Analytical Quantum Drain Current Model in Undoped Cylindrical Surrounding-Gate MOSFETS

Fatimah A. Noor, Christoforus Bimo, and Khairurrijal

Physics of Electronic Materials Research Division,
Faculty of Mathematics and Natural Sciences,
Institut Teknologi Bandung,
Jalan Ganesa 10, Bandung 40132, Indonesia.
fatimah@fi.itb.ac.id

***Abstract*—Analytical potential model for cylindrical surrounding-gate or gate-all-around metal oxide semiconductor field effect transistors (MOSFETs) has been developed. The model presented here takes quantum confinement effects into account in which embodied by two physical parameters, namely, (1) threshold voltage shift and (2) inversion layer centroid. These parameters have been incorporated into the classical procedure as modifications for the gate work function and the inversion layer capacitance to obtain the quantum version of drain current. The model has been able to reproduce drain current vs. gate voltage characteristics obtained from self-consistent calculation. Therefore, it is suitable to use it in the context of circuit simulator.**

***Index Terms*—Analytical Model; Cylindrical Surrounding Gate MOSFETs.**

I. Introduction

Conventional bulk MOSFETs has approaching its scaling limit [1]. As the size of the device are continue to shrink, several detrimental effects such as gate oxide tunneling and short channel effects began to affects device performance [2-11]. One of the solutions to overcome those undesirable effects is to improve the channel electrostatics control. To get better control over the channel, multi-gate based technology, like double gate, triple gate, and surrounding gate, incorporate more than one gate into the channel [6]. Surrounding gate (SG) MOSFETs offers the best gate control compared to the others since the channel is basically fully surrounded by the gate.

In this paper, we propose a simple analytical model for obtaining the I_ds (V_g,V_ds) characteristic of cylindrical SG MOSFET. Since the channel is fully surrounded by the gate and as the size is approaching the nanometers regime, quantum confinement effects have a major influence into the inversion charge of SG MOSFET. Therefore, to be able to describe more accurate characteristics of the device, this quantum confinement effects have to be included in the current model [2, 4, 13]. The quantum confinement effects can manifest into the behavior of the device characteristics in two aspects. First, since the energy levels are quantized, more band bending would be required to populate the subbands therefore the threshold voltage become higher. Second, the inversion layer formed near surface would reduce the total gate capacitance [4].

Roldan et al. [2] obtained a great result for an analytical I-V characteristic model for cylindrical SG MOSFETs that include quantum confinement effects by performing modification on classical charge control based approach [3]. Although the analytical model developed there can match numerical simulation at low and high gate voltages, it is not quite accurate particularly on the transition region near threshold voltage. Therefore, a more accurate model is needed since this transition region become more and more important as the power supply voltage is scaled down [7].

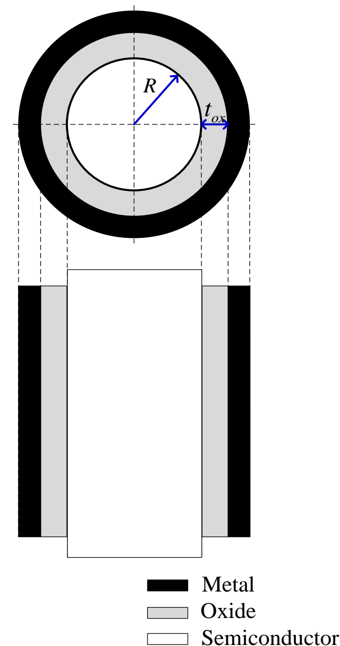


Figure 1: Cross section of a cylindrical SG MOSFET

In this paper, we will demonstrate that an analogous formulation of quantum correction or modification for classical potential based approach [4] as proposed by Wang et al. for the double-gate (DG) MOSFET [5] can be carried out for the cylindrical SG MOSFET [6, 7]. Classical potential based approach that we used here is an analytical method proposed by Yu et al. [7]. This method proved to be more correct mathematically and more accurate compared to classical charge control based method [7]. Therefore, hopefully by applying this method as our base analytical model, we can get more accurate result compared to the one obtained by [2]. For this purpose, we organize this letter as follows: In Section 2 solutions to the coupled Schrödinger and Poisson equations in cylindrical coordinate system are developed using a one-dimensional Poisson-Schrödinger solver. Then the Influences of quantum confinement on

device characteristics, namely, threshold voltage shift and inversion centroid are expressed as closed form functions of device radius and inversion charge density. In Section 3, quantum effects are implemented in the classical potential based compact model for generating I-V curves. The advantages when compared to with the previous model from [2] will be covered in Section 4. The final conclusions are drawn in Section 5.

