# Simultaneous Computation of Model Order and Parameter Estimation for Arx Model Based on Single Swarm and Multi Swarm Simulated Kalman Filter

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***Abstract*—Motivated by the estimation capability of Kalman filter, a new meta-heuristic optimization algorithm known as Simulated Kalman Filter (SKF) has been introduced recently. According to the components of Kalman filtering, which includes prediction, measurement, and estimation, the global minimum/maximum can be estimated. Measurement process, which is needed in Kalman filtering, is mathematically modeled and simulated. Agents interact among them to modify and enhance the solution throughout the search process. Simultaneous Model Order and Parameter Estimation (SMOPE) and Simultaneous Model Order and Parameter Estimation based on Multi Swarm (SMOPE-MS) are two techniques of implementing meta-heuristic algorithm to iteratively establish an optimal model order and parameters simultaneously for an unknown system. The performance of SMOPE and SMOPE-MS has been examined through the utilization of Particle Swarm Optimization (PSO) and Gravitational Search Algorithm (GSA). The objective of this paper is to test the effectiveness of SKF in solving system identification problem throughout SMOPE and SMOPE-MS. Experiments are conducted on six system identification problems. The obtained outcomes showed that the performance of SMOPE-MS(SKF) is better than SMOPE (SKF).**

***Index Terms*—Simulated Kalman Filter; Single Swarm; Multi Swarm; System Identification.**

## I. INTRODUCTION

Meta-heuristic optimization algorithms are well-established techniques to address those problems which are difficult to solve through traditional optimization methods.

Among the various kinds of optimization algorithms, Simulated Kalman Filter (SKF) is a new population-based optimization algorithm based on estimation method of Kalman Filter which has been recently introduced by Ibrahim *et al*. [1] in 2015.

System identification is a method employed to obtain a mathematical model of a system by performing analysis on input-output behaviour of the system. Fundamental steps of system identification procedure are generally summarized into four main stages. The primary stage is collection of experimental data. Following that, the model order is selected. The next stage is to approximate the parameters of the model and lastly, the mathematical model is validated.

Auto-Regressive Model with Exogenous Inputs (ARX) is the most basic model in linear black box identification [2]. Conventionally, in addressing the system identification problem of ARX model, the model order selection and parameter estimation are done separately.

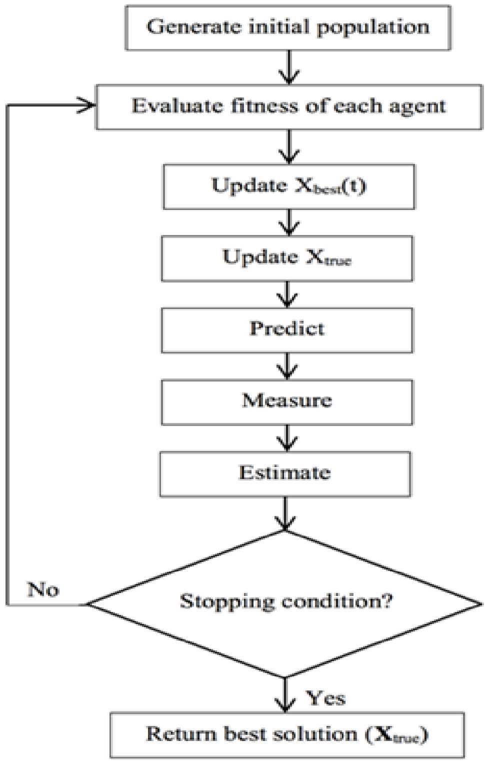


Figure 1: The Simulated Kalman Filter (SKF) algorithm

There are some techniques reported in literature in solving system identification problems. Hansson *et al*. presented a subspace system identification method based on weighted nuclear norm approximation [3,4]. Moreover, there are some methods proposed to address system identification problem based on meta-heuristic algorithm but it mainly focus on parameter estimation only [5,6].

