Backstepping and Sliding-Mode Methods for Stabilizing an Underactuated X4-AUV

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***Abstract*—In this paper, we are interested principally in dynamic modelling of an autonomous underwater vehicle (X4-AUV) while taking into account the high order nonholonomic constraints in order to develop a new control scheme as well as the various physical phenomena, which can influence the dynamic of a swimming structure. We deal with the design of two controllers, based on backstepping and sliding-mode control techniques to stabilize altitude and attitude of an underactuated X4-AUV. The designed controllers are: full backstepping control for attitude and altitude control, and partially sliding-mode control for attitude combine with altitude backstepping control. Some numerical simulations are conducted to demonstrate the effectiveness of the proposed controllers.**

***Index Terms*—Underactuated System; X4-AUV; Backstepping Control, Sliding-Mode Control.**

I. INTRODUCTION

Systems having less number of control inputs than the degrees of freedom available are known as underactuated systems. Underactuated systems exist in a wide range of real time applications such as aerospace [1], robotics [2][3], underwater vehicles [4][5] and flexible systems [6]. A system can become underactuated due to its inherent dynamics [7], induced by the actual design method [8], malfunctioning of actuators, artificially induced for experimentation and research purpose or the mathematical model used for the design process [9]. As underactuated systems require few number of actuators, their cost and complexity are low and consume less energy. Research involving study, analysis and control of underactuated systems has been continuing since long [10–15]. For fully actuated systems, a good number of control techniques are available which can be applied to the entire class of the system. However, for underactuated systems, except for only a few methods [15], the control techniques vary from system to system and cannot be applied in general to an entire class of the system. Also most of the underactuated systems are nonholonomic due to the presence of non-integrable differential constraints. Hence controlling an underactuated system is a challenging problem.

The autonomous underwater vehicle (X4-AUV) is an example of a nonlinear and unstable underactuated system. The X4-AUV with an ellipsoid hull shape was studied by Zain [16], in which it makes only use of four thrusters to control the vehicle without using any steering rudders. This vehicle falls into the class of underactuated AUVs since it has 6-DOFs (position (x, y, z), pitch, roll and yaw).

Various researches about the control technique of underactuated systems have been achieved up to now. Among them, it is very often to use canonical models such as a chained form, a power form, a double integrator model, etc. Astolfi [17] made a canonical model discontinuous, and then he proposed the technique of performing continuous feedback control. Khennouf et al. [18] carried out well use of the structure of a chained form, and proposed the switching control that performs two steps of control by an invariant manifold. Furthermore, Khennouf et al. [19] also proposed the technique called quasi-continuous exponential stabilization control. As an approach for robust control, sliding mode control has been applied to the trajectory control of robot manipulators [20]. The advantages of using sliding mode control include fast response, good transient performance and robustness with regard to parameter variations.

The combination of sliding mode control and the backstepping procedure is an attractive approach for developing robust controllers for nonlinear systems. The most common approach is only to use sliding mode control in the last step of the backstepping [21]. However, a new robust control technique which uses backstepping to design virtual sliding mode controllers at each recursive step has been developed by Zinober and Liu [22] for a class of SISO systems only with unstructured uncertainty. For MIMO system, Bouabdallah and Siegwart [23] combined backstepping and sliding-mode control to stabilize a quadrotor system.

Note however that among them major research in underactuated area is for controlled object with two-inputs and therefore there is restricted research for controlled object with three or more inputs. One of causes is that there is no definite method of transforming the original model into a canonical model to the case of the controlled system with three or more inputs.

In this paper, we will present backstepping and combined backstepping and sliding mode control approach for stabilizing an X4-AUV, a class of MIMO nonlinear systems. The controllers are devised to stabilize the X4-AUV, and simulation results demonstrate their effectiveness.

Chapters are organized as follows. In section 2, the coordinate system of an AUV is presented. The dynamic system of an X4-AUV is discussed in Section 3. Sections 4 and 5, we present the control strategy to stabilize the X4-AUV with the simulation results for backstepping controller and combined backstepping and sliding-mode controller. Section 6 concludes the paper.

