Multiple Symbol Double Differential Transmission for Amplify-and-Forward Cooperative Diversity Networks in Time-Varying Channel

Sylvia Ong Ai Ling, Hushairi Zen, Al-Khalid Othman, Khairuddin Ab Hamid
Faculty of Engineering, Universiti Malaysia Sarawak
sylviaong7121gmail.com

***Abstract*— In the cooperative diversity wireless networks, the task to perform cooperation communication amongst neighbouring nodes is very challenging. Subjected to rapidly increasing mobility of the nodes i.e. wireless devices in fast moving vehicles and trains, at the destination end the receiver may not ideally estimate the channel characteristics and frequency offsets. Due to these circumstances which results in time-varying channels, the performance network degrades drastically. In order to enhance the performance in such environment, Double Differential (DD) modulation employing multiple symbol based detection is proposed which takes mobility environment of different nodes into consideration. By utilizing the DD transmission approach, the channel properties and frequency offset estimation is omitted in the amplify-and-forward cooperative networks. The MATLAB simulation and numerical analysis on Bit Error Rate (BER) are carried out with consideration on considering flat-fading (i.e. the frequency non-selective) Rayleigh channels and when frequency offsets. The results depict that the proposed method over fading channels without channel estimation requirements and in the presence of frequency offsets performs better as compared to the conventional DD transmission. Optimized power allocation is also carried out to enhance the network performance by minimizing the BER analytical expression. It is demonstrated that the proposed power allocation scheme offers enhancement over the equally distributed power allocation approach.**

***Index Terms*— Double Differential Transmission; Frequency Offsets; Multiple-Symbol Differential Detection; Sphere Decoding.**

I. INTRODUCTION

Cooperative or relay communications are an active area of research and have been widely explored in wireless communication systems. In cooperative communications, users exploit the broadcast characteristic of the wireless channel by cooperating among each other to transmit messages. Owing to its capability to provide untethered connectivity, coverage enhancement, throughput and mobile access, this technology has been studied and integrated into many modern wireless applications, such as cognitive radio and has progressed toward the next generation wireless communication standards, such as Long Term Evolution Advanced (LTE-A), WiMAX IEEE 802.16j and IEEE 802.11s. Accordingly, transmission reliability and high throughput will be a strong demand to support the high speed communication systems (e.g. mobile users in high speed rail system). Yet, in real-world wireless networks, the performance of the networks is greatly influenced by the channel fading and the mobility of the users.

In general, majority of the existing literatures assume that the relay(s) and destination channel coefficients knowledge are perfectly estimated (i.e. coherent detection). But, it is a difficult as well as challenging task for the nodes to estimate the channel perfectly in practice, particularly in time-varying environment due to the variation effects of the communication channels. Thus, non-coherent detection is introduced by exploiting the benefits of bypassing channel knowledge in the networks or in circumstances for which the channel knowledge is inaccessible. Single Differential (SD) transmission scheme is investigated in [1–4] and these schemes, the channel is fixed for at least two symbol periods. However, in the presence of frequency offsets the channel is unable to maintain fixed over the two symbol periods because of the mismatch of the source and receiver oscillator or relative movement among the nodes in time-varying channel and as a result the networks' performance is affected substantially [5]. There are two noteworthy methods to overcome the effect of frequency offset in the wireless communication channels.

One way to deal with the effect of frequency offset is to design excellent estimators that estimate and then compensate for the frequency offset as investigated in [5–9]. These schemes, however, increase the computational complexity and reduce the data rates transmission due to the use of extra pilot symbols that act as the reference in the estimation process. Hence, numerous studies on Double Differential (DD) modulation for Differential Amplify-and-Forward (DAF) in [4,10,11,12] and piecewise linear decoder for Differential Decode-and-Forward (DDF) in [14] are suggested, due to its attractive characteristic which is insensitive to frequency offset. Therefore, the estimation of frequency offset is not required at the destination. The detection employing DD transmission can be achieved by using the previously received symbols without requiring the accurate channel knowledge. Cano et al. in [5] demonstrated that the DD modulation is performed at the relays where a simple heuristic detector is designed in the distributive network using diagonal space-time (ST) unitary codes. The said scheme, however, requires the knowledge of the previous source. In addition the transmission ordering protocol (i.e. the protocol that determine the ordering of the nodes, such as which nodes acts as online transmitters, the first node source in the network nodes order and the destination node) is also required so that the source acknowledges the corresponding relay[15]. As the result, the scheme may be ineffective in an ad-hoc network as the moving node may joins or terminates from the network [5].

