Numerical Optimization of PID-Regulator for Object with Distributed Parameters

Vadim Zhmud, Alexander Liapidevskii, Boris Pyakillya
Novosibirsk State Technical University, Novosibirsk, Russia;
Novosibirsk Institute of Program Systems, Novosibirsk, Russia &
National Research Tomsk Polytechnic University, Tomsk, Russia.
oao_nips@bk.ry

***Abstract*—The control of dynamic objects is very important for technologies and technical sciences. The difficulty of the control depends on the complexity of object mathematical model. Objects with distributed parameters are most difficult for control with feedback loop. The mathematical model of such object is much more complicated. There are two method of calculating regulator. The first method is analytical calculation of the coefficients. It can be used only for relatively simple object. The second method is numerical optimization. It can be used even with complex object. But object with distributed parameters has such complex model that in this case the use of the method of numerical optimization is a difficult problem. Only approximate model of such object can be used for the optimization with the help of the simulation program. Newest results in numerical optimization are based on some specific cost functions. Various composite cost functions are effective tools for regulator calculation. But even these tools are not sufficient. The paper researches possibilities of the use of approximate mathematical model of objects with distributed parameters and various cost functions, including new ideas, for the calculation of the regulator for system with such object. With all known methods, it was impossible to reduce the overshooting less than 22%. The proposed new cost function allows reducing of the overshoot to a value of about 11%, which is be preferred for many applications. The proposed method adds an arsenal of techniques of control of complex dynamic objects.**

***Index Terms*—Feedback Control; Objects with Delay Link; Object with Distributed Parameters; Regulator Numerical Optimization; Suppress of Overshooting.**

I. INTRODUCTION

The precise control of objects can be done only in the loop with negative feedback. It is successfully used with objects having simple mathematical models. Controllers for such systems, also called “regulators”, can be calculated analytically. If the object model is complex, analytical methods does not work.

Traditionally, the mathematical model of control object is a transfer function, that is, the ratio of the Laplace transforms of the signal at the output to this at its input.

For example, it can be as following:

$$W_1(s) = \frac{M(s)}{N(s)}\exp(-\tau s) = = \frac{a_m s^m + a_{m-1}s^{m-1} ... + a_0}{b_n s^n + b_{n-1}s^{n-1} ... + b_0}\exp(-\tau s) \quad (1)$$

Here $M(s)$ and $N(s)$ are polynomials, m and n – are integer values, $m < n$, a_i and b_j are coefficients of the regulator model, τ – is time constant of delay link, s – is Laplace transform argument. Such object can be easily simulated in program *VisSim* 6.0, for example. Typical frequency characteristic consists of parts with approximately constant integer slope, divisible by 20 *dB/dec*, as Figure 1 shows.

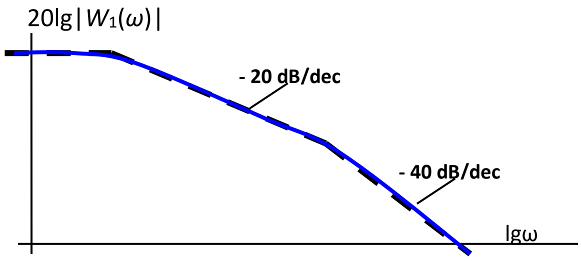

Figure 1: Typical frequency characteristic of object according (1)

Objects with distributed parameters has model with fractional exponents in (1). The model of such object has the following form:

$$W_2(s) = \frac{a_m s^{q_m} + a_{m-1}s^{q_{m-1}} ... + a_0}{b_n s^{r_n} + b_{n-1}s^{r_{n-1}} ... + b_0}\exp\{-\tau s\} \quad (2)$$

Here q_i, r_j are fractional values. Typical frequency characteristic of such object is shown at *Fig.* 2. The characteristic can deviate from the integer slope. The slope is not always divisible by 20 *dB/dec*.

