Variable Speed Control of Two-Mass Wind Turbine with Unknown Stiffness

N. S. F. M. Murad[1], M. N. Kamarudin[1], S. Md. Rozali[2], N. M. Shaharudin[1], A. Khamis[1] and A. R. Husain[3]
[1]*Faculty of Electrical Engineering*
[2]*Faculty of Engineering Technology*
Centre for Robotics and Industrial Automation (CeRIA)
Universiti Teknikal Malaysia Melaka (UTeM), Hang Tuah Jaya, 76100 Durian Tunggal, Melaka, Malaysia
[3]*Faculty of Electrical Engineering, Universiti Teknologi Malaysia (UTM), 81310 Skudai, Johor, Malaysia*
syazafarmurad@student.utem.edu.my

***Abstract*— This paper presents a variable speed control of two-mass wind turbine with unknown stiffness. The aim of the algorithm is to control rotor speed in order to maintain optimum tip-speed ratio and obtain maximum power output of the turbine with the appearance of unknown stiffness. The algorithm is formulated using full-state feedback which stabilizes in the sense of Lyapunov. Adaptation law is embedded into the variable speed algorithm to assure adaptability towards unknown stiffness. To validate the effectiveness of proposed speed control, a simulation in MATLAB/SIMULINK® is carried out. The results show that optimum tip speed ratio is obtained via the proposed controlled rotor speed regardless.**

***Index Terms*— Lyapunov; Rotor Speed; Stiffness; Variable Speed.**

I. Introduction

Wind turbine converts kinetic energy from the wind into electrical energy. Wind turbine has two configurations; horizontal axis wind turbine (HAWT) and vertical axis wind turbine (VAWT). In HAWT, main rotor shaft, gearbox, and generator are located on top of a tower. The orientation of main rotor shaft is parallel to the wind direction. In VAWT, the main rotor shaft is perpendicular to the wind direction and located on top of the tower, while generator and gearbox are located near the ground [1,2].Wind energy conversion system (WECS) consists of a mechanical power control (MPC) side and electrical power control (EPC) side. WECS are dependent on wind flow dynamics which are highly nonlinear, non-deterministic and have chaotic behavior. Plus, the most striking characteristic of wind flow that bother control engineers is its variability [3]. EPC side of the system demanded maximum mechanical power from the MPC side despite wind intermittent and seasonal interference. As such, the need to develop a variable speed algorithm for a modern WECS is crucial. In modern WECS, research in variable speed wind turbine is getting blossom. In a constant speed wind turbine, the rotor speed remain constant for all wind speed profile. Due to wind intermittent, the inherent problems of the constant speed wind turbine becomes more pronounced. Alternatively, variable speed wind turbine allows the rotor and wind speed to be matched in order to maintain its optimum tip-speed-ratio (TSR) for maximum efficiency. For a fixed pitch variable speed wind turbine, preserving the optimum TSR guarantees the maximum output power of the turbine [4,5]. As such, the need for asymptotic tracking for the rotor speed is a must, so that the optimum tip-speed ratio (TSR) can be achieved. It is crucial to develop control algorithm for a variable speed wind turbine with unknown parameter in the system dynamics. The mechanical dynamics involve two-mass rotational system which consists of a rotor-generator inertia, rotor-generator external damping, and rotor-generator stiffness.

To date, researchers came out with variable speed wind turbine control structure using linear parameter varying with anti-windup turbine [6], sliding mode control [7], nonlinear static and dynamic state feedback controller [8], full state-feedback [9,10, 20] and many more. Researcher in [7] used the similar wind turbine drive train dynamics as in early publications in [9]and [10]. Researcher in [9]addresses the advantages of variable speed wind turbines such as the possibility to control the turbine rotor speed, hence allowing the system to operate constantly near to its optimum tip-speed ratio. As such, increasing energy capture and maximizing energy generation. Variable speed wind turbines also reduce drive train loads. Most of the approaches in previous research [6–10] focused on nacelle (or MPC) with some assumptions to assist the design phase. Moreover in [7] and[8] the lumped rotor stiffness and the generator stiffness has been neglected as the presence of the stiffness will introduce an integral-term to the system dynamic that is difficult to handle by control engineers. Thus, the propose method in this paper is considering the stiffness as unknown parameter.

