & 0.1107 \\ 0.0077 & 0.9802 & -0.0173 \\ 0.0142 & 0 & 0.8953 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, G = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

In numerical simulations, an unknown sensor fault is assumed to be ramp-up from $i = 100$ to $i = 250$, and then step, and ramp-down from $i = 450$ to $i = 600.$ Noise covariances are set by $Q = 0.01^2 I$ and $R = 0.02^2 I$, respectively. The threshold value is set by 7.88 to analyze results for high threshold. Table 1 shows detection latency for each simulation run according to diverse window lengths. The simulation result with $M = 10$ is superior to other cases, which means that the proposed algorithm can have smaller detection latency as M decreases and thus improve the fast detection performance. However, the estimation performance in this case can be unsatisfactory. Thus, if both fast detection ability and noise-suppressing estimation ability are considered simultaneously, the windows length $M = 30$ or $M = 50$ can be better. It is shown from numerical simulation results that detection latency can be adjusted via the window length.

Table 1
Simulation results

Window length	Mean of test variable	Detection latency (Samples)
M = 10	11.4386	170
M = 30	8.2703	180
M = 50	5.7246	200
M = 70	4.1571	230

III. CONCLUSION

A fault detection algorithm has been proposed using multiple residual generation filters for discrete-time systems with multiple sensors. Residual are generated from the estimation error between the reference filter and multiple residual generation filters. These filters utilize only the most recent finite sensor observation. The reference filter gives optimal state estimates based on all sensors. One the other hand, one of multiple residual generation filters can give the sub-optimal state estimates which can be independent of faulty sensor. Then, the fault detection rule has been developed to indicate presence of fault by checking the agreement of multiple residuals. Multiple test variables for the detection rule have been defined using the chi-squared distribution with one degree of freedom. The proposed algorithm has been verified through extensive numerical simulations for the aircraft engine system.

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