The calculating of regulator for such object meets two difficulties:

1. It is difficult to use simple software for simulation of working of such object as (2).
2. The analytical method is difficult to use because of delay link in (2).

The use of complex software as *MATLAB* is not advisable because it can give not reliable results because it uses too ideal model of regulator, for example pseudo-delay link with positive coefficient, or infinite frequency response model, and so on. Our earlier researches has demonstrated that with *MATLAB* results can be better in the theory, but with *VisSim* the results are more agree with the real object and regulator working [1–8].

Therefore, it is important to simulate system with object with distributed parameters with the helps of more appropriate software as *VisSim* with the goal of calculation of the optimal regulator. The typical structure of the system is shown at Figure 3. In this structure $V(s)$ is setting, $E(s)$ is error, $U(s)$ is controlling signal, $X(s)$ is state of object, $H(s)$ is unknown disturbance, $Y(s)$ is object output value. Each Laplace function dependant on argument s corresponds to the real signal dependant on the time t, $Y(s) \Leftrightarrow y(t)$, and so on.

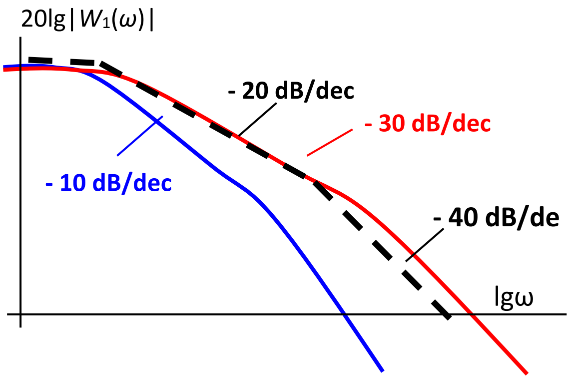

Figure 2: Possible view of frequency characteristic of object with distributed parameters

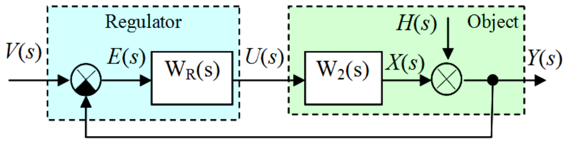

Figure 3: Typical structure of feedback control system

The goal of the controlling is y(t) ≈ v(t). The high static accuracy mean:

$$\lim_{t\to\infty} y(t) = v(t) \tag{4}$$

The most common model of regulator is the following:

$$W_R(s) = k_P + \frac{k_I}{s} + k_D s \tag{5}$$

Here k_P, k_I and k_D are coefficient of proportional, integrative and derivative links. Regulator according (5) is called PID-regulator, or simple PID.

It is necessary to calculate coefficients of the transient function (5), which would provide good static accuracy, according (4), and small overshooting, with the possibly small duration of the transient process.

II. The Choice of the Object Approximate Model

Transfer function of the form (2) can be approximately modeled by the function of the following form:

$$W_4(s) = \frac{(\tau_1 s+1)\ldots(\tau_m s+1)k}{(T_1 s+1)\ldots(T_n s+1)s} \cdot e^{-\tau s} \tag{6}$$

For example, it can approximate the following transient function:

$$W_5(s) = \frac{k}{\sqrt{(T_0 s+1)}} \cdot e^{-\tau s} \tag{7}$$

This function corresponds to the frequency response with the slope – 30 *dB/dec* (Figure 4).

