Conditioned Reverse Path Method on Frame Structure for Damage Detection

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Abstract—**Nonlinear system identification has received great interests from researchers especially when related to structural damage detection. Generally, damaged system tends to exhibit nonlinear characteristics. Therefore, it is essential to have a reliable method which can exploit the nonlinear characteristics for detecting damage at an early stage as an effort to ensure the integrity of structural systems. In this paper, a spectral approach called Conditioned Reverse Path (CRP) method is used to identify the nonlinear behavior thus obtain the physical meaning with the possible damage occurs in the studied system. The system chosen is a 4 degree-of-freedom frame structure tested in laboratory environment. The CRP can detect damage and extract the nonlinear coefficients if an adequate nonlinear function was provided. Smaller gap makes the structure more sensitive to damage.**

Index Terms—**Conditioned Reverse Path; Damage Detection; Nonlinear System Identification; Nonlinear Coefficient.**

I. Introduction

Structural Health Monitoring (SHM) is the term referring to the process of implementing damage detection strategy for structural systems such as civil or mechanical infrastructures. There are several types of damage; open and closed cracks under dynamic loads (or breathing crack), loose joints and friction. These "real-world" damages are assumed to change the stiffness or mass distribution of the structure which leads the systems' transition from linear to nonlinear behavior. Therefore, it is logical to associate damage with nonlinearity and exploit nonlinear system identification (NSI) process to detect damages [1, 2].

The conventional methods of H_1 and H_2 are at disposal with the presence of nonlinearity. The Reverse Path (RP) formulation is a spectral approach which offers simple calculation and instinctive interpretation regarding systems with nonlinearity [3]. The RP method works by mathematically reversing the input-to-output path of the given system and applies spectral analysis to extract the underlying linear FRF [4]. The Conditioned Reverse Path (CRP) method is developed from the RP method and generalized to multi degree-of-freedom (DOF) systems [5]. The CRP works by separating the nonlinear elements of the response and finding the true FRF matrix of the underlying linear system utilizing conditioned spectral analysis. The CRP method has been proved to be efficient and has been applied tremendously in NSI area [6-11], so far none has integrate this method with damage detection. This is may be due to the need of specifying the nonlinear terms *a priori* in the studied system's equation of motion.

Recently, there are some developments of method in damage detection. The frequency- and mode shape-based damage detection (FBDD and MBDD) methods locate damages from changes in natural frequency and modal strain energy, respectively [12]. Cyclostationary method uses the stochastic process to calculate the magnitude of frequency of breathing cracks [13]. The frequency shift path (FRESH) method applies the discrete time Fourier transform (DTFT) to obtain the frequency shifting and amplitude changing thus create a damage index called the FRESH curvature [14]. These methods are capable to detect nonlinearity and damage, only the process is quite complicated.

Spectral approach is easier to implement since the frequency response function (FRF) can give direct interpretation of information about the system's response. In this paper, a few steps have been suggested to tackle the challenge since the CRP is a spectral approach and has a great potential in damage detection process. The proposed steps are validated on experimental data-sets from three-story aluminum frame structure with random excitation.

II. Experimental

A. Experimental Setup

The data used in this study is the experimental data sets from a nonlinear 4-DOF frame structure tested at the Los Alamos National Laboratory (LANL). There are many studies on this data set that have been previously published [15-18]. A more detailed report on this LANL test setup is available in [19].

Figure 1 shows the experimental setup of a three-story shear-building structure which consists of four aluminum plates (0.305 x 0.305 x 0.025 m) and four aluminum columns (0.177 x 0.025 x 0.006 m) at each floor. The columns were assembled to the plates using bolted joins forming a 4-DOF system which only moves in the *y*-direction. Another center column (0.15 x 0.025 x 0.025 m) and an adjustable bumper were introduced in the system to simulate damage by inducing nonlinear behavior when impacted during excitation. The gap between the center column and bumper was adjusted accordingly to simulate different severity of damage.

A shaker was used to excite the structure at the base floor with a band-limited random base excitation of 20-150 Hz to avoid rigid body modes that present below 20 Hz. One force sensor (Channel 1) and four accelerometers (Channel 2 to 5) were mounted at the centerline of each floor to measure the input force and the system's responses, respectively. The data was processed using data acquisition system and the signals were discretized into 8192 number of data with a sampling frequency of 320 Hz and the time interval was taken as 3.125 ms.