II. DEFINITION OF COORDINATE SYSTEM

In order to describe the underwater vehicle's motion, a special reference frame must be established. There have two coordinate systems: i.e., inertial coordinate system (or fixed coordinate system) and motion coordinate system (or body-fixed coordinate system). The coordinate frame $\{E\}$ is composed of the orthogonal axes $\{E_x\ E_y\ E_z\}$ and is called as an inertial frame. This frame is commonly placed at a fixed place on Earth. The axes E_x and E_y form a horizontal plane and E_z has the direction of the gravity field. The body fixed frame $\{B\}$ is composed of the orthonormal axes $\{X, Y, Z\}$ and attached to the vehicle. The body axes, two of which coincide with principle axes of inertia of the vehicles, are defined in Fossen [13] as follows:

X is the longitudinal axis (directed from aft to fore)
Y is the transverse axis (directed to starboard)
Z is the normal axis (directed from top to bottom)

Figure 1 shows the coordinate systems of AUV, which consist of a right-hand inertial frame $\{E\}$ in which the downward vertical direction is to be positive and right-hand body frame $\{B\}$.

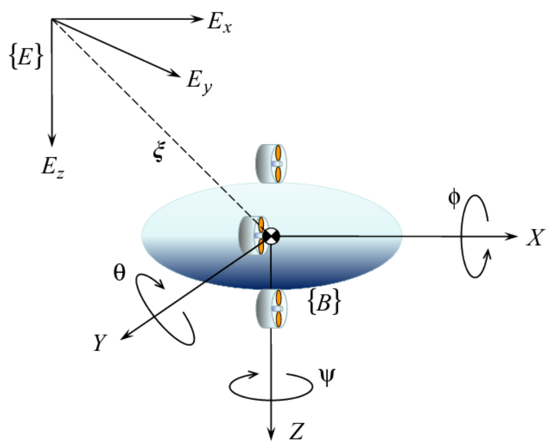


Figure 1: Coordinate systems of AUV

Letting $\boldsymbol{\xi} = [x \quad y \quad z]^T$ denote the mass center of the body in the inertial frame, defining the rotational angles of X-, Y- and Z-axis as $\boldsymbol{\eta} = [\phi \quad \theta \quad \psi]^T$, the rotational matrix R from the body frame {B} to the inertial frame {E} can be reduced to:

$$R = \begin{bmatrix} c\theta c\psi & s\phi s\theta c\psi - c\phi s\psi & c\phi s\theta c\psi + s\phi s\psi \\ c\theta s\psi & s\phi s\theta s\psi + c\phi c\psi & c\phi s\theta s\psi - s\phi c\psi \\ -s\theta & s\phi c\theta & c\phi c\theta \end{bmatrix} \tag{1}$$

where $c\alpha$ denotes cos α and $s\alpha$ is sin α.

III. SYSTEM DESCRIPTION

Defining $\boldsymbol{q} = [\boldsymbol{\xi}^T\ \boldsymbol{\eta}^T]$, the dynamical model of an X4-AUV is described in the following matrix form:

$$M(\boldsymbol{q})\ddot{\boldsymbol{q}} + V_m(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + G(\boldsymbol{q}) = B(\boldsymbol{q})\boldsymbol{\tau} \tag{2}$$

where $M(\boldsymbol{q}) \in \Re^{6\times6}$ is the symmetric, positive definite inertia matrix, $V_m(\boldsymbol{q}, \dot{\boldsymbol{q}}) \in \Re^{6\times6}$ is the centrifugal and Coriolis matrix, $G(\boldsymbol{q}) \in \Re^{6}$ is the gravitational vector, $B(\boldsymbol{q}) \in \Re^{6\times4}$ is the input transformation matrix, and $\boldsymbol{\tau} \in \Re^{4}$ is a generalized force vector consisting of force or torque components.

Note also that each matrix in the dynamical model can be reduced to:

$$M(\boldsymbol{q}) = \begin{bmatrix} m_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & m_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_x & 0 & 0 \\ 0 & 0 & 0 & 0 & I_y & 0 \\ 0 & 0 & 0 & 0 & 0 & I_z \end{bmatrix}$$

$$B(\boldsymbol{q}) = \begin{bmatrix} c\theta c\psi & 0 & 0 & 0 \\ c\theta s\psi & 0 & 0 & 0 \\ -s\theta & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & l & 0 \\ 0 & 0 & 0 & l \end{bmatrix}$$

$$V_m(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I_z\dot{\psi} & -I_y\dot{\theta} \\ 0 & 0 & 0 & -I_z\dot{\psi} & 0 & J_t\Omega + I_x\dot{\phi} \\ 0 & 0 & 0 & I_y\dot{\theta} & -J_t\Omega - I_x\dot{\phi} & 0 \end{bmatrix}$$

$$\mathrm{diag}(m_1, m_2, m_3) = m_b I + M_f$$
$$\mathrm{diag}(I_x, I_y, I_z) = J_b + J_f$$