Simultaneous Model Order and Parameter Estimation (SMOPE) was proposed to address system identification problem efficiently using meta-heuristics algorithms [7]. The technique enabled the computation of model order and parameters values to be done concurrently. This is achievable through the way the problem is encoded in the search agents.

Furthermore, SMOPE could also successfully be adapted to fit with other meta-heuristic algorithm like Gravitational Search Algorithm (GSA) [8].

A new computation model termed as Simultaneous Model Order and Parameter Estimation based on Multi-Swarm approach (SMOPE-MS) is proposed by Mohd Azmi *et al* [9] to improve the capability of SMOPE. The strategy is by assigning each swarm of meta-heuristic algorithm to each model order of ARX mathematical equation. The results reported that the performance of SMOPE-MS is better than original SMOPE in term of solution quality.

The performance of SMOPE and SMOPE-MS has been examined through the utilization of Particle Swarm Optimization (PSO) and Gravitational Search Algorithm (GSA) only, and no reported that SKF has been used in both techniques. Therefore, in this paper, the implementation of SMOPE and SMOPE-MS based on SKF is studied and compared. Six ARX system identification problems are used for verification. The results showed that the performance of SMOPE-MS(SKF) is better than SMOPE (SKF).

The remainder of this paper is organized as follows: Section 2 reviewed the SKF algorithm. Section 3 explains the SMOPE and SMOPE-MS technique based on SKF respectively. Section 4 and Section 5 provide the experimental settings and discusses the experimental results respectively. Section 6 concludes the paper.

## II. SIMULATED KALMAN FILTER ALGORITHM

The simulated Kalman filter (SKF) algorithm is shown in Figure 1. Regard $n$ number of agents, SKF algorithm starts with initialization of $n$ agents, in which the states of each agent are given randomly. The maximum number of iterations, $t_{max}$, is defined. The initial value of error covariance estimate, $P(0)$, the process noise value, $Q$, and the measurement noise value, $R$, which are required in Kalman filtering, are also defined during initialization stage. Then, every agent is subjected to fitness evaluation to produce initial solutions $\{X_1(0), X_2(0), X_3(0), \dots, X_{n-2}(0), X_{n-1}(0), X_n(0)\}$. The fitness values are compared and the agent having the best fitness value at every iteration, $t$, is stored as $X_{best}(t)$. For function maximization problem,

$$X_{best}(t) = \max_{i \in 1,\dots,n} fit_i(X(t)) \tag{1}$$

whereas, for function minimization problem,

$$X_{best}(t) = \min_{i \in 1,\dots,n} fit_i\big(X(t)\big) \tag{2}$$

The-best-so-far solution in SKF is registered as $X_{true}$. The $X_{true}$ is updated only if the $X_{best}(t)$ is better ($X_{best}(t) < X_{true}$ for minimization problem, or $X_{best}(t) > X_{true}$ for maximization problem) than the $X_{true}$.

The next calculations are according to the predict-measure-estimate steps in Kalman filter. In the prediction step, the following time-update equations are computed.

$$X_i(t|t) = X_i(t) \tag{3}$$

$$P(t|t) = P(t) + Q \tag{4}$$

where $X_{i(t)}$ and $X_{i(t|t)}$ are the previous state and transition/predicted state, respectively, and P(t) and P(t|t) are previous error covariant estimate and transition error covariant estimate, respectively. Note that the error covariant estimate is affected by the process noise, Q.

The next stage is measurement, which act as feedback to estimation process. Measurement is modeled such that its output may take any value from the predicted state estimate, $X_{i\,(t|t)}$, to the true value, $X_{true}$. Measurement, $Z_{i(t)}$, of each individual agent is simulated according to the following equation:

$$Z_i(t) = X_i(t|t) + \sin(rand \times 2\pi) \times |X_i(t|t) - X_{true}| \tag{5}$$

The $\sin(rand \times 2\pi)$ term provides the stochastic aspect of SKF algorithm and $rand$ is a uniformly distributed random number in the range of [0,1].