In this paper, the variable speed algorithm is formulated using full-state feedback. The speed of the turbine is bijective mapped into error dynamic in order to solve the tracking control as a regulation case. The stability is guaranteed in a sense of Lyapunov. The adaptability of the algorithm towards unknown stiffness is achieved via adaptation law that embedded into variable speed algorithm. Therein, the estimated stiffness is augmented in the Lyapunov function. The lie derivative of the function is made negative semi-definite via the non-negative control parameters. The algorithm is formulated to control the rotor speed in order to maintain optimum TSR, and hence obtaining maximum power output of the turbine. The results show that optimum tip speed ratio is obtained via the proposed controlled rotor speed regardless.

This paper is organized as follows. Section 2 describes the wind turbine dynamic, Section 3 explains the design of variable speed control, Section 4 discusses the results obtained, and Section 5 concludes the finding.

II. Wind Turbine Dynamic

The dynamic model of two-mass horizontal axis wind turbine is referring to its mechanical power control side only that consists of rotor model and aero-turbine model. The turbine rotor consists of blades, hubs, and pitch. The aero-turbine consists of brake, gearbox, rotor dynamics (low speed shaft and high speed shaft), and mechanical dynamics of the generator [3,4,11–14]. Figure 1 shows the two-mass wind turbine structure and Table 1 tabulates the definition of symbols consist in Figure 1 [11].

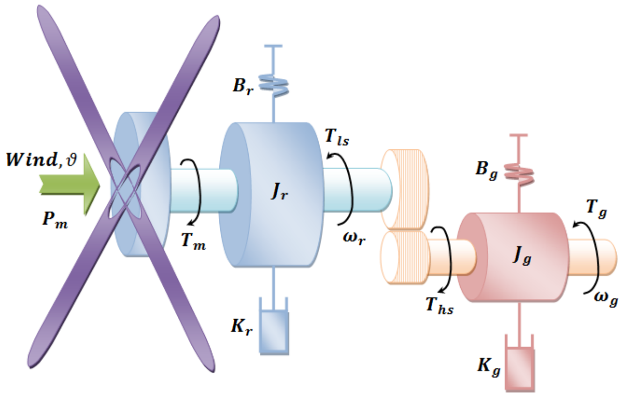

Figure 1: Two-mass wind turbine structure

Table 1
Nomenclature

Symbols	Definition
R	Rotor blade radius (m)
v	Wind speed $(m.s^{-1})$
ρ	Air density $(Kg.m^{-3})$
$C_p(\lambda,\beta)$	Power coefficient
λ	Tip speed ratio
β	Pitch angle (deg)
γ	Gearing ratio
ω_r	Rotor speed $(rad.s^{-1})$
ω_g	Generator speed $(rad.s^{-1})$
J_r	Rotor inertia $(Kg.m^2)$
J_g	Generator inertia $(Kg.m^2)$
K_r	Rotor external damping $(N.m.rad^{-1}.s^{-1})$
K_g	Generator external damping $(N.m.rad^{-1}.s^{-1})$
B_r	Rotor stiffness $(N.m.rad^{-1})$
B_g	Generator stiffness $(N.m.rad^{-1})$
T_m	Aerodynamic torque $(N.m)$
T_g	Generator torque $(N.m)$
T_{hs}	High-speed shaft torque $(N.m)$
T_{ls}	Low-speed shaft torque $(N.m)$
θ_g	Generator-side angular deviation (Rad)
θ_r	Rotor-side angular deviation (Rad)

The aero-dynamic power produced by the turbine is depends on instantaneous wind power,P_{wind} and the power coefficient of the unique wind turbine $C_p(\lambda,\beta)$,

$$P_m = P_{wind} C_p(\lambda,\beta) \tag{1}$$

where

$$P_{wind} = \frac{1}{2}\rho\pi R^2 v^3 \tag{2}$$

and

$$\lambda = \frac{R\omega_r}{v} \tag{3}$$

is the tip-speed ratio.