Here, m_1, m_2 and m_3 is a total mass in the x-, y- and z-direction, I_x, I_y and I_z is a total inertia in the x-, y- and z-direction, J_t is a total thruster inertia, l is a horizontal distance from the propeller center to the center of gravity, m_b is a mass of the vehicle, J_b is an inertia matrix of the vehicle, I denotes the unit matrix, M_f is an added mass matrix, and J_f is an added moment of inertia matrix. Assuming that the fluid density is ρ and the present AUV form is ellipsoid, it is found that suitable M_f and J_f are obtained [24][25]. Furthermore assume that the X4-AUV is in the state of neutral buoyancy to neglect the potential energy, so that $G(\boldsymbol{q}) = 0$. From the rotational matrix (1), the kinematic equation for X4-AUV.

$$\dot{\boldsymbol{q}} = S(\boldsymbol{q})\boldsymbol{v} \tag{3}$$

can be reduced to:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} c\theta c\psi & 0 & 0 & 0 \\ c\theta s\psi & 0 & 0 & 0 \\ -s\theta & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x}_b \\ \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \tag{4}$$

because the lateral type X4-AUV has only the total thrust in the X-direction, where $v = \begin{bmatrix} \dot{x}_b & \dot{\phi} & \dot{\theta} & \dot{\psi} \end{bmatrix}^T$, where $\dot{x}_b$ denotes the X-directional translational velocity and $\begin{bmatrix} \dot{\phi} & \dot{\theta} & \dot{\psi} \end{bmatrix}^T$ is the rotational angular velocity vector in the body frame.

Therefore, the dynamic equations of motion for an X4-AUV in Equation (2) can be written as:

$$\begin{aligned} m_1\ddot{x} &= \cos\theta\cos\psi u_1 \\ m_2\ddot{y} &= \cos\theta\sin\psi u_1 \\ m_3\ddot{z} &= -\sin\theta u_1 \\ I_x\ddot{\phi} &= \dot{\theta}\dot{\psi}(I_y - I_z) + u_2 \\ I_y\ddot{\theta} &= \dot{\phi}\dot{\psi}(I_z - I_x) - J_t\dot{\psi}\Omega + lu_3 \\ I_z\ddot{\psi} &= \dot{\phi}\dot{\theta}(I_x - I_y) - J_t\dot{\theta}\Omega + lu_4 \end{aligned} \tag{5}$$

IV. BACKSTEPPING CONTROL OF AN X4-AUV

The model (5), can be rewritten in a state-space form $\dot{X} = f(X,U)$ by introducing $X = (x_1 \cdots x_{12})^T \in \Re^{12}$ as state vector of the system as follows:

$$\begin{aligned} &x_1 = x && x_7 = \phi \\ &x_2 = \dot{x}_1 = \dot{x} && x_8 = \dot{x}_7 = \dot{\phi} \\ &x_3 = y && x_9 = \theta \\ &x_4 = \dot{x}_3 = \dot{y} && x_{10} = \dot{x}_9 = \dot{\theta} \\ &x_5 = z && x_{11} = \psi \\ &x_6 = \dot{x}_5 = \dot{z} && x_{12} = \dot{x}_{11} = \dot{\psi} \end{aligned} \tag{6}$$

where the inputs $U = (u_1 \cdots u_2)^T \in \Re^4$.