The final stage is the estimation. During this stage, Kalman gain, $K(t)$, is calculated as follows:

$$K(t) = \frac{P(t|t)}{P(t|t) + R} \tag{6}$$

Then, the estimation of next state, $\boldsymbol{X_i}(t+1)$, is computed based on Equation 7.

$$X_i(t+1) = X_i(t|t) + K(t) \times (Z_i(t) - X_i(t|t)) \tag{7}$$

and the error covariant is updated based on Equation 8.

$$P(t) = \big(1 - K(t)\big) \times P(t|t) \tag{8}$$

Lastly, the next iteration is executed until the maximum number of iterations, $t_{max}$, is reached.

## III. SIMULTANEOUS COMPUTATION OF MODEL ORDER AND PARAMETER ESTIMATION BASED ON SINGLE SWARM AND MULTI SWARM SIMULATED KALMAN FILTER

Contrary to other system identification techniques, SMOPE and SMOPE-MS obtain the optimal system order and the parameters values simultaneously. The key of these techniques is the encoding of the search agents. For that reason, by applying same encoding, SMOPE and SMOPE-MS can simply be integrated to other meta-heuristic algorithms. The agent's encoding employed in SMOPE and SMOPE-MS is shown in Table 1 and Table 2 respectively.

Table 1
Agent encoding for SMOPE

| Dimension | 1 | 2 | 3 | … | D+1 | D+2 | D+3 | … | 2D+1 |
|---|---|---|---|---|---|---|---|---|---|
| Variable in ARX | Order, *n* | $a_1$ | $a_2$ | … | $a_D$ | $b_1$ | $b_2$ | … | $b_D$ |

Table 2
Agent encoding for SMOPE-MS

| Dimension | 1 | 2 | 3 | … | D | D+1 | D+2 | D+3 | … | 2D |
|---|---|---|---|---|---|---|---|---|---|---|
| Variable in ARX | $a_1$ | $a_2$ | $a_3$ | … | $a_9$ | $b_1$ | $b_2$ | $b_3$ | … | $b_9$ |

Each of the agents in SMOPE represents the ARX parameters values. Assuming maximum system order under consideration is $D$, the agents dimension should be $2D$+1. The first dimension of each agent represents the system order, $n$, while second dimension to $D$+1 represents the possible values of poles parameters and dimension $D$+2 to 2$D$+1 are reserved for the zeros parameters.

Different with SMOPE, the dimension used in SMOPE-MS is 18 instead of 19. The SMOPE-MS does not required to tune the model order before continuing with fitness evaluation of possible ARX mathematical model such in SMOPE. This computation model has already assigned specific swarm to corresponding model order and its related ARX mathematical model. Assuming maximum system order under consideration is $D$, the agents dimension should be 2$D$. The first dimension to $D$ represents the possible values of poles parameters and dimension $D$+1 to 2$D$ are reserved for the zeros parameters.

The transfer function of ARX model used in SMOPE and SMOPE-MS is as follow:

$$G(z)=\frac{Y(z)}{U(z)}=\frac{b_1z^{-1}+b_2z^{-2}+\ldots+b_mz^{-m}}{1+a_1z^{-1}+a_2z^{-2}+\ldots+a_nz^{-n}} \quad (9)$$

where $m$ and $n$ are the number of numerator and denominator orders of the transfer function respectively and $a_n$ and $b_m$ are the pole and zero parameters that will be tuned by optimization algorithm.

In SMOPE and SMOPE-MS, maximum order of 9th is taken into account. To determine the parameter '$a$' and '$b$', the constraint $n \geq m$ is considered. This is based on the transfer function form which the order value of poles ($n$ value) must be the same or greater than the order of zeroes ($m$ value).