Here, in the case of (7), the time constant of the numerator and denominator are in turn in size, for example,

$$T_1 > \tau_1 > T_2 > \tau_2 > T_3 > \tau_3 > \ldots > T_n > \tau_m \tag{8}$$

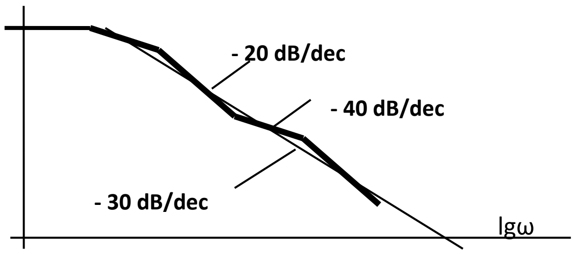

Figure 4: Typical frequency characteristic of object according (8)

Therefore, we can propose the mean for the approximate modelling of object with distributed parameters with the helps of software VisSim, because the form (6) is suitable for the modelling, whereas the form (7) is not suitable.

III. The Structure for the Numerical Optimization of the Regulator

For the regulator optimization it is necessary to use according structure in software *VisSim*. This structure must contain the full system model according Figure 3. In addition the structure must contain cost function estimator, optimization unit, oscilloscope and formers of start values of the regulator parameters and step unit jump of the setting value $V(t)$. Figure 5 presents the total structure for the said task.

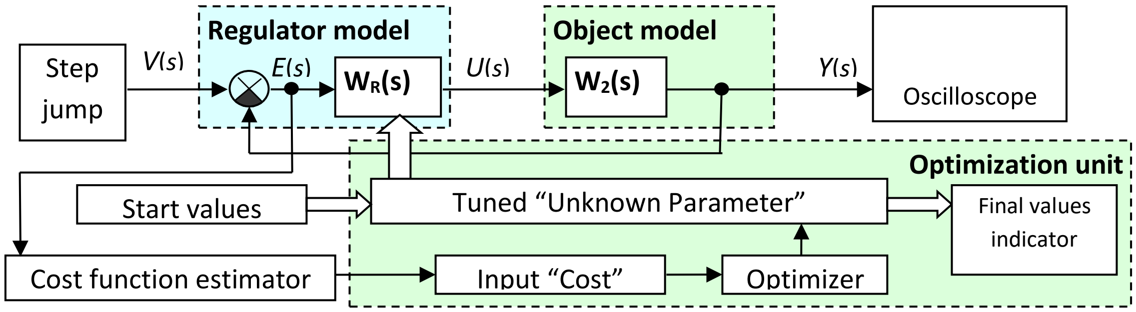

Figure 5: Typical structure of feedback control system

IV. Cost Function Tools

The most evident but not the best variant of the cost function is the following:

$$\Psi_1(T) = \int_0^{\Theta} e^2(t)dt \quad (8)$$

Here Θ is duration of the modeling transient response. More effective cost function is:

$$\Psi_2(T) = \int_0^{\Theta} |e(t)|\, dt \quad (9)$$

Nevertheless, function (9) is also not the best choice. The best cost function can be calculated by the following general relationship:

$$\Psi(\Theta) = \int_{t=0}^{\Theta} w_q \sum_{q=1}^{Q} \psi_q dt \quad (10)$$

Here the cost function is defined as the time integral of positively defined functions ψ_q, from the beginning of the transient process $t = 0$ to the end of it, when $t = \Theta$. If the number of terms ψ_q in function (10) is more than one, it is necessary to use the weighting coefficients, which allow establish the significance of the contribution of each of these functions.

One of the effective functions ψ_q for (7) is a module error $e(t)$, multiplied by the time t from the start of the transient process [1]:

$$\psi_1(t) = |e(t)|\, t \quad (11)$$

This reduces the module of the error. Also it gives more intensive reduction of this value with the development of the process, because the factor t plays the role of weight that is continuously increases linearly.

Weakness of the cost function based only on the term (11) in the equation (10) is that oscillations often exist in the transient response on the resulting system.

One effective way to suppress oscillations in the transient processes is the use of detector of error growth [2]:

$$\psi_2(t) = \max\{0, e(t)\frac{de(t)}{dt}\} \quad (12)$$

Here function max{0, f} is limiter:

$$\max\{0, f\} = \begin{cases} 0, if \;\; f < 0 \\ f, if \;\; f \geq 0 \end{cases} \quad (13)$$

The product of the error and its derivative must be negative for the best process. In this case, i.e. if the error and its derivative have different signs, the module of error is reducing during the process. The function (12) is zero, and its contribution to the cost function (10) is zero too. This situation corresponds to the desired development of the process.