DD transmission is further investigated in [11] for Analog Network Coding (ANC) utilizing the symbol-by-symbol based and multiple symbol based transmission by [16]. The study showed that the latter detection yields better performance, however with the cost of increased complexity. Hence sphere decoding has been devised by Lamp et al. in [17] and integrated into [18]. So far, there has been little study on the implementation of multiple symbol-based DD transmissions in a time-varying channel for AF relaying networks and this is the motivation behind the present study. The plots demonstrate that the Multiple Symbol Double Differential (MSDD) detection outperforms the conventional DD transmission with the effect of frequency offsets.

The remaining manuscript is structured accordingly: Section II introduces the system model and differential transmission. In Section III, a conventional DD transmission is presented. Section IV explains in detail the proposed MSDD with sphere decoding. Section V provides the numerical analysis for both the conventional DD and proposed MSDD schemes. The simulation and numerical analysis results in terms of BER are shown in Section VI and the final section concludes the research and analysis of the results.

II. RELATED WORK

A dual-hop relayed communication network with a direct link, which comprises of a source, a relay, and a destination nodes as portrayed in Figure 1 is considered. Assume that each node is moving in an independent time-varying Rayleigh channel. All nodes are installed with an antenna employing a half-duplex transmission approach (i.e. each node is allowed to transmit and receive signal but not simultaneously at a time). In this cooperative assisted network, source communicates with destination via relay that acts as an AF gain relay in a Time Division Multiple Access (TDMA) manner. Following the TDMA approach, assuming information bit sequence of , the transmission duration is allocated into two time slot intervals. In the first phase, the source communicates with the relay and at the same time transmitting the source signals towards the destination via the direct link. Then, for the second phase, the relay retransmits the amplified signal towards destination with source remaining idle. All transmission link is perturbed with frequency offset caused by the Doppler effect – the change in frequency between all nodes. However, with the effect of frequency offset, the channel of the differential schemes which is assumed to stay constant for at least two consecutive transmission times is violated [12].

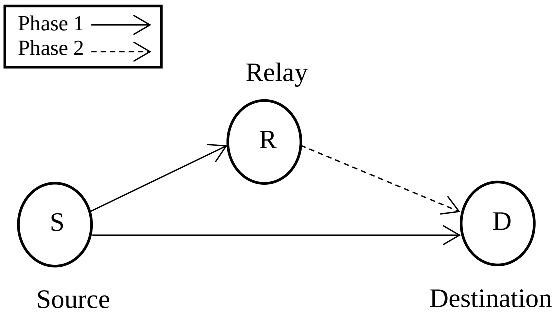


Figure 1: A Dual-Hop Relayed Network With Direct Link [19]

A. Channel Model

It is assumed that all communication links are flat-fading Rayleigh channel with the following Probability Density Function (PDF) [20] where γ denotes the instantaneous of the received SNR and $\bar{\gamma}$ represents the average SNR at the destination.

$$P_\gamma(\gamma) = \frac{1}{\bar{\gamma}} e^{-\gamma/\bar{\gamma}} \tag{1}$$

In order to characterize the fading channel, Gauss-Markov model is simple and effective as suggested in [21], wherein, the autocorrelation function can be incorporated in the Gauss-Markov model to characterize the channel PDF. The channel statistic of Gauss-Markov process can be defined as:

$$h[k] = \alpha h[k-1] + \sqrt{1-\alpha^2}\, u[k], \quad k = 1,2,\ldots \tag{2}$$

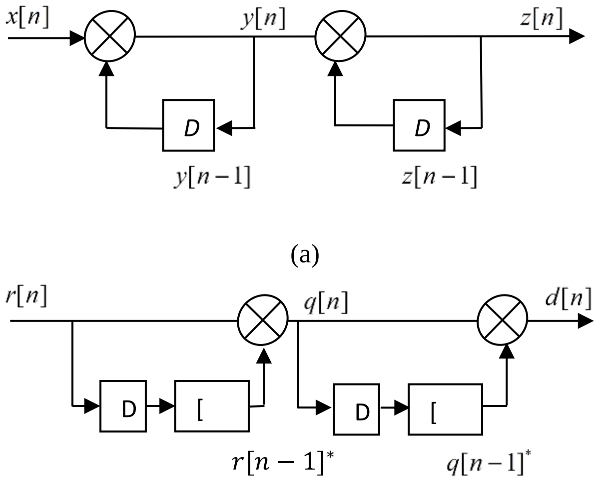
where α corresponds to the correlation parameter. It is noted that the value of α lies in between the value of 0 and 1, which is subjected to the channel Doppler shift and the transmission bandwidth. When $\alpha \ll 0$, it indicates fast-fading channel and when $\alpha \approx 1$, it represents slow-fading channel. Because of the mobility of the nodes, the channel varies with time (i.e. time-varying) resulting in correlated channel fading. The time-varying channel can be modelled as the normalised autocorrelation function R based on Jake's model and is defined as:

$$R = \sigma_h^2 J_0(2\pi f_D T_s) \tag{3}$$

where $J_0(\cdot)$ represents the zero order Bessel function of the first kind, $f_D = \frac{v}{\lambda}$ is the channel maximum normalized Doppler frequency with v indicating the velocity of the nodes, T_s denotes the symbol duration and λ represents the wavelength. In order to generate the simulation for the channel, a Sum of Sinusoids (SoS) based Rayleigh faded mobile-to-mobile channels narrowband complex envelope in [22] is applied.