From (5) and (6), we obtain:

$$f(X,U) = \begin{pmatrix} x_2 \\ (\cos\theta\ \cos\psi)\dfrac{1}{m_1}u_1 \\ x_4 \\ (\cos\theta\ \sin\psi)\dfrac{1}{m_2}u_1 \\ x_6 \\ (-\sin\theta)\dfrac{1}{m_3}u_1 \\ x_8 \\ x_{10}x_{12}\left(\dfrac{I_y - I_z}{I_x}\right) + \dfrac{l}{I_x}u_2 \\ x_{10} \\ x_8x_{12}\left(\dfrac{I_z - I_x}{I_y}\right) - \dfrac{J_t}{I_y}x_{12}\Omega + \dfrac{l}{I_y}u_3 \\ x_{12} \\ x_8x_{10}\left(\dfrac{I_x - I_y}{I_z}\right) + \dfrac{J_t}{I_z}x_{10}\Omega + \dfrac{l}{I_z}u_4 \end{pmatrix} \tag{7}$$

with:

$$\begin{aligned} &a_1 = (I_y - I_z)/I_x && b_1 = 1/I_x \\ &a_2 = (I_z - I_x)/I_y && b_2 = l/I_y \\ &a_3 = J_t/I_y && b_3 = l/I_z \\ &a_4 = J_t/I_z \\ &a_5 = (I_x - I_y)/I_z \\ &u_y = \cos x_9 \sin x_{11} \\ &u_z = \sin x_{11} \end{aligned}$$

It is worthwhile to note in the latter system that the angles and their time derivatives do not depend on translation components. On the other hand, the translations depend on the angles. We can ideally imagine the overall system described by (7) as constituted of two subsystems, the angular rotations and the linear translations, see Figure 2.

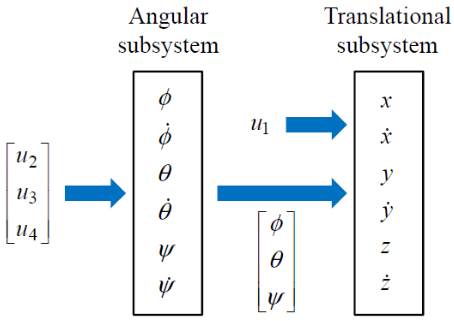


Figure 2: Connection of rotational and translational subsystems

A. *Backstepping Control of the Rotations Subsystem*

Using the backstepping approach, one can synthesize the control law forcing the system to follow the desired trajectory. For the first step, we consider the tracking-error:

$$z_1 = x_{7d} - x_7 \tag{8}$$

And we use the Lyapunov theorem by considering the Lyapunov function z_1 positive definite and it's time derivative negative semi-definite:

$$V(z_1) = \tfrac{1}{2}{z_1}^2 \tag{9}$$

$$\dot{V}(z_1) = z_1(\dot{x}_{7d} - x_8) \tag{10}$$

The stabilization of z_1 can be obtained by introducing a virtual control input x_8:

$$x_8 = \dot{x}_{7d} + \alpha_1 z_1 \quad \text{with } : \alpha_1 > 0 \tag{11}$$

The equation (6) is then:

$$\dot{V}(z_1) = -\alpha_1 {z_1}^2 \tag{12}$$

Let us proceed to a variable change by making:

$$z_2 = x_8 - \dot{x}_{7d} - \alpha_1 z_1 \tag{13}$$

For the second step we consider the augmented Lyapunov function:

$$V(z_1, z_2) = \tfrac{1}{2}{z_1}^2 + \tfrac{1}{2}{z_2}^2 \tag{14}$$

And it's time derivative is then:

$$\begin{aligned}\dot{V}(z_1, z_2) &= z_2(a_1 x_{10} x_{12} + b_1 u_2) \\ &- z_2(\ddot{x}_{7d} - \alpha_1(z_2 + \alpha_1 z_1)) - z_1 z_2 \\ &- \alpha_1 {z_1}^2\end{aligned} \tag{15}$$

The control input u_2 is then extracted $(\ddot{x}_{1,2,3d} = 0)$, satisfying $\dot{V}(z_1 z_2) < 0$:

$$u_2 = \frac{1}{b_1}(z_1 - a_1 x_{10} x_{12} - \alpha_1(z_2 + \alpha_1 z_1) - \alpha_2 z_2) \tag{16}$$