II. Quantum Correction

The goal of this section is to model the quantum mechanical aspects (threshold voltage shift and capacitance degradation) that would affect the device characteristics.

A. Self-consistent Solution of Schrödinger-Poisson Equation

Figure 1 shows the geometrical dimensions of a cylindrical SG MOSFET. A silicon nanowire with radius R is surrounded by thin gate oxide with thickness t_{ox}. For a long-channel SG-MOSFET, quantum confinement effects arise both from field potential from gate and from 2-D structural confinement from the surrounding oxide barrier. To obtain the quantum electrical characteristics of the MOSFET, one needs to solve the coupled Poisson and Schrödinger equations self-consistently. The Poisson equation connecting the electrostatic potential ψ(r) to the electron density n(r) is [2, 6, 7].

$$\frac{d^2\psi(r)}{dr^2}+\frac{1}{r}\frac{d\psi(r)}{dr}=\frac{q}{\varepsilon_{si}}n(r). \quad (1)$$

Instead of using classical Boltzmann's statistics, which is given as $n=n_i \exp q\psi/kT$ in the classical model, n is modeled quantum mechanically as [2]

$$n(r)=\sum_{v,n_r,n_\phi} g_v\left(\frac{2m_{d,v}^* kT}{\pi^2\hbar^2}\right)^{\frac{1}{2}}\left|\Psi_{v,n_r,n_\phi}(r)\right|^2, \times F_{-\frac{1}{2}}\left(\frac{E_f-E_{v,n_r,n_\phi}}{kT}\right) \quad (2)$$

due to the application of Fermi-Dirac statistics. Here q is the electronic charge, ε_{si} is the silicon permitivitty, k is Boltzmann's constant, T is temperature, E_f is the Fermi level, g_v and $m_{d,v}^*$ are the degeneracy and density-of-state effective mass in the vth valley, respectively, $\hbar$ is Planck's constant, $\Psi_{v,n_r,n_\phi}(r)$ is the normalized wave functions and E_{v,n_r,n_ϕ} is the energy of the n_rth and n_ϕth subband in the vth valley satisfying the following Schrödinger equation.

$$-\frac{\hbar^2}{2m_{r,v}^*}\left[\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\phi^2}\right]\Psi(r,\phi) +E_c(r)\Psi(r,\phi)=E_{v,n_r,n_\phi}\Psi(r,\phi). \quad (3)$$

Here $m_{r,v}^*$ is the electron effective mass in the vth valley in the radial direction and the conduction band $E_c(r)$ is coupled to the electrostatic potential ψ(r) through $E_c=E_g/2-q\psi(r)$.

The boundary condition relates the potential and electric field at the silicon-oxide interfaces to the applied gate voltage as follows [4, 6].

$$-\frac{\hbar^2}{2m_{r,v}^*}\left[\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\phi^2}\right]\Psi(r,\phi) +E_c(r)\Psi(r,\phi)=E_{v,n_r,n_\phi}\Psi(r,\phi). \quad (4)$$

where $C_{ox}=\varepsilon_{ox}/R\ln(1+t_{ox}/R)$ is the oxide capacitance per unit area, Δϕ is the work function difference between the gate electrode and the intrinsic silicon, V_g is gate voltage, and Q_i is the electron charge sheet density. The wave function is forced to be zero at the silicon-oxide interface mimicking impenetrable barrier but continues at the center (r=0).

We have developed numerical self-consistent calculation to solve the Poisson Equation (1) and Schrödinger Equation (3). The illustration of our quantum self-consistent calculation when compared to the classical calculation will be covered in the result section (Section 5) as a minor result.