In some research papers [8,15–17,20,21], the power coefficient is obtained via look-up-table which is provided by each turbine manufacturer. However, previous studies from [4,18,19] presented the empirical power coefficient model that is expressed as

$$C_p(\lambda,\beta) = 0.5(116\frac{1}{\phi} - 0.4\phi\beta - 5)e^{-21\frac{1}{\phi}} \tag{4}$$

where the function ϕ is given as

$$\frac{1}{\phi} = \frac{1}{\lambda + 0.08\beta} - \frac{0.035}{1+\beta^3} \tag{5}$$

In this research, the pitch angle β is regulated at $0°$ in order to facilitate fixed-pitch system. Hence the power coefficient can be expressed as

$$C_p(\lambda, 0°) = 0.5(\frac{116}{\lambda + 0.0001} - 9.06)e^{-\frac{21}{\lambda+0.0001}+0.735} \tag{6}$$

As a result, the maximum power coefficient is obtained as $C_{pmax} = 0.4109631031$ when the optimum tip-speed ratio is $\lambda_{opt} = 7.953925991$ as shown in Figure 2.

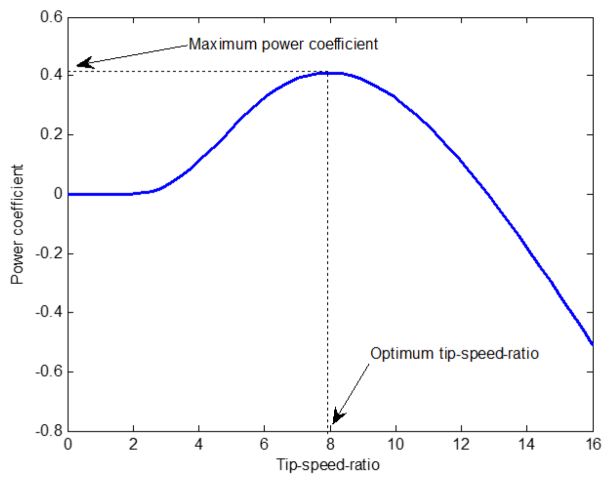

Figure 2: Typical power coefficient characteristic for fixed pitch wind turbine

Aero-turbine model has two inertias corresponding to turbine and generator. According to Figure 1, the torque produced by low speed shaft can be devised as

$$T_m - T_{ls} = J_r\dot{\omega}_r + K_r\omega_r + B_r\theta_r \tag{7}$$

The transmission output torque at the high speed shaft is devised as

$$T_{hs} - T_g = J_g\dot{\omega}_g + K_g\omega_g + B_g\theta_g \tag{8}$$

The gear ratio can be expressed as

$$\gamma = \frac{\omega_g}{\omega_r} = \frac{T_{ls}}{T_{hs}} \tag{9}$$

that yields

$$\omega_g = \gamma\omega_r \tag{10}$$

From (9) and (10), equation (8) can be expressed as

$$T_{ls} = \gamma T_g + J_g\gamma^2\dot{\omega}_r + K_g\gamma^2\omega_r + \gamma B_g\theta_g \tag{11}$$

The generator side angular deviation, θ_g can be expressed as$\theta_g = \gamma\int \omega_r d(t)$, then substitute into equation (11) to yield

$$T_{ls} = \gamma T_g + J_g\gamma^2\dot{\omega}_r + K_g\gamma^2\omega_r + \gamma^2 B_g\int \omega_r d(t) \tag{12}$$

Finally, the dynamic of wind turbine can be expressed as

$$T_m - \gamma T_g = (J_g\gamma^2 + J_r)\dot{\omega}_r + (K_g\gamma^2 + K_r)\omega_r + (\gamma^2 B_g + B_r)\int \omega_r d(t) \tag{13}$$

Defining

lumped inertia, $J = \gamma^2 J_g + J_r$
lumped external damping, $K = \gamma^2 K_g + K_r$
lumped external stiffness, $B = \gamma^2 B_g + B_r$