The term $\alpha_2 z_2$ with $\alpha_2 > 0$ is added to stabilize z_1. the same steps are followed to extract u_3 and u_4

$$\begin{aligned}u_3 &= \frac{1}{b_2}((z_3 - a_2 x_8 x_{12} - a_3 x_{12}\Omega) \\ &- \alpha_3(z_4 + \alpha_3 z_3) - \alpha_4 z_4)\end{aligned} \tag{17}$$

$$\begin{aligned}u_4 &= \frac{1}{b_3}((z_5 - a_5 x_8 x_{10} - a_4 x_{10}\Omega) \\ &- \alpha_5(z_6 + \alpha_5 z_5) - \alpha_6 z_6)\end{aligned} \tag{18}$$

with:

$$\begin{cases} z_3 = x_{9d} - x_9 \\ z_4 = x_{10} - \dot{x}_{9d} - \alpha_3 z_3 \\ z_5 = x_{11d} - x_{11} \\ z_6 = x_{12} - \dot{x}_{11d} - \alpha_5 z_5 \end{cases} \tag{19}$$

Note that this technique also used for a Quadrotor studied in [5].

B. Backstepping Control of the Linear Subsystem

The second part of the application is highlighting the regions, which have the same HSV value as the centre.

a. Altitude Control

The altitude control u_1 is obtained using the same approach described in the backstepping control of the rotational subsystem.

$$u_1 = \frac{m_1}{\cos x_9 \ \cos x_{11}}[z_7 - \alpha_7(z_8 + \alpha_7 z_7) - \alpha_8 z_8] \tag{20}$$

with:

$$\begin{cases} z_7 = x_{1d} - x_1 \\ z_8 = x_2 - \dot{x}_{1d} - \alpha_7 z_7 \end{cases} \tag{21}$$

b. Linear y and z Motion Control:

From the model (5) one can see that the motion through the axes y and z depends on u_1. In fact u_1 is the total thrust vector oriented to obtain the desired linear motion. If we consider u_y and u_z the orientations of u_1 responsible for the motion through y and z axis respectively, we can then extract from (7) the roll and pitch angles necessary to compute the controls u_y and u_z satisfying $\dot{V}(z_1 z_2) < 0$. The yaw control is then given as a desired angle.

$$u_y = \left(\frac{m_2}{u_1}\right)(z_9 - \alpha_9(z_{10} + \alpha_9 z_9) - \alpha_{10} z_{10}) \tag{22}$$

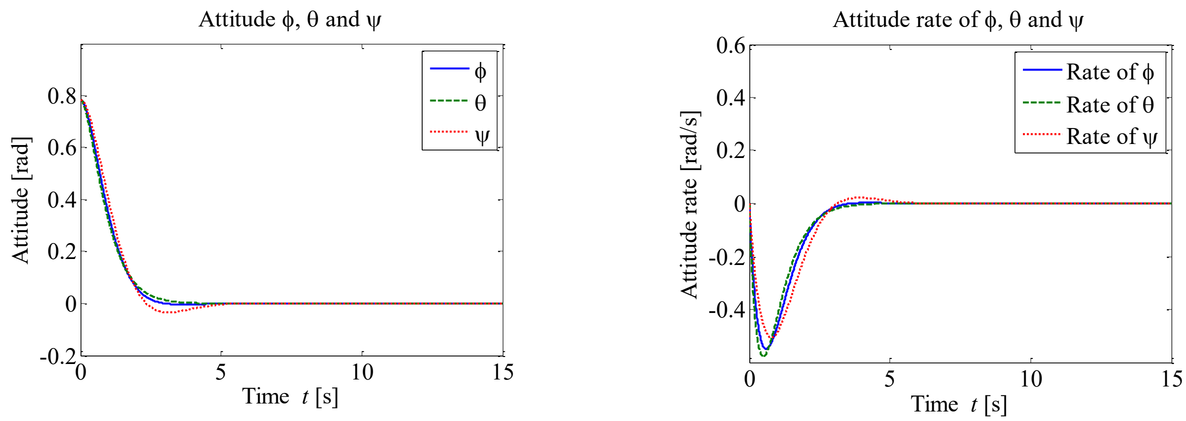
$$u_z = \left(-\frac{m_3}{u_1}\right)(z_{11} - \alpha_9(z_{12} + \alpha_{11} z_{11}) - \alpha_{12} z_{12}) \tag{23}$$