B. Threshold Voltage Shift

In order to convert those complex self-consistent calculations into analytical one, we followed the work by Roldan et.al. [2] and our previous work [13] by the following. First, we modeled the trends of the threshold voltage shift obtained from self-consistent calculation by empirical equation given by

$$\Delta V_{th}=0.03001\,(V)+\frac{0.1187\times10^{-18}(V/m^2)}{R^2}. \quad (5)$$

This procedure is quite different with the work by Wang et al. for the DG-MOSFET [4], in which the threshold voltage shift can be modeled only by the lowest subband.

C. Gate Capacitance Degradation

It is known that the gate capacitance is degraded due to quantum quantization effects [2, 4, 13]. This gate capacitance degradation can be modeled by correcting the oxide capacitance C_ox. the quantum charge distribution in the semiconductor through the determination of the inversion layer centroid is characterized to calculate the corrected oxide capacitance. In order to get the correction for C_ox, first we gathered self-consistent data of inversion layer centroid. We then used the following empirical equation to model those data [2, 13].

$$\frac{1}{z_I}=\frac{1}{a+2bR}+\frac{1}{z_{I0}}\left(\frac{Q_i}{qN_{I0}}\right)^n, \quad (6)$$

where a=0.55 nm, b=0.198, z_I0=5.1 nm, $N_{I0}=7\times10^{16}-4.9\times10^{24}/\mathrm{m}$, and n=0.75 are constants.

The classical oxide capacitance, Cox, was replaced by another capacitance called as corrected oxide capacitance, Cox*, to incorporate the effects of zI for the evaluation of the inversion charge concentration in Equation (4) which formulated as follows:

$$\frac{1}{C_{ox}^*}=\frac{1}{C_{ox}}+\frac{1}{C_{centroid}}, \quad (7)$$

where $C_{centroid}$ is calculated as follows [2, 13]:

$$C_{centroid}=\frac{\varepsilon_{si}}{(R-z_I)\,ln\left(1+\frac{z_I}{R-z_I}\right)}. \quad (8)$$

These two semi-empirical equations (Equation (5) and Equation (6)) are proven to be consistently fit with the data for various range of radius and gate voltage. The illustration can be found elsewhere [13].

III. Implementation of Quantum Correction for SG-MOSFETS for Inversion Charge Model

Finally, in order to obtain the quantum version of analytical approach, the quantum mechanical correction factors (the threshold voltage shift and the gate capacitance degradation) are implemented in a classical analytic potential model for SG MOSFETs proposed by Ref [6-7]. There, in the analytic potential model, Poisson's equation is rigorously solved to obtain an analytical expression for potential in the silicon film [6-7].

$$(r) = V - \frac{2kT}{q} * ln\left[\frac{R}{2}\sqrt{\frac{q^2 n_i}{2\varepsilon_{si} kT(1-\alpha)}} \times \left(1 - \frac{(1-\alpha)r^2}{R^2}\right)\right] \quad (9)$$

where V is the electron quasi-Fermi potential at a point along the channel, n_i is the intrinsic carrier density, and the dimensionless parameter α which is a function of V to be determined from the boundary condition in Equation (4) with V_g replace by $V_g - V$. For this analytical consideration, the boundary condition can be appropriately implemented by substituting Equation (9) into Equation (4) to obtain the following boundary equation:

$$\frac{1}{2} ln(1-\alpha) - ln\,\alpha + s\frac{1-\alpha}{\alpha} = \frac{q(V_g - \Delta\phi - V)}{2kT} - ln\left(\frac{2}{R}\sqrt{\frac{2\varepsilon_{si} kT}{q^2 n_i}}\right) \quad (10)$$

where $s = 2\varepsilon_{si}/RC_{ox}$. This equation is a non linear equation that need to be solve to find the appropriate value of α.