Then, the dynamic model is expressed as

$$\dot{\omega}_r = \frac{1}{J}\left(T_m - \gamma T_g - K\omega_r - B\int \omega_r d(t)\right) \tag{14}$$

III. VARIABLE SPEED CONTROL

Variable speed control is formulated using state-feedback control, Lyapunov redesign and adaptive control. Equation (14) can be written as

$$\begin{gathered}\dot{\omega}_r = -\frac{K}{J}\omega_r - \frac{B}{J}\int \omega_r d(t) + \frac{\rho\pi R^5 C_p\omega_r^2}{2\lambda^3 J} - \frac{\gamma}{J}T_g \\ \dot{\omega}_r = A\omega_r - \frac{1}{J}B\theta_r + C\omega_r^2 + DT_g\end{gathered} \tag{15}$$

where $A = -\frac{K}{J}$,
$C = \frac{\rho\pi R^5 C_p}{2\lambda^3 J}$,
$D = -\frac{\gamma}{J}$

and B is the unknown lumped stiffness

For a maximum power capture of the wind turbine in equation (18), the rotor speed ω_r should be regulated at

$$\omega_r^* = \left(\frac{\lambda_{opt}}{R}\right)v \tag{16}$$

Therefore, the rotor tracking error is devised as $z = \omega_r - \omega_r^*$. Then, the error dynamics become

$$\dot{z} = \dot{\omega}_r - \dot{\omega}_r^* \tag{17}$$

$$\begin{aligned}\dot{z} = {} & Az - \tfrac{1}{J}B\int z\, d(t) + Cz^2 + 2Cz\omega_r^* + \\ & DT_g + A\omega_r^* - \tfrac{1}{J}B\int \omega_r^*\, d(t) + C\omega_r^{*2} - \\ & \dot{\omega}_r^*\end{aligned} \tag{18}$$

Let consider $(A + C\omega_r^*)\omega_r^* - \frac{B}{J}\int \omega_r^*\, d(t) - \xi(\omega_r^*, \dot{\omega}_r^*, B) = \dot{\omega}_r^*$ be the uncertainty subjected by wind toward rotor speed. Thus, the error dynamic can be expressed as

$$\dot{z} = Az - \frac{1}{J}B\int z\, d(t) + Cz^2 + 2Cz\omega_r^* + DT_g + \xi(\omega_r^*, \dot{\omega}_r^*, B) \tag{19}$$

In designing the control algorithm, let consider the error dynamic without the uncertainty as

$$\dot{z} = Az - \frac{1}{J}B\int z\, d(t) + Cz^2 + 2Cz\omega_r^* + DT_g \tag{20}$$

Since B is unknown stiffness, thus adaptation law is formulated. Let

$$\tilde{B} = B - \hat{B} \tag{21}$$

where $\hat{B}$ is the estimate of B with Lyapunov function

$$V(z, \tilde{B}) = \frac{1}{2}z^2 + \frac{1}{2}\tilde{B}^{\,T}\Gamma^{-1}\tilde{B}, \Gamma^{-1} > 0 \tag{22}$$

The adaptation law is devised as

$$\dot{\hat{B}} = -\frac{\Gamma^{-1}}{J}z\int z\, d(t) \tag{23}$$

with Lyapunov function in (22), there exists control parameter $K > 0$ that yield control law

$$T_{g_{nom}} = \frac{1}{D}\left(-Kz - Az + \frac{1}{J}\hat{B}\int z\, d(t) - Cz^2 - 2C\omega_r^* z\right) \tag{24}$$

To ensure robustness towards uncertainties, consider error dynamic in (19) with Lyapunov function in (22), the robust controller T_{g_r} can be devised as

$$T_{g_r} = -\xi_g(\omega_r^*, \dot{\omega}_r^*, B)\frac{Dz\xi_g(\omega_r^*, \dot{\omega}_r^*, B)}{Dz\xi_g(\omega_r^*, \dot{\omega}_r^*, B) + \varepsilon e^{-at}} \tag{25}$$