Table 3 specifies which ARX equation parameters should be considered for any assigned number of order, $n$. Thus, a set of 45 mathematical models are tested according to $n$ value and SKF will be employed to search for the best mathematical model.

As an example, if the model order value is selected 2, all possible mathematical models related to the second order are subjected to fitness calculation. In that case, the computations focus on two mathematical models, which are:

$$\frac{b_1z^{-1}}{1+a_1z^{-1}+a_2z^{-2}}$$

and

$$\frac{b_1z^{-1}+b_2z^{-2}}{1+a_1z^{-1}+a_2z^{-2}}.$$

Another example, if the model order is 3, then the computations involve three mathematical models, which are:

$$\frac{b_1z^{-1}}{1+a_1z^{-1}+a_2z^{-2}+a_3z^{-3}},$$

$$\frac{b_1z^{-1}+b_2z^{-2}}{1+a_1z^{-1}+a_2z^{-2}+a_3z^{-3}}$$

and

$$\frac{b_1z^{-1}+b_2z^{-2}+b_3z^{-3}}{1+a_1z^{-1}+a_2z^{-2}+a_3z^{-3}}.$$

Note that up to ninth order mathematical model of ARX is considered for the purpose of this research.

In detail, the SMOPE and SMOPE-MS begin with initialization of $n$ agents, in which the states of each agent are given randomly. Note that, for SMOPE-MS, there are 9 swarms of agents will be generated as shown in Figure 2. The maximum number of iterations, $t_{max}$, the initial value of error covariance estimate, $P(0)$, the process noise value, $Q$, the measurement noise value, $R$, are also defined during initialization stage. After the initialization stage is complete, the fitness function is evaluated as in Equation 10.

After that, $\boldsymbol{X}_{\mathbf{best}}(t)$ and $\boldsymbol{X}_{\mathbf{true}}$ are updated according to SKF algorithm. In SMOPE-MS, for each swarm, every agent is subjected to fitness evaluation, thus there will be 9 $X_{best}$ ($t$) and 9 $X_{true}$ are going to be updated.

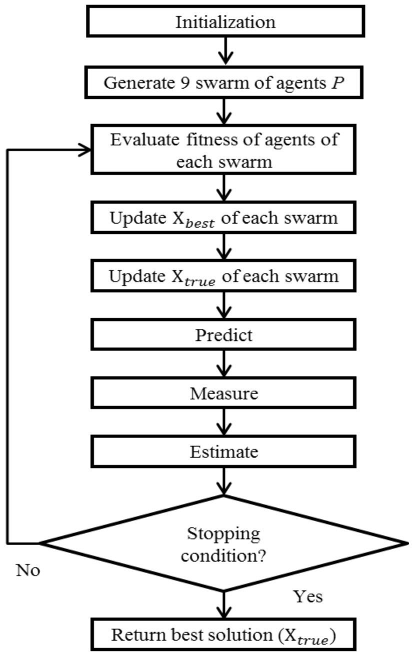


Figure 2: Flowchart of SKF for SMOPE-MS

$$best\ fit=100\left[1-\frac{norm(y_{(actual)}-y_{(estimated)})}{norm(y_{(actual)}-y_{(mean)})}\right]\% \quad (10)$$

The algorithm continues with measurement and estimation similar to SKF using Equation 5 to Equation 8. For SMOPE-MS, each swarm will generate their own measurement and estimation process occurs in respective swarm. The next iteration is executed until the maximum number of iterations, $t_{\max}$, is reached. When the algorithm process ends, the final optimum solution, $OS$, which is the best solution among 9 $X_{true}$ of each swarm is reported as shown in Equation 11.

$$OS=\max(X_{true}^{n}) \quad (11)$$

where $X_{true}^{n}$ is the true value in the $n$th swarm.

## IV. EXPERIMENTS

The experimental data used for the benchmarking, is Database for Identification of Systems, (DaISy) [10]. The data for heating system, hair dryer system, ball beam system, robot arm system and exchanger system are produced from laboratory works while the data of wing flutter system is obtained from industry.