If the error and its derivative have the same sign, the error in magnitude in this area increases, product of error and its derivative is positive, the function (12) is positive too. Then the cost function (10) with (12) increases due to the integration of the positive function (12). The optimization procedure will seek to find such values of regulator coefficients that will minimize (10). Therefore, the procedure minimizes the parts of transient process, in which (12) is small.

This term (12) does not provide a complete absence of areas of the transient process in which the error is increasing, but it makes the presence of such parts minimal.

In the Equation (8) the according term is:

$$\psi_3(t) = [e(t)]^2 \quad (14)$$

It works not effectively. At the beginning of any process error is big. If the setting jump is $v(0) = 1$, then the initial error is $e(0) = 1$. During the rest part of the process module of the error is much less than 1, hence, the term (14) has initial component which is cannot be reduced. In order to cut off the initial error from the cost function, limiter can be used. It can cut off the positive part of the error and leave only its negative part, namely:

$$\psi_4(t) = [\min\{0, e(t)\}]^2 \quad (15)$$

Some other terms can be used in (10) with different effectiveness in the different cases.

V. The Results of the Regulator Calculation

For example, the object model (7) can be approximated with the following function:

$$W_4(s) = \frac{\prod_{i=1}^{4}(\tau_i s + 1)}{s\prod_{j=1}^{4}(T_j s + 1)} \cdot e^{-s} \quad (16)$$

Here the parameters are $\tau_1 = 100$, $\tau_2 = 10$, $\tau_3 = 1$, $\tau_4 = 0.1$, $T_1 = 300$, $T_2 = 30$, $T_3 = 3$, $T_4 = 0.3$.

Figure 6 shows the structure of the project for the modelling and optimization. This structure is made in the program *VisSim* 6.0. It contains compound blocks, named “PI-Regulator”, “Object”, “Optimizer” and “Cost estimator”.

Figure 7 shows the structure of object model according (16). Figure 8 shows the model of the structure of PID-regulator according (5). The structure of the block for optimization should contain such quantity of blocks “*parameterUnkmnown*”, as there are variables, which must be optimized in the result of the optimization procedure. The form of this model is shown in Figure 9, where the said blocks are connected to the generators of the starting values for optimization. These values are $k_p = 1$, $k_i = 0$ and $k_d = 0$, accordingly.

Also block “*Cost*” must present in the model. It analyses the cost function for calculating new values of optimized parameters. In addition, the structure for the calculation of the actual cost function is necessary. Its output is the result of the calculation; it should be connected to the entrance of the block “*Cost*”. Figure 10 shows block for calculation of the cost function on the base of (10) containing (11) and (12).