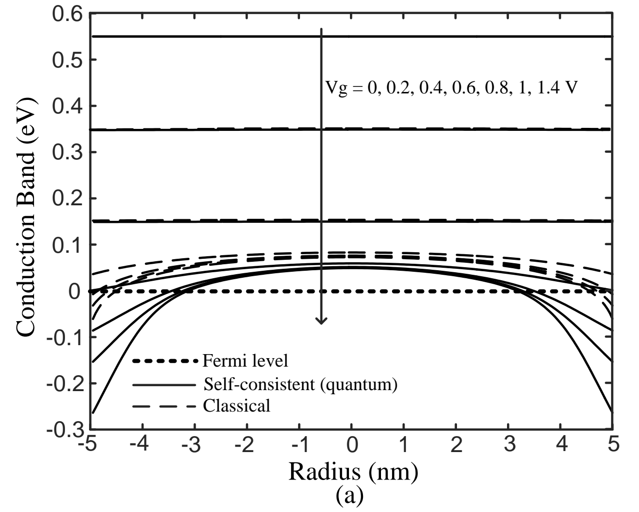
From Gauss' law, the classical charge density is $Q_i = \varepsilon_{si}(d\psi/dr)_{r=R}$, which is simply

$$Q_i = \frac{4\varepsilon_{si} kT}{qR}\frac{(1-\alpha)}{\alpha}. \quad (11)$$

By integrating the current continuity equation [9], a continuous, analytical drain current expression is obtained for all regions of MOSFET operation [8]

$$I_{ds} = \mu\frac{8\pi\varepsilon_{si}}{L}\left(\frac{kT}{q}\right)^2 [f(\alpha_d) - f(\alpha_s)], \quad (12)$$

where μ is the electron mobility, L is the channel length, α_s and α_d are solutions of Equation (10) corresponding to $V = V_s$ and $V = V_d$, respectively, and $f(\alpha) = -2/\alpha - \ln\alpha + s(-1/\alpha^2 + 2/\alpha)$.



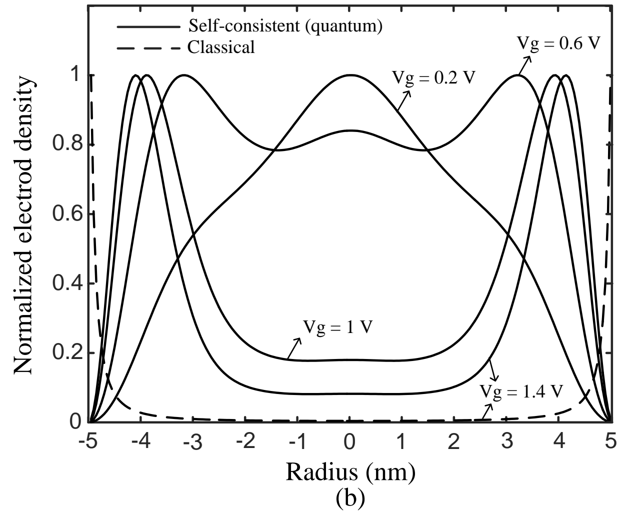


Figure 2: (a) Conduction band along the radial direction and (b) normalized electron density of SG-MOSFETs with radius of silicon body R=5 nm and oxide thickness t_{ox} =1nm.

To incorporate the quantization effects into the above classical analytical potential model, the following procedure was performed.

i. Implement the threshold voltage shift by changing the gate work function in Equation (10) from $\Delta\phi$ to $\Delta\phi + \Delta V_{th}$, with ΔV_{th} is given by Equation (5), then solve Equation (10) to find α. A simple explicit procedure to calculate α can be found in [7].
ii. Compute the inversion charge density Q_i by substituting α into Equation (11).
iii. Compute the inversion layer centroid z_I by putting Q_i found in (b) into Equation (6) then find the corrected oxide capacitance C_{ox}^* by substituting z_I into Equation (8) and then Equation (7).
iv. Put this new corrected oxide capacitance into Equation (10) inside the s variable then compute the second value of α as step (a). We name this α as α^{QM} since its value determines all quantum-mechanically corrected parameters.
v. Compute the drain current I_{ds} by substituting α^{QM} and C_{ox}^* into Equation (12) for the final result of the compact quantum I-V model.