The data is equally separated for training and testing. The training data is used to find the best mathematical model based on ARX model while the testing data is used to assess the quality of mathematical model obtained.

For each system, the numbers of data points are separated equally into the proportions of 50% for training samples and 50% for testing samples from the entire dataset. As an example, for heating system, 400 number of samples are used for training and another 400 number of samples are used for testing. The similar procedure has been employed by L. Ljung in conventional ARX [1].

Table 3
ARX parameters selected for the calculation of *best fit* ($n$=1,2,3,4,5,6,7,8,9)

| Order, $n$ | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $a_6$ | $a_7$ | $a_8$ | $a_9$ | $b_1$ | $b_2$ | $b_3$ | $b_4$ | $b_5$ | $b_6$ | $b_7$ | $b_8$ | $b_9$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | X | | | | | | | | | X | | | | | | | | |
| 2 | X | X | | | | | | | | X | | | | | | | | |
| 2 | X | X | | | | | | | | X | X | | | | | | | |
| 3 | X | X | X | | | | | | | X | | | | | | | | |
| 3 | X | X | X | | | | | | | X | X | | | | | | | |
| 3 | X | X | X | | | | | | | X | X | X | | | | | | |
| 4 | X | X | X | X | | | | | | X | | | | | | | | |
| 4 | X | X | X | X | | | | | | X | X | | | | | | | |
| 4 | X | X | X | X | | | | | | X | X | X | | | | | | |
| 4 | X | X | X | X | | | | | | X | X | X | X | | | | | |
| 5 | X | X | X | X | X | | | | | X | | | | | | | | |
| 5 | X | X | X | X | X | | | | | X | X | | | | | | | |
| 5 | X | X | X | X | X | | | | | X | X | X | | | | | | |
| 5 | X | X | X | X | X | | | | | X | X | X | X | | | | | |
| 5 | X | X | X | X | X | | | | | X | X | X | X | X | | | | |
| 6 | X | X | X | X | X | X | | | | X | | | | | | | | |
| 6 | X | X | X | X | X | X | | | | X | X | | | | | | | |
| 6 | X | X | X | X | X | X | | | | X | X | X | | | | | | |
| 6 | X | X | X | X | X | X | | | | X | X | X | X | | | | | |
| 6 | X | X | X | X | X | X | | | | X | X | X | X | X | | | | |
| 6 | X | X | X | X | X | X | | | | X | X | X | X | X | X | | | |
| 7 | X | X | X | X | X | X | X | | | X | | | | | | | | |
| 7 | X | X | X | X | X | X | X | | | X | X | | | | | | | |
| 7 | X | X | X | X | X | X | X | | | X | X | X | | | | | | |
| 7 | X | X | X | X | X | X | X | | | X | X | X | X | | | | | |
| 7 | X | X | X | X | X | X | X | | | X | X | X | X | X | | | | |
| 7 | X | X | X | X | X | X | X | | | X | X | X | X | X | X | | | |
| 7 | X | X | X | X | X | X | X | | | X | X | X | X | X | X | X | | |
| 8 | X | X | X | X | X | X | X | X | | X | | | | | | | | |
| 8 | X | X | X | X | X | X | X | X | | X | X | | | | | | | |
| 8 | X | X | X | X | X | X | X | X | | X | X | X | | | | | | |
| 8 | X | X | X | X | X | X | X | X | | X | X | X | X | | | | | |
| 8 | X | X | X | X | X | X | X | X | | X | X | X | X | X | | | | |
| 8 | X | X | X | X | X | X | X | X | | X | X | X | X | X | X | | | |
| 8 | X | X | X | X | X | X | X | X | | X | X | X | X | X | X | X | | |
| 8 | X | X | X | X | X | X | X | X | | X | X | X | X | X | X | X | X | |
| 9 | X | X | X | X | X | X | X | X | X | X | | | | | | | | |
| 9 | X | X | X | X | X | X | X | X | X | X | X | | | | | | | |
| 9 | X | X | X | X | X | X | X | X | X | X | X | X | | | | | | |
| 9 | X | X | X | X | X | X | X | X | X | X | X | X | X | | | | | |
| 9 | X | X | X | X | X | X | X | X | X | X | X | X | X | X | | | | |
| 9 | X | X | X | X | X | X | X | X | X | X | X | X | X | X | X | | | |
| 9 | X | X | X | X | X | X | X | X | X | X | X | X | X | X | X | X | | |
| 9 | X | X | X | X | X | X | X | X | X | X | X | X | X | X | X | X | X | |
| 9 | X | X | X | X | X | X | X | X | X | X | X | X | X | X | X | X | X | X |