As such, the overall variable speed algorithm can be expressed as

$$\begin{aligned}T_g &= T_{g_{nom}} + T_{g_r} \\ &= \frac{1}{D}\left(-Kz - Az + \frac{1}{J}\hat{B}\int z\, d(t) - Cz^2 - 2C\omega_r^* z\right) \\ &\quad - \xi_g(\omega_r^*, \dot{\omega}_r^*, B)\frac{Dz\xi_g(\omega_r^*, \dot{\omega}_r^*, B)}{Dz\xi_g(\omega_r^*, \dot{\omega}_r^*, B) + \varepsilon e^{-a}}\end{aligned} \tag{26}$$

The asymptotic stability reflect the asymptotic tracking of the rotor speed with the proof of Lyapunov function in (22) as

$$\dot{V} = -Kz^2 + \frac{Dz\xi_g(\omega_r^*, \dot{\omega}_r^*, B)\varepsilon e^{-at}}{Dz\xi_g(\omega_r^*, \dot{\omega}_r^*, B) + \varepsilon e^{-at}} \leq -Kz^2 + \varepsilon e^{-at} \tag{27}$$

IV. RESULT AND DISCUSSION

The proposed control method has been implemented in MATLAB with SIMULINK® toolbox. A wind turbine with 21.65 meter blade radius is simulated along with a gearbox with 43.165 gearing ratio. To observe the adaptability of the adaptation law and the variable speed algorithm, the unknown stiffness is varied within $0 \leq B \leq 100000\, N.m.rad^{-1}$. The maximum power coefficient is 0.4109631031 when the pitch angle $\beta = 0°$ which indicated that only 41.09% of the wind is captured by the turbine to be converted to electrical energy. The average wind speed, $\bar{v}$ is around 18.15 m/s.

The simulation results are shown in Figure 3, Figure 4 and Figure 5 respectively. Asymptotic tracking of the demanded rotor speed ω_r guarantees the optimum tip speed ratio λ_{opt} of the closed-loop wind turbine system as ω_r is bounded within asymptotic tracking region. Figure 3 shows the demanded rotor speed and the actual rotor speed of the turbine with different stiffness value. The higher stiffness value, the bigger transient response recorded. However, the controller took less than 30 second to precisely asymptotically track the demanded rotor speed for any stiffness value which validates that optimum tip-speed ratio is maintained even though with appearance of stiffness. The asymptotic tracking behavior also proves by integral of error recorded in Figure 4 as the integral of error gives a small magnitude value. The performance of adaptation law is shown in Figure 5.Adaptation response for system with any values of stiffness recorded very small magnitude ($-3.302e^{-10}$ to $9.726e^{-11}$) and settled in less than 10 seconds. Thus, it is verified that the adaptation law is adaptable to the variable speed algorithm with appearance of stiffness.