Five different structural conditions are considered in this paper. The first case is the undamaged condition and there were no impacts between the center column and bumper during excitation. The remaining cases are the damaged conditions and four different types of damages were used here (see, Table 1). The different gaps were meant to simulate "real-world" damage with different severities which stimulate the transitions from linear to nonlinear response of a system. The gap in the frame structure represents a breathing crack or loose joint that clatters under dynamic loads.

Table 1
Summary of structural state conditions

Label	Condition	Description
Case 1	Undamaged	Baseline condition
Case 2	Damaged	Gap 0.20 mm
Case 3	Damaged	Gap 0.15 mm
Case 4	Damaged	Gap 0.10 mm
Case 5	Damaged	Gap 0.05 mm

B. Identification of the nonlinear frame structure

The steps taken for identifying and quantifying nonlinearity of the 4-DOF frame structure is summarized in Figure 2.
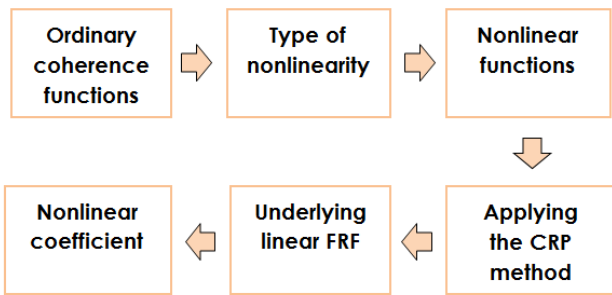

Figure 2: Block diagram of the methodology

The CRP method has been well documented in many publications [5-10] thus only a brief summary is presented here. The vibrations of a general nonlinear system are governed by the following equation where $B(\omega)$ is the linear dynamic stiffness matrix, $Y(\omega)$ and $X(\omega)$ are the Fourier transforms of output and input signal, respectively. The term Z_j is the nonlinear function vector and A_j is the coefficients of the nonlinear terms. The spectrum will be conditioned and the linear FRF $H(\omega)$ can be estimated. The nonlinear coefficient can now be computed using the equation below.

$$B(\omega)Y(\omega) + \sum_{j=1}^{n} A_j Z_j(\omega) = X(\omega) \qquad (1)$$

$$A_j^T H^T = S_{ii(-1:j-1)}^t \left(\begin{array}{c} S_{iX(-1:j-1)} H^T - S_{iY(-1:j-1)} \\ - \sum_{j=i+1}^{n} S_{ii(-1:j-1)} A_j^T H^T \end{array} \right) \qquad (2)$$

The ordinary coherence functions for each condition are also calculated in this paper to ensure that nonlinearity is present in the data sets. The ordinary coherence function is given as [20]

$$\gamma^2_{XY}(\omega) = |S_{XY}(\omega)|^2 / S_{XX}(\omega) S_{YY}(\omega) \qquad (3)$$

where S_{XY} is the cross-spectral density function, while S_{XX} and S_{YY} are the auto spectral density functions of the input and output vectors, accordingly. The function is always between 0 and 1 thus may be considered as a measure of model accuracy.

The type of nonlinearity will be identified by plotting the excitation force (Channel 1) versus the displacement of each floor. There are significant forms of nonlinearity which are commonly seen in structural engineering [21, 22]. Figure 3 shows the idealized forms of simple structural nonlinear force curves with their known names. Based on this forms, the type of nonlinearity for the 4-DOF aluminum frame structure could be identified.

Once the type of nonlinearity has been identified, the nonlinear function of the system will be investigated. As the first attempt to model the nonlinearity, a grounded symmetrical nonlinearity of type $|y|^\alpha$ sign (y) was used in the spectral analysis [23].

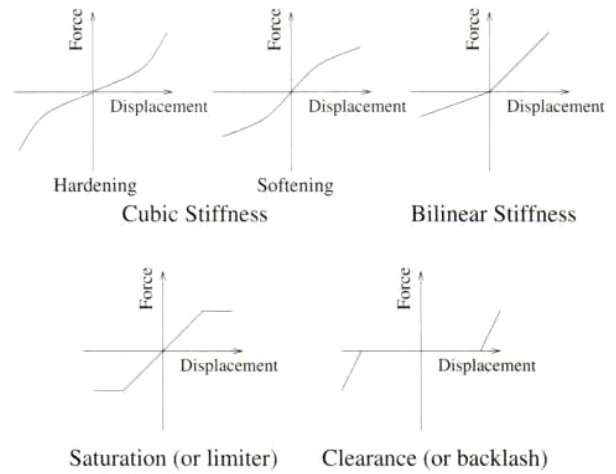$$z_1(y) = a|y|^3 \text{ sign } (y) \qquad (4)$$

Figure 3: Idealized forms of structural nonlinearity

There are good physical fundamentals to consider the nonlinearity to be cubic, therefore the exponent of the nonlinear term used in the spectral analysis was taken as $\alpha = 3$. Now that the parameters needed in the spectral analysis is adequate, the data sets of 4-DOF frame structure will be conditioned thus identify the underlying linear FRF and nonlinear coefficients using the CRP method. The nonlinear coefficients are frequency dependent, therefore a spectral mean need to be calculated to find the single value for the coefficients.