C. Backstepping Controller Simulation

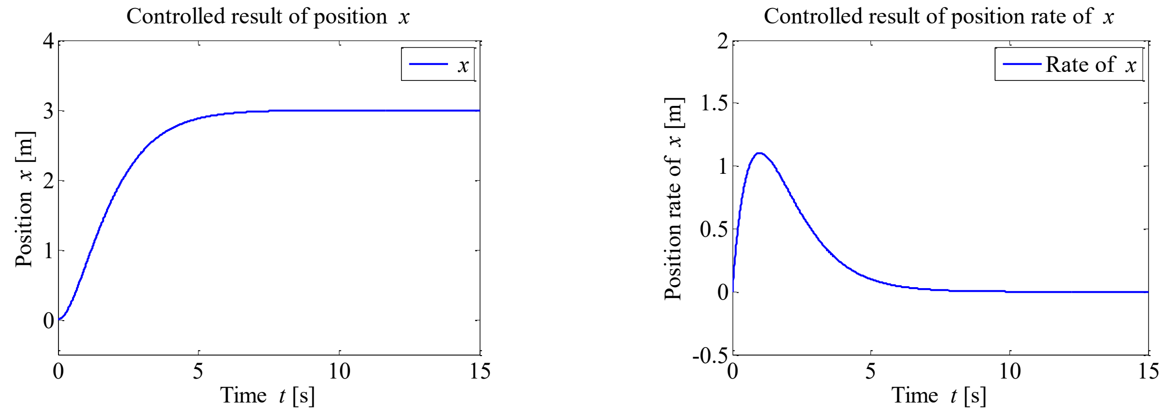
The controllers have been implemented on MATLAB and the simulation results for stabilizing an X4-AUV are shown in Figure 3. The system started with an initial state $X_0 = (0,0,0,0,0,0,\frac{\pi}{4},0,\frac{\pi}{4},0,\frac{\pi}{4},0)^T$ and we wanted the final x-positions, at 3 m with all zero orientation angles. As shown in Figure 3, it is seen that all orientation angles, and x-positions converge to the targets, where $\alpha_1 = 8$, $\alpha_2 = 2$, $\alpha_3 = 8$, $\alpha_4 = 2$, $\alpha_5 = 4$, $\alpha_6 = 2$, $\alpha_7 = 3$, $\alpha_8 = 1$. The physical parameters for X4-AUV that has been used for simulating the dynamic model presented in Table 1. Note that the simulations for stabilizing the X4-AUV in x-, y- and z-positions were implemented independently. The other results for y- and z-position are not included in this paper.

Table 1
Physical parameters for X4-AUV

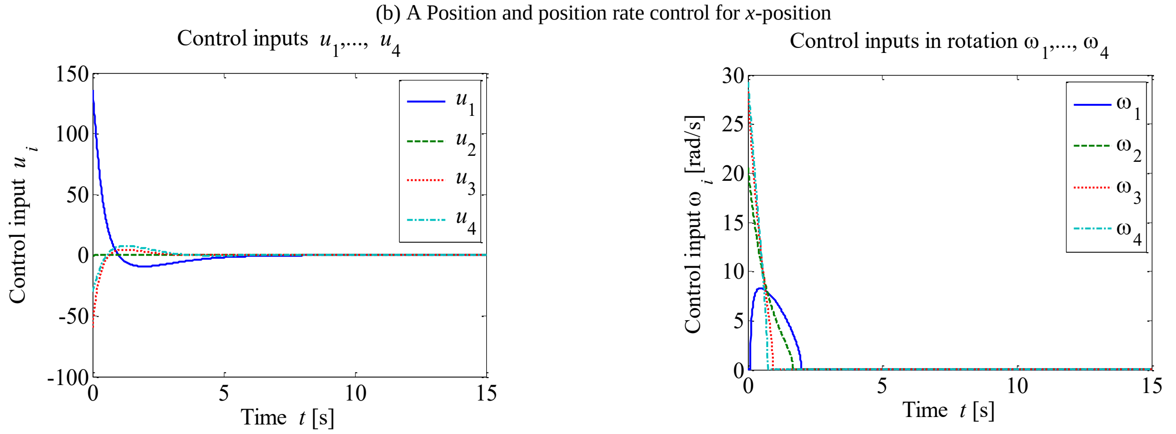
Parameter	Description	Value	Unit
m_b	Mass	21.43	Kg
ρ	Fluid density	1023.0	kg/m^3
l	Distance	0.1	M
r	Radius	0.1	m
b	Thrust factor	0.068	N·s^2
d	Drag factor	3.617e^{-4}	N·m·s^{-2}
J_{bx}	Roll inertia	0.0857	kg·m^2
J_{by}	Pitch inertia	1.1143	kg·m^2
J_{bz}	Yaw inertia	1.1143	kg·m^2
J_t	Thrust inertia	1.1941e^{-4}	N·m·s^{-2}