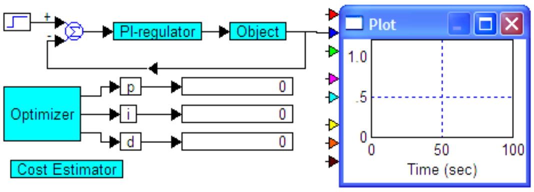

Figure 6: The model of the system in VisSim

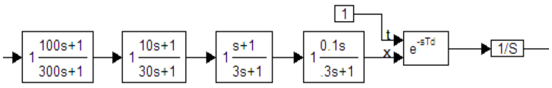

Figure 7: The objects model in VisSim

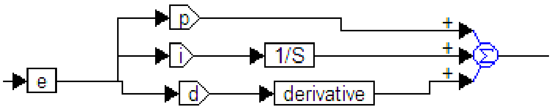

Figure 8: The model of regulator in VisSim

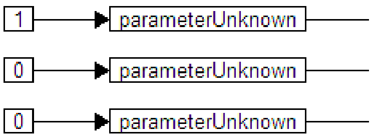

Figure 9: The model of block “Optimizer” in VisSim

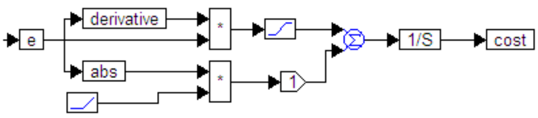

Figure 10: The model of block “Cost Estimator” in VisSim

Software optimizes coefficients of (5) with the use of the structure of Figure 6 containing compound blocks shown at Figure 7–10. Red line shows this response with the use of (10) and (11), blue line corresponds the result with the addition of (12).

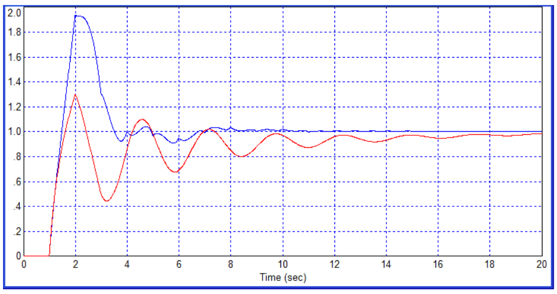

Figure 11: Type transient processes obtained with different cost function: red line is using only (10) and (11); blue line is when using these and (12)

When cost estimator uses only Equation (10) with the only term (11), the resulting system has transient process with many oscillations and the initial overshoot reaches 30% (red line in Figure 11). The result of joint application of functions (11) and (12) in the cost function (10) gives the system, which transient process has overshooting about 90 % with few oscillations (blue line in Figure 11).

Hence, in this case the introduction of the term (12) into (10) is not good tool. The process shown with blue line can be more attractive only in very specific case, if the shortening of the process duration is much more important then minimizing of the overshooting. In the most cases overshooting not more then 10–15 % can be acceptable.

It is necessary to reduce the value of overshooting without the growth of the number of the oscillations.

The introducing of (14) into (10) gives the system with the transient process, shown at Figure 12.

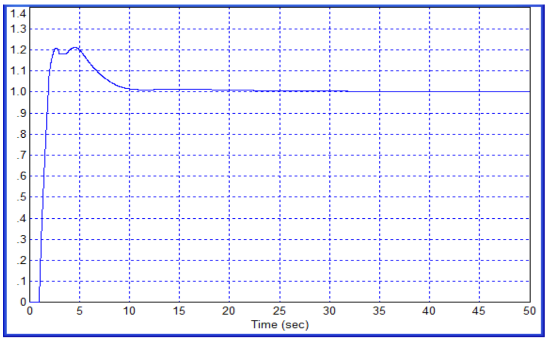

Figure 12; The transient process in the system obtained in the result of the optimization with the use of (10) with (11), (12) and (14)

The introducing of the term (14) has allowed reducing the overshoot in the transient process to the value about 20%. This process is better than any process shown in Figure 11. Nevertheless, the further decreasing of overshooting is desirable.

VI. The Use of the New Cost Function

A new method for the further decreasing of the overshooting has been proposed and tested.

This method is based on the use of the reference transient process, which is chosen as the desirable function $q(t)$. It should start in the same time, when all the processes begin on the previous graphs, since it is impossible to overcome the delay of the object with delay time $\tau = 1$ *s*.

Further, it is desirable that the shape of the graph was similar to the exponential dependence. The time constant of the exponential must be chosen depending on the speed of the system. The experimentally found time constant is 10 *s*. Hence, the transfer function of ideal system is:

$$W_Q(s) = \frac{\exp(-s)}{10s+1} \tag{17}$$

It corresponds to the ideal output signal:

$$q(t) = \begin{cases} 0, \; if \; t < 1\,s \\ 1 - \exp(t-1), \; if \; t \geq 1\,s \end{cases} \tag{18}$$

Cost estimator in this case must contain former of signal (18) and subtractor of the error from this signal. The difference can be squared and integrated to produce the following additional term to (10):

$$\psi_5(t) = [e(t) - q(t)]^2 \tag{19}$$

Figure 13 shows the corrected structure, which contains former of signal q(t) according (17). Figure 14 shows the structure for calculating of the cost function (10) including terms (11), (12), (14) and (19) with different weighting factors.