III. RESULT AND DISCUSSION

Figure 4 shows the results of ordinary coherence functions for every condition. As stated previously, the frequency of interest is chosen between 20 to 160 Hz to avoid rigid body modes that present below 20 Hz. From Figure 4, the nonlinearity has interrupted the coherence functions for the damaged conditions and significant drops can clearly be seen at frequency range between 50 to 80 Hz. It is observed that as the damage severity decreases, the coherence functions are getting more interrupted in the low frequency range. This is due to the rapid repetition of impact between the center column and the bumper when the gap is the smallest. The

frame structure is most sensitive to damage when excited within the low frequency range.

The plot between excitation force and the displacement for mass 4 (where nonlinearity exists) at every condition is shown in Figure 5. The displacement of mass 4 for Case 1 (undamaged) is relatively small compared to the other conditions. It is expected that the largest displacement is displayed when the gap is the smallest (Case 5), however from Figure 5 it shows that the displacement is the largest at gap 0.15 mm (Case 3). It is assumed that the frame structure's material contributes to this behavior. If this force versus displacement plot is being compared with the idealized forms in Figure 3, it is understood that the nonlinearity induced in the 4-DOF frame structure is the cubic type nonlinearity. After the verification of the nonlinearity type, the suggested function of nonlinearity (Equation 4) was used in the CRP formulation.
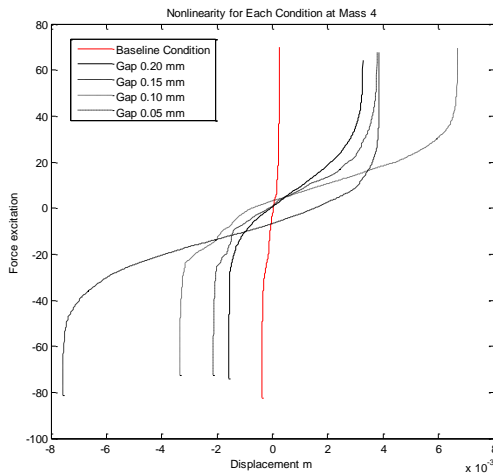

Figure 5: The excitation force versus the displacements of mass 4 for each condition

The conditioned spectral analysis using the CRP method was done to obtain the underlying linear FRF $H_1(\omega)$ thus calculates the nonlinear coefficient a. The underlying linear frequency response functions for each condition are shown in Figure 6. The FRFs extracted from the CRP method are able to distinct three modes from the data sets and the trends for every condition are in a good agreement. The resonance frequencies for the conditioned FRF are 31, 55 and 72 Hz, respectively. However, some bias (artefacts) can be seen in the FRF especially at the higher frequency range and they are slightly shifted when compared to the undamaged linear FRF (Case 1). It is believed that the nonlinear function chosen was not fitting with the damage under study. It is possible that there is more than one nonlinearity occurs in the frame structure and a more suitable nonlinear function need to be investigated in the future works.

Another reason that may cause the slight shift in the underlying linear system and the bias is the absence of the displacement data from the experimental stage which is required in the nonlinear equation. Based on experience, the displacement data measured from the displacement sensor and the displacement data obtained by integrating the acceleration data produce different values. The data measured directly from the displacement sensor is more accurate and may produce better nonlinear function vector *Z*. The displacement data is not available in the LANL test set up, hence new sets of experimental testing is required if the displacement data is needed.
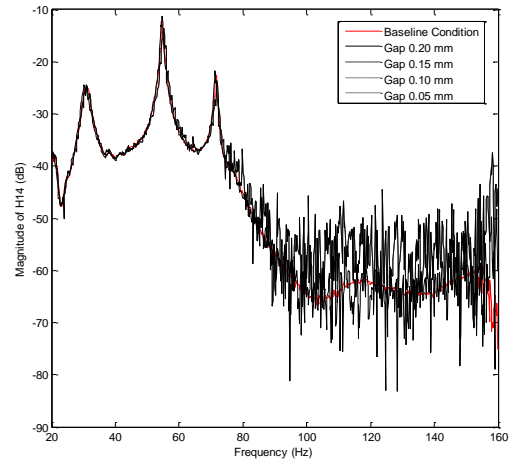