In the SMOPE and SMOPE-MS based on SKF, each agent determines a suitable model order and parameters of the ARX model from first order up to ninth order. The parameters setting used in this study are shown in Table 7. The algorithm will stop when the iteration count exceeds 2000. Each of the experiment is repeated 50 times and the results are averaged.

## V. Result and Discussion

Using MATLAB for simulation, the results obtained from the experiment are shown in Table 4 and Table 5. Table 4 shows the summary result of SMOPE (SKF) for all six dataset while Table 5 shows result obtained based on SMOPE-MS (SKF). The comparison between these two algorithms can be based on the average best fit value acquired at the testing stage as shown in Table 6. Based on the result, it clearly shows that the performance of SMOPE-MS (SKF) is better than SMOPE (SKF) for all six dataset.

Table 4
SKF parameters value

| Parameters | Value |
|---|---|
| Number of agents | 100 |
| Initial error covariance estimate, $P(0)$ | 1000 |
| Process noise, $Q$ | 0.5 |
| Measurement noise, $R$ | 0.5 |
| Number of iterations | 2000 |
| Number of run | 50 |

Table 5
Summary result of SMOPE (SKF) for six dataset

| Data Set | Best Fit (Training) (%) | | Average Best Fit (Training) (%) | STDEV (Training) | Best Fit (Testing) (%) | | Average Best Fit (Testing) (%) | STDEV (Testing) |
|---|---|---|---|---|---|---|---|---|
| | Min | Max | | | Min | Max | | |
| Heating System | 96.88 | 98.87 | 98.32 | 0.52 | 94.98 | 98.36 | 97.37 | 0.87 |
| Hair Dryer System | 67.44 | 93.86 | 82.04 | 6.11 | 65.70 | 93.67 | 80.96 | 6.49 |
| Ball Beam System | 92.92 | 97.31 | 96.33 | 1.04 | 90.32 | 97.76 | 96.15 | 1.84 |
| Robot Arm System | 79.83 | 91.13 | 86.39 | 2.34 | 79.05 | 90.81 | 85.88 | 2.43 |
| Wing Flutter System | 84.85 | 96.67 | 92.87 | 2.59 | 66.10 | 89.26 | 81.75 | 5.32 |
| Exchanger System | 58.40 | 80.92 | 77.13 | 3.81 | 0.04 | 50.22 | 42.34 | 8.71 |

Table 6
Summary result of SMOPE-MS (SKF) for six dataset

| Data Set | Best Fit (Training) (%) | | Average Best Fit (Training) (%) | STDEV (Training) | Best Fit (Testing) (%) | | Average Best Fit (Testing) (%) | STDEV (Testing) |
|---|---|---|---|---|---|---|---|---|
| | Min | Max | | | Min | Max | | |
| Heating System | 98.51 | 99.02 | 98.80 | 0.12 | 97.68 | 98.66 | 98.24 | 0.24 |
| Hair Dryer System | 84.05 | 94.81 | 92.65 | 2.08 | 82.81 | 94.82 | 92.36 | 2.26 |
| Ball Beam System | 96.95 | 97.34 | 97.16 | 0.10 | 97.21 | 97.79 | 97.55 | 0.14 |
| Robot Arm System | 89.15 | 95.00 | 91.74 | 1.37 | 88.80 | 94.88 | 91.48 | 1.42 |
| Wing Flutter System | 94.34 | 97.33 | 95.72 | 0.70 | 78.38 | 92.03 | 86.38 | 2.81 |
| Exchanger System | 77.21 | 80.97 | 79.70 | 0.78 | 43.00 | 51.17 | 48.28 | 1.68 |