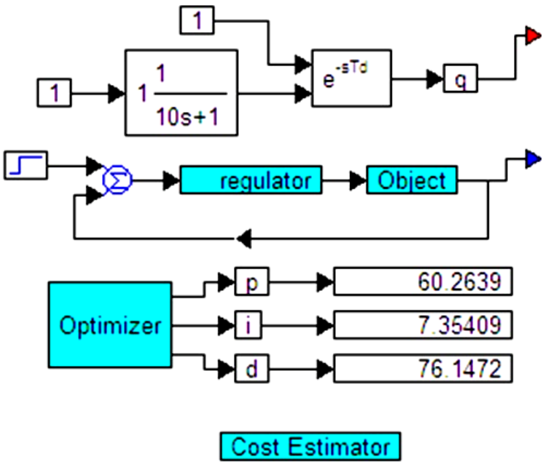

Figure 13: Model structure for optimization and the result of its use in the form of the resulting coefficients for (5)

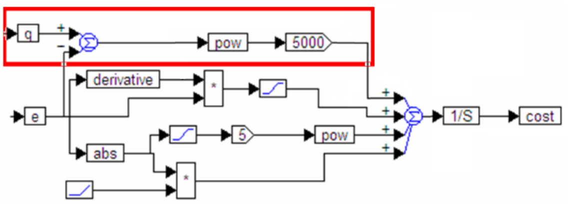

Figure 14: Model structure for the cost function (10) with (11), (12), (14) and (19) components as terms under the integral

The weigh factors has been chosen empirically on the base of analysis of obtained transient processes. Figure 14 also shows the result of optimization in the form of three coefficients obtained for the PID-regulator, namely: $k_p = 60.26$; $k_i = 7.35$ and $k_p = 60.26$; $k_i = 7.35$. Figure 15 shows the resulting transient processes as blue line. For comparison, the red line shows a process corresponding to an ideal error process (18).

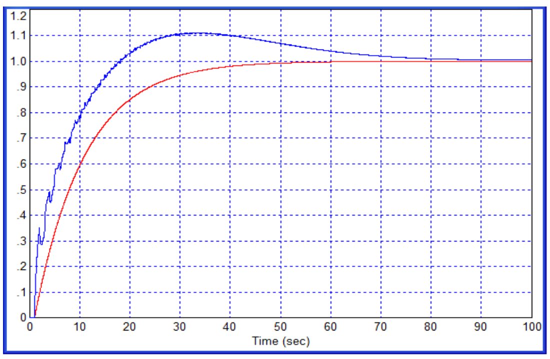

Figure 15: The view of the reference function (red line) and the resulting transient process (blue line) in the system because of optimization with the helps of the reference function

The resulting transient processes in comparison with the process in Figure 12 have both advantages and disadvantages. The obvious advantage of this process is the smallest value of the overshoot, namely 11% versus 22%. Another advantage is the relatively smooth form, except for small variations in the initial stage, when it is not reached half the prescribed value. If large overshooting is contraindicated for the system, this process should be preferred in comparison with the previous ones, and to further reduce overshooting, the proposed way may be the most effective.

VII. Conclusion

Based on the investigations, the following conclusion can be made.

Numerical optimization is an effective tool for the regulator calculation even if the object model is not suitable for the simulation of its action in the necessary software. The model given by (7) as well as any model with fractional power of parameter s can be approximately simulated with the model similar to (16).