Figure 6: The calculated FRFs from the CRP method using grounded symmetrical nonlinear function

The nonlinear coefficients for damaged cases (undamaged Case 1 was excluded) were calculated using the underlying linear FRF obtained from the CRP. Although the FRF contains some bias, it is anticipated to test the developed formulation of the CRP to estimate the nonlinear coefficients hence correlates the value with the physical damage. As stated previously, a spectral mean need to be calculated to obtain the single value of the estimated coefficients and the results were summarized in Table 2. It is observed that the nonlinear coefficients are getting smaller with decreasing gaps. The negative values of nonlinear coefficients might imply that the physical damage in the frame structure is quite severe. The smaller the gap, the more sensitive the frame structure towards damage.

Table 2
Estimated nonlinear coefficients for damaged conditions from the CRP method

Label	Condition	Nonlinear Coefficient a
Case 2	Gap 0.20 mm	$4.06 \times 10^{10} - i\ 7.97 \times 10^{10}$
Case 3	Gap 0.15 mm	$1.25 \times 10^{10} - i\ 2.09 \times 10^{10}$
Case 4	Gap 0.10 mm	$-6.02 \times 10^{9} - i\ 2.73 \times 10^{10}$
Case 5	Gap 0.05 mm	$-6.12 \times 10^{9} + i\ 1.05 \times 10^{9}$

As stated previously, the different gaps were intended to simulate the "real-world" damage. In this case, the gap in the frame structure represents a breathing crack or loose joint that clatters under dynamic loads. To correlate the nonlinear coefficients obtained with the breathing cracks, a large breathing crack gives little effect to the nonlinear behavior of the system as the impact when the crack is open and close under loading is small. On the other hand, a small breathing crack impacts more frequently when the crack opens and closes under loading. This contributes to the severe nonlinearity behavior and gives the large negative value of nonlinear coefficient.

IV. Conclusion

Steps of identifying unknown nonlinearity forms and application of nonlinear system identification using the CRP was proposed and validated on LANL data sets of a 4-DOF three-story aluminum frame structure. Several conclusions can be made from present study.
- The steps proposed in this paper are able to detect and identify the type of nonlinearity present in the frame

structure.

- The nonlinear function used in this study was not adequate to model the damage under study. The possibility of the structure containing several nonlinearities should be considered when choosing a suitable nonlinear function. The exponent of the nonlinear function may have affected the performance of the CRP. Further investigations on the relationship between cumulative coherence function [24] and exponent of nonlinear function will be done in the future.

- The capability of the CRP method was not fully exploited in this study since the nonlinear function could not be clearly identified. A more vigorous work is planned in order to correctly model the damage and obtain unbiased underlying linear FRF.

- The nonlinear coefficients gave certain value corresponding to the severity of physical damage in the frame structure. The coefficients can be improved by calculating the spectral mean from a frequency range with less bias. The negative value may imply different meaning, therefore more numerical study will be conducted in the future to verify the claims.

Present works will be continued to tackle the difficulties faced in this study on identifying unknown nonlinearity forms. Once the correct nonlinear function could be identified, it is planned to test another nonlinear system identification algorithm which was recently developed named Orthogonalised Reverse Path (ORP) method [3, 9-11]. The ORP method is a time domain approach and it also has the potential as a damage-sensitive feature. It is expected that the ORP method can give the same good performance as the CRP method.