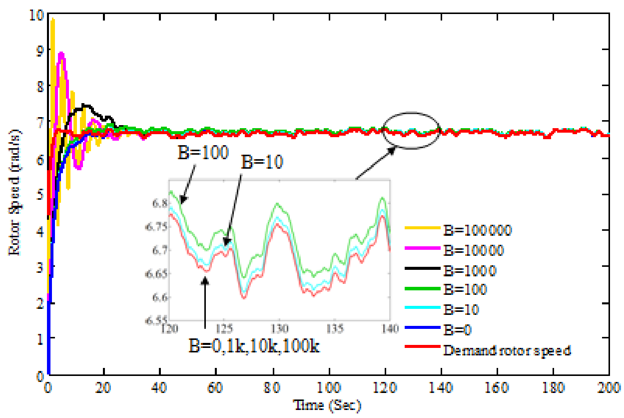

Figure 3: Demanded rotor speed versus and actual rotor speed

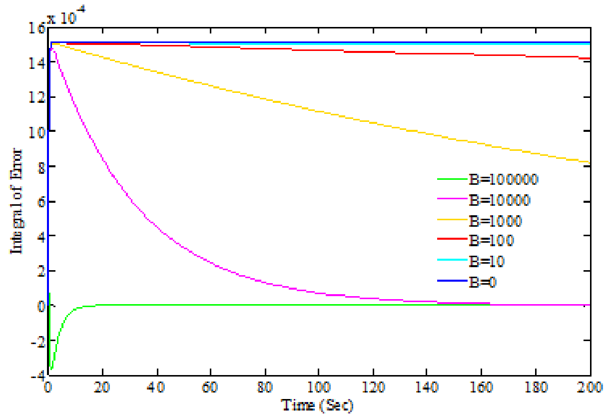

Figure 4: Integral of error

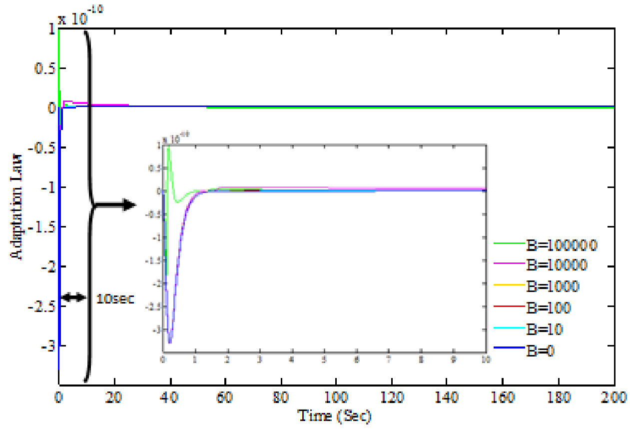

Figure 5: Adaptation Law

V. CONCLUSION

This paper has presented the design of variable speed control of two-mass wind turbine with unknown stiffness. The results show that optimum tip speed ratio is obtained via the proposed controlled rotor speed. The rotor speed is stable in the sense of Lyapunov and the unknown stiffness is adapted to the algorithm with the aid of adaptation law. Thus, as power output depends on tip speed ratio, the optimum tip speed ratio obtained leads to maximum power output of the turbine. In future, the proposed design technique can be applied to higher mass wind turbine drive-train. As higher mass wind turbine drive-train increases the system order, the mathematical model can be represented in strict-feedback system and might requires nonlinear control technique such as backstepping. Moreover, if the speed is unmeasurable, the observer can be designed beforehand.

ACKNOWLEDGMENT

We acknowledge the Center for Robotics and Industrial Automation, Universiti Teknikal Malaysia Melaka (UTeM) for research facilities, research collaboration and research grant funding Vote No: FRGS/1/2015/TK04/FKE/03/F00259 and PJP/2015/FKE(21C)S01641.

REFERENCES

[1] K. Cheng, Z. Wang, Y. He, and G. Yang, "The Comparison of Theoretical Potential Application of Two Types of Wind Turbines in Northern Shaanxi," *Asia-Pacific Power Energy Eng. Conf. APPEEC*, pp. 2–5, 2012.

[2] T. Al-Shemmeri, *Wind Turbines*. T. Al-Shemmeri & Ventus

Publishing ApS, 2010.

[3] M. Kamarudin, A. Husain, M. Ahmad, and Z. Mohamed, “Model and Analysis of Wind Speed Profile using Artificial Neural Network-Feasibility Study in Peninsular Malaysia,” *J. Teknol. (Sciences Eng.*, vol. 74, no. 1, pp. 77–81, 2015.

[4] A. W. Manyonge, R. M. Ochieng, F. N. Onyango, and J. M. Shichikha, “Mathematical Modelling of Wind Turbine in a Wind Energy Conversion System : Power Coefficient Analysis,” *Appl. Math. Sci.*, vol. 6, no. 91, pp. 4527–4536, 2012.

[5] G. Ofualagba and E. Ubeku, “The Modeling and Dynamic Characteristics of a Variable Speed Wind Turbine,” *J. Energy Technol. Policy*, vol. 1, no. 3, pp. 10–22, 2011.

[6] F. A. Inthamoussou, F. D. Bianchi, H. De Battista, and R. J. Mantz, “LPV wind turbine control with anti-windup features covering the complete wind speed range,” *IEEE Trans. Energy Convers.*, vol. 29, no. 1, pp. 259–266, 2014.