It is shown that the object with distributed parameters can be quite complicated for regulator optimization; known methods with known cost function for numerical optimization did not allow effectively solve the problem of regulator design for a system with such an object. The additional modification of the cost function was necessary.

The cost function of the form (10) can be successfully updated with new term, such as (19). This term is useful to reduce the overshooting in the system. This paper first proposed, justified and used the term of the form (19) in the cost function of the form (10); results of simulation and optimization have shown the usefulness and effectiveness of this term to reduce overshoot and to ensure the overall smoothness of the transient process.

Acknowledgement

The work was supported by the Russian Foundation for Basic Researches (RFBR) [4], project № 15-38-50594 “Development and research of algorithms of identification of control objects with lumped and distributed parameters, in order to build adaptive control systems and regulation”.

References

[1] Zhmud V. A. Modelirovanie, Issledovanie I Optimizacija Zamknutyh Sistem Avtomaticheskogo Upravlenija. *Monografija*. Novosibirsk, Izd-vo NGTU, 2012. – 335 s.

[2] Zhmud V. A. Modelirovanie, Issledovanie I Optimizacija Zamknutyh Sistem Avtomaticheskogo Upravlenija. *Monografija*. Novosibirsk, Izd-vo NGTU, 2012. – 335 s.

[3] Zhmud V.A., Zavorin A.N. Struktura Modeli Dlja Optimizacii Sistemy S Obratnoj Svjaz'ju. Patent RF RU 2554291 C1. Opubl. 27.06.2015. Bjull. № 18.

[4] V.A. Zhmud, L.V. Dimitrov, O.D. Zavorin. Calculation of Regulators for The Problems Of Mechatronics By Means Of Numerical Optimization Method. 2014. 12-th *International Conference on Actual Problems of Electronic Instruments Engineering (APEIE)* 34006. Proceedings. Novosibirsk. 2014. Vol.1. P. 739 – 744.

[5] Vadim A. Zhmud, Dashi B. Imekov, Kirill Yu. Lastochkin and Anna V. Lukicheva. New Structure of Regulator for Controlling of Object with Oscillation Features and its Modelling Testing. *Proceedings of International Conference on Engineering and Applied Sciences Optimization (OPT-i-2014).* Kos Island, Greece, 4-6 June 2014.

[6] Vadim Zhmud. New Modeling Tested Method for Controlling of Object with Oscillation Features. *Proceeding of 2014 International Conference on Mechanical Design, Manufacture and Automation Engineering (MDMAE 2014).* ISBN: 978-1-60595-156-0. Thailand, Phuket. Part II. .P.289 – 296.

[7] Zhmud V.A., Zavorin A.N. Metodi di ottimizzazione del controllo numerico su una modelli troncati. Italian Science Review. 2014; 4(13). PP. 686-689. Available at URL: http://www.ias-journal.org/archive/2014/april/Zhmud.pdf and URL: http://www.ias-journal.org/archives/april-2014

[8] V. Zhmud, A. Zavorin. Fractional order PID-regulators and method of their simplification with the increase of the controlling effect. Proceedings Of The 8-Th International Forum On Strategic Technology 2013 (IFOST-2013), vol. II, 28 June – 1 July. Mongolian

University of Science and Technology, Ulaanbaator, Mongolia. IEEE organized. 2013. P. 250 – 255. http://www.must.edu.mn/IFOST2013/

[9] V. Zhmud, O. Yadrishnikov. Numerical optimization of PID-regulators using the improper moving detector in cost function. Proceedings Of The 8-Th International Forum On Strategic Technology 2013 (IFOST-2013), vol. II, 28 June – 1 July. Mongolian University of Science and Technology, Ulaanbaator, Mongolia. IEEE organized. 2013. P. 265 – 270. http://www.must.edu.mn/IFOST2013/

[10] Russian Foundation for Basic Research, URL; http://www.rfbr.ru/rffi/eng.