(a) Attitude and attitude rate control for *x*-position



(b) A Position and position rate control for *x*-position



(c)A control inputs and control inputs in rotation

Figure 3: Backstepping controller: A case for stabilizing the orientation angles and *x*-axis position

V. Sliding-Mode Control of an X4-AUV

A. Sliding-mode Control of the Angular Rotation

The mapping (7) is partially used to design the sliding-mode controller for the rotations subsystem of the X4-AUV. The first step in this design is similar to the one for the backstepping approach, except for the Equation (11) were S_2 (Surface) is used instead of z_2 for more clearance.

$$s_2 = x_8 - \dot{x}_{7d} - \alpha_1 z_1 \tag{24}$$

For the second step we consider tha augmented Lyapunov function:

$$V(z_1, s_2) = \frac{1}{2}(z_1^2 + s_2^2) \tag{25}$$

The chosen law for the attractive surface is the time derivative of (24) satisfying $(s\dot{s}) < 0)$:

$$\begin{aligned} \dot{s}_2 &= -k\,sign(s_2) - k_2 s_2 \\ &= \dot{x}_2 - \ddot{x_{1d}} - \alpha_1 \dot{z}_1 \\ &= \alpha_1 x_4 x_6 + \alpha_2 x_4 \Omega + b_1 U_2 - \ddot{x_{1d}} + \alpha_1 (z_2 + \alpha_1 z_1) \end{aligned} \tag{26}$$

As for the backstepping approach, the control U_2 is extracted:

$$u_2 = \frac{1}{b_1}(-a_1 x_{10} x_{12} - \alpha_1^2 z_2 - k_1 sign(s_2) - k_2 s_2) \tag{27}$$

The same steps are followed to extract U_3 and U_4.

$$u_3 = \frac{1}{b_2}(-a_2 x_8 x_{12} - a_3 x_{12}\Omega - \alpha_2^2 z_3 - k_3 sign(s_3) - k_4 s_3) \tag{28}$$

$$u_4 = \frac{1}{b_3}(-a_5 x_8 x_{10} - a_4 x_{10}\Omega - \alpha_3^2 z_5 - k_5 sign(s_4) - k_6 s_4) \tag{29}$$

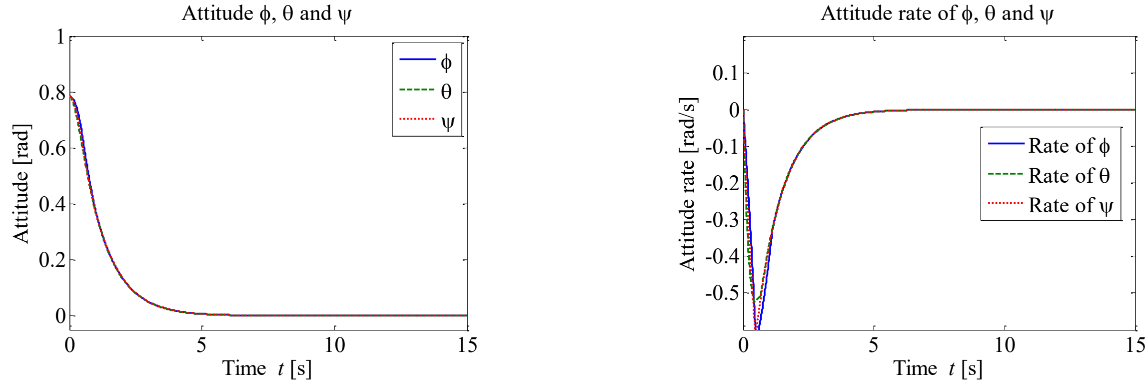
with:

$$\begin{cases} z_3 = x_{9d} - x_9 \\ s_3 = x_{10} - \dot{x}_{9d} - \alpha_2 z_3 \\ z_5 = x_{11d} - x_{11} \\ s_6 = x_{12} - \dot{x}_{11d} - \alpha_3 z_5 \end{cases}$$