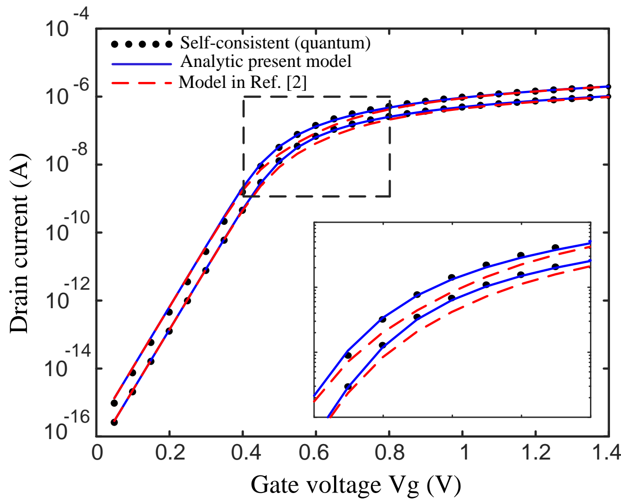


Figure 3: Comparison of drain current I_{ds} vs. gate voltage V_g between self-consistent data (dots), model from [2] (red dashed line), and our present model (blue line). Our analytic present model shows a better accuracy compared to [2] at the transition region near threshold voltage (square dashed box).

IV. Results And Discussion

Before we go to the main result, we need to recall our self-consistent Poisson and Schrödinger numerical simulation that have been discussed in section 2.1. To see how quantum consideration affects the electrons inside the channel, we can simply comparing some findings in our numerical simulation of quantum self-consistent method with the classical (non quantum) one. The purpose of this comparison is simply to give a sense that quantum and classical calculation would gives a different results. One-dimensional coupled Poisson-Schrödinger solver was developed to solve equations Equation (1) and (3) using a simple finite difference method. To illustrate the difference between the classical (non-quantum) and the quantum self-consistent method, the potential profile in the form of conduction band is presented in Figure 2. (a). It is shown that in the subthreshold region, the conduction band of the silicon is essentially flat and move along with the applied gate voltage (volume inversion). It is also shown that in general the quantum self-consistent method would results more conduction band bending in the surface. The difference between these two methods can also be seen in Figure 2(b), in which the normalized electron densities are plotted for different gate voltages. It can be seen that the electrons are spread throughout the silicon cylinder and get their maximum values at different points within the cylinder depending on the gate voltage. For low gate voltages, most of the charge is located at the center. As the gate voltage rises, the inversion charge gets closer to the surface. This quantum charge distribution makes it very different with its classical counterpart which always has maximum value at the surface.

Moving to our major result, the advantage of our explicit model would be discussed in detail. A direct way to understand the accuracy of our I-V model is to compare it with I-V model obtained from self-consistent calculation. We also can compare our result to the work done by [2] since as we have mentioned before that Roldan et. al. [2] had previously proposed an I-V compact model that include quantum confinement effects for SG-MOSFETs. Figure 3 gives the drain current (I_{ds}) vs. gate voltage (V_g) calculated by our present model with those obtained from self-consistent simulation and also the model from [2] to show the accuracy of the model. For calculation of the present model, we use the procedures described in section 3 from step (a) to (e) thoroughly. It is shown that our model and model from [2] is in a good agreement with the self-consistent calculation. Compared to the model from [2], our model has a better behavior in the area around threshold voltage. This is due to the characteristics of classical charge control based approach proposed by Iniguez et.al [3], which has been used by [2] as the base analytical equation. This classical charge control was described by Yu et.al. [7] as not mathematically accurate. In the other way, in our model, we used the classical potential based approach [7] as our base analytical equation. This approach has higher degree of mathematical correctness compare to charge control based approach.

V. Conclusion

An analytic compact model with quantum corrections has been developed for cylindrical surrounding-gate MOSFETs. Two distinctive quantum effects have been extracted from numerical solutions obtained from a 1-D Poisson-Schrödinger solver in the cylindrical coordinate system. First, the threshold voltage shift as a function of silicon radius has been implemented as an effective change in the gate work function. Second, quantum degradation of the gate capacitance has been expressed as a correction to the oxide capacitance by introduction of quantum inversion layer centroid. Quantum effects on the inversion charge density, capacitance and current are then incorporated into an analytic potential SG compact model by a simple procedure. The current versus voltage curves generated by compact model with quantum modifications were in excellent agreement with the self-consistent Poisson-Schrödinger simulation.

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