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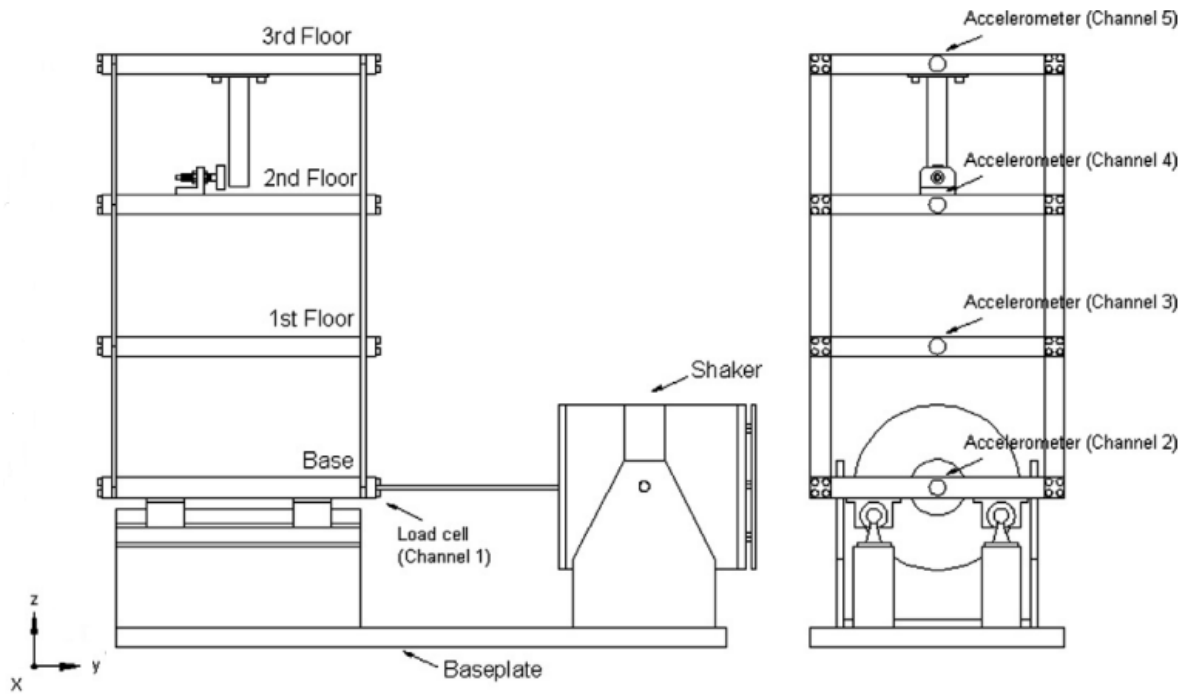
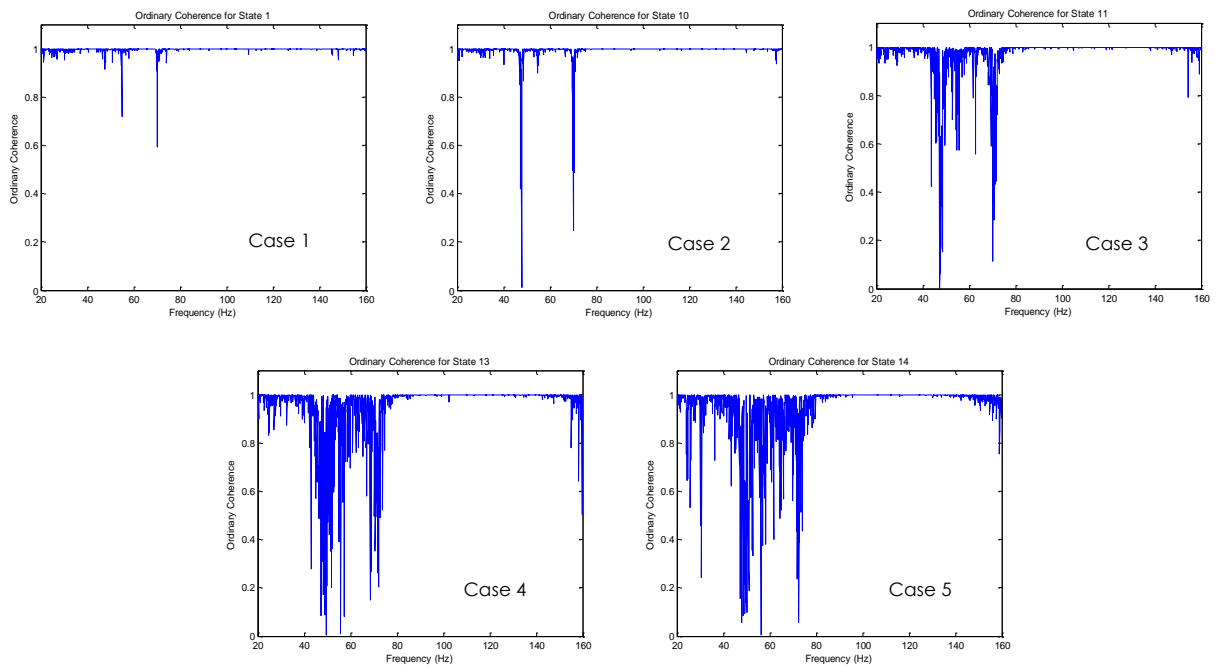Figure 1: Schematic drawing of LANL 4 degree-of-freedom frame structure

Figure 4: Ordinary coherence functions for every structural condition