[7] B. Beltran, T. Ahmed-Ali, and M. Benbouzid, “High-Order Sliding-Mode Control of Variable-Speed Wind Turbines,” *IEEE Trans. Ind. Electron.*, vol. 56, no. 9, pp. 3314–3321, 2009.

[8] B. Boukhezzar and H. Siguerdidjane, “Nonlinear Control of a Variable-Speed Wind Turbine Using a Two-Mass Model,” *IEEE Trans. ENERGY Convers.*, vol. 26, no. 1, pp. 149–162, 2011.

[9] U. Ozbay, E. Zergeroglu, and S. Sivrioglu, “Adaptive backstepping control of variable speed wind turbines,” *Int. J. Control*, vol. 81, no. 6, pp. 910–919, 2008.

[10] Y. D. Song, B. Dhinakaran, and X. Y. Bao, “Variable speed control of wind turbines using nonlinear and adaptive algorithms,” *J. Wind Eng. Ind. Aerodyn.*, vol. 85, no. 3, pp. 293–308, 2000.

[11] M. N. Kamarudin, A. R. Husain, M. Noh, and Z. Mohamed, “Development of A Strict Feedback Two-Mass Horizontal Axis Wind Turbine Model With Empirical Power Coefficient And External Stiffness,” *J. Adv. Manuf. Technol.*, vol. 9, no. 1, 2015.

[12] P. Ashwini and T. Archana, “Mathematical Modeling of Wind Energy System Using Two Mass Model Including Generator Losses,” *Interrnational J. Emerg. Trends Electr. Electron.*, vol. 12, pp. 18–23, 2016.

[13] S. Rajendran and D. Jena, “Backstepping Sliding Mode Control for Variable Speed Wind turbine Saravanakumar,” in *2014 Annual IEEE India Conference (INDICON)*, 2014, no. 2, pp. 1–5.

[14] Z. Zhenyu, S. Yongduan, L. I. Peng, W. Wenliang, and Q. I. N. Ming, “Adaptive and Robust Variable-Speed Control of Wind Turbine Based on Virtual Parameter Approach,” in *Proceedings of The 32nd Chinese Control Conference*, 2013, pp. 8886–8890.

[15] A. G. Aissaoui, A. Tahour, M. Abid, N. Essounbouli, and F. Nollet, “Power control of wind turbine based on fuzzy controllers,” *Energy Procedia*, vol. 42, pp. 163–172, 2013.

[16] B. Boukhezzar, L. Lupu, H. Siguerdidjane, and M. Hand, “Multivariable control strategy for variable speed, variable pitch wind turbines,” *Renew. Energy*, vol. 32, no. 8, pp. 1273–1287, 2007.

[17] V. Calderaro, V. Galdi, A. Piccolo, and P. Siano, “A Fuzzy Controller for Maximum Energy Extraction from Variable Speed Wind Power Generation Systems,” *Electr. Power Syst. Res.*, vol. 78, no. 6, pp. 1109–1118, 2008.

[18] P. M. Anderson and A. Bose, “Stability Simulation of Wind Turine Systems,” *IEEE Trans. Power Appar. Syst.*, vol. PAS-102, no. 12, pp. 3791–3795, 1983.

[19] H. Dharmawardena and K. Uhlen, “Modeling Variable Speed Wind Turbine for Power System Dynamic Studies,” *2015 IEEE Students Conf. Eng. Syst.*, vol. 2, no. 1, pp. 1–6, 2015.

[20] S. Md. Rozali, M. Nizam Kamarudin, M. F. Rahmat, A. R. Husain, "Asymptotic Tracking Position Control for Nonlinear Systems using BacksteppingTechnique," *Procedia Engineering*, vol. 53, pp. 255–263, 2013.

[21] M. Nizam Kamarudin, S. Md.Rozali, A. Rashid HusainObserver-Based Output Feedback Control with Linear Quadratic Performance," *Procedia Engineering*, vol.53, pp. 233–240, 2013.