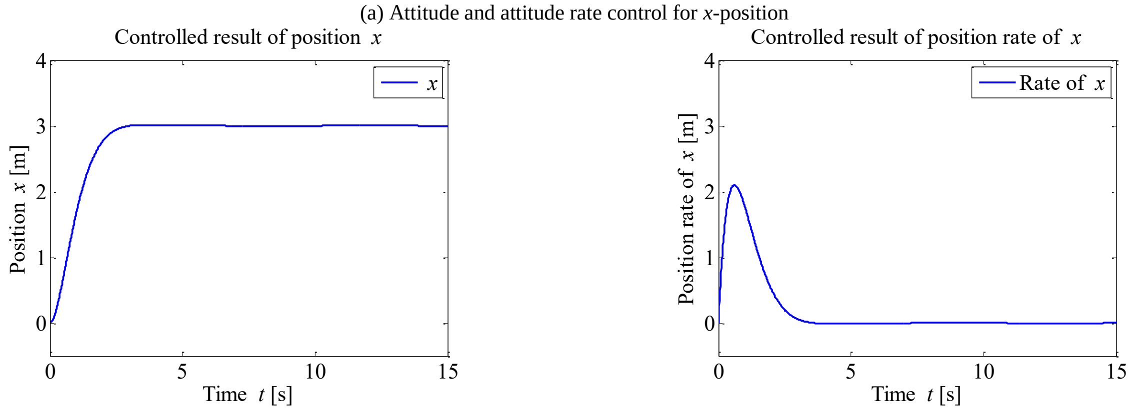
where $\alpha_2, \alpha_3, k_1, k_3, k_5$ is a positive constant.

B. Sliding-mode Controller Simulation

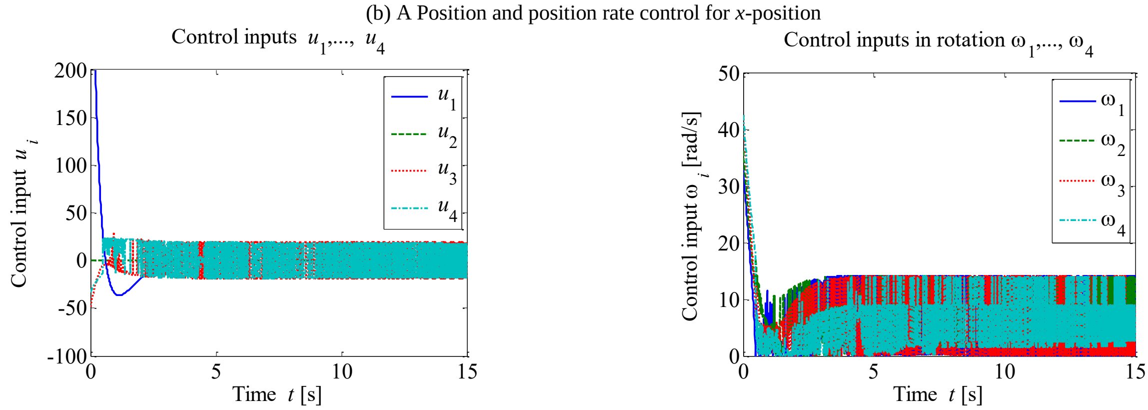
For these simulations, we considered only the angular rotations subsystem in sliding-mode control. As shown in Figure 4, it is seen that all orientation angles, and x-positions converge to the targets, where $\alpha_1 = 1$, $\alpha_2 = 1$, $\alpha_3 = 3$, $\alpha_7 = 1$, $\alpha_8 = 2$, $k_1 = 1$, $k_2 = 1$, $k_3 = 1.0$, $k_4 = 3$, $k_5 = 1$ $k_6 = 2$. The same physical parameters for X4-AUV that has been used for simulating the dynamic model presented in Table 1.



(a) Attitude and attitude rate control for *x*-position



(b) A Position and position rate control for *x*-position



(c) A control inputs and control inputs in rotation

Figure 4: Partially sliding-mode controller: A case for stabilizing the orientation angles and *x*-axis position.

VI. Conclusion

In this paper, we presented two different control techniques "Backstepping" and "Sliding-mode" to stabilize an X4-AUV. As it can be seen from the simulations, partially sliding-mode method gives better performance compared to fully backstepping method in stabilizing an X4-AUV dynamic model. Our future work is to develop a fully sliding-mode controller for an X4-AUV.

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