Table 7
Average best fit value comparison at testing stage between SMOPE (SKF) and SMOPE-MS (SKF)

| Data Set | SMOPE (SKF) | SMOPE-MS (SKF) |
|---|---|---|
| Heating System | 97.37 | 98.24 |
| Hair dryer System | 80.96 | 92.36 |
| Ball beam System | 96.15 | 97.55 |
| Robot arm System | 85.88 | 91.48 |
| Wing flutter System | 81.75 | 86.38 |
| Exchanger System | 42.34 | 48.28 |

## VI. CONCLUSION

This paper intends to test the effectiveness of SKF in solving system identification problem throughout SMOPE and SMOPE-MS. The overall performance is evaluated based on six case studies. According to the experimental results, it was observed that the SMOPE-MS (SKF) has better performance compared to SMOPE (SKF). For future research, different optimization algorithm shall be considered to validate further this finding.

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## REFERENCES

[1] Z., Ibrahim, N.H, Abdul Aziz, N. A., Ab Aziz, S., Razali, M.I., Shapiai, S.W., Nawawi,, and M.S., Mohamad. A Kalman filter approach for solving unimodal optimization problems. 2015. ICIC Express Letters. 9(12): 3415-3422.

[2] L. Ljung. 1999. System identification theory for the user. 2nd ed. Prentice Hall. CA: Linkoping University Sweden.

[3] A., Hansson , Z., Liu, and L., Vandenberghe. 2012. Subspace system identification via weighted nuclear norm optimization. Proceedings of the 51st IEEE Conference on Decision and Control. Maui, HI: IEEE. pp. 3439-3444.

[4] Z., Liu, A., Hansson and L., Vandenberghe. 2013. Nuclear norm system identification with missing inputs and outputs. Systems and Control Letters. 62(8): 605-612.

[5] B., Luitel and G.K.,Venayagamoorthy. 2010. Particle swarm optimization with quantum infusion for system identification. Engineering Applications of Artificial Intelligence. 23(5): 635-649.

[6] P., Upadhyay, R., Kar, D., Mandal and S.P., Ghoshal. 2014. IIR system identification using differential evolution with wavelet mutation. Engineering Science and Technology, an International Journal. 17(1): 8-24.

[7] Z., Ibrahim , S.W., Nawawi, S., Razali , B., Muhammad, K.Z., Mohd Azmi , Z., Aspar and N.A., Ab Aziz. 2015. Simultaneous computation of model order and parameter estimation of a heating system based on particle swarm optimization for autoregressive with exogenous model. ICIC Express Letters. 9(4): 1159–1165.

[8] K.Z., Mohd Azmi, Z., Ibrahim, D., Pebrianti, S.W., Nawawi and Ab N.A., Aziz. 2015. Simultaneous computation of model order and parameter estimation of a heating system based on gravitational search algorithm for autoregressive with exogenous inputs. ARPN Journal of Engineering and Applied Sciences. 10(2): 633–642.

[9] K.Z., Mohd Azmi, Z., Ibrahim Z. and D., Pebrianti. 2015. Simultaneous computation of model order and parameter estimation for ARX model based on multi-swarm particle swarm optimization. ARPN Journal of Engineering and Applied Sciences. 10(22): 17191–17196.

[10] http://www.esat.kuleuven.ac.be/sista/daisy.