B. Double Differential Transmission

The DD modulation and demodulation blocks are portrayed in Figure 2(a) and (b). Let $x[n]$ denotes a group of $\log_2 M$ information to be modulated in Pulse Shift Keying (PSK) at the time n and D represents the delay. The transmitted signal $z[n]$ is obtained as $x[n] \times y[n-1] \times z[n-1]$. Referring to Figure 2(b), by assuming a point-to-point communication channel with the presence of frequency offset, the signal received at the destination is expressed as $he^{wn} r[n] + e[n]$ where h is the channel fading coefficient, w denotes the frequency offset while $e[n]$ represents the Additive White Gaussian Noise (AWGN). At the destination, $\hat{x}[n]$ is decoded and performed as $q[n]q[n-1]^* x[n]^*$ where $q[n] = r[n]r[n-1]^*$.



(b)
Figure 2: Double Differential (DD) Block Diagram
(a) Modulation and (b) Demodulation.

III. DOUBLE DIFFERENTIAL TRANSMISSION FOR AF COOPERATIVE DIVERSITY

The concept of DD transmission as detailed in [23] is developed for the time varying AF relayed link. At the first transmission phase, the source transmits the double differentially modulated signal towards the relay and the destination. Thus, the signals received at the relay and the desired destination is expressed as:

$$r_{s,r}(n)=\sqrt{P_s}h_{s,r}e^{jw_{s,r}n}z(n)+e_{s,r}(n) \tag{4}$$

$$\begin{gathered} r_{s,d}(n)=\sqrt{P_s}h_{s,d}e^{jw_{s,d}n}z(n)+e_{s,d}(n), \\ n=0,1,\ldots \end{gathered} \tag{5}$$

P_s denotes the transmitted power at the source, $h_{s,r}$ and $h_{s,d}$ represent the channel fading coefficients, $w_{s,r}$ and $w_{s,d}$ denote the frequency offsets, $e_{s,r}$ and $e_{s,d}$ are the AWGN for the source-relay and source-destination links.

During the second phase, only the amplified signal of the relay is retransmitted towards the destination and at the same time the source stays silent. The received signal at the destination is given as:

$$\begin{gathered} r_{r,d}(n)=Gh_{r,d}e^{jw_{r,d}n}r_{s,r}(n)+e_{r,d}(n), \\ n=0,1,\ldots \end{gathered} \tag{6}$$

where $h_{r,d}$ represents the channel fading coefficients, $w_{r,d}$ denote the frequency offsets, $e_{r,d}$ is the AWGN for the relay-destination links. G represents the constant amplification factor at the relay and is defined as $G=\frac{P_R}{P_s\sigma^2+N}$, where P_s and P_R denotes the power transmitted by the source and the relay. It is assumed that the total transmitted power is defined as $P_T=P_s+P_R$. For simplicity, n is omitted from the equations. Hence, substituting (4) into (5), the simplified received signals at the destination are given as:

$$r=G\sqrt{P_s}he^{jw}z+e \tag{7}$$

where h indicates double-Rayleigh channel (i.e. the multiplication of two complex Gaussian channel of $h_{s,r}$ and $h_{r,d}$)[24].

At the destination, the received relayed and directly transmitted signals are combined based on the DD demodulation and given as:

$$\begin{aligned} d(l)=\alpha_1(r_{r,d}[n-1])(r_{r,d}[n-1]r^*_{r,d}[n-2])^* \\ +\alpha_2 r_{s,d}[k]r^*_{s,d}[k-1](r_{s,d}[k-1]r^*_{s,d}[k-2])^* \end{aligned} \tag{8}$$

where $l=n=k$ and $\alpha_1=\alpha_2=1$ represent the factor for the direct combiner [25]. Then the final combined signal is decoded using ML decoder as:

$$\hat{x}(n)=\arg\max_{x\in\mathrm{E}}\mathrm{Re}\{d(l)x^*\} \tag{9}$$

According to the principle of DD modulation and demodulation, the received signals can be detected based on the previously received consecutive symbols. Hence the knowledge of the channel and frequency offset is not required. However, due to the mobility of the nodes and symbol-based transmission, the DD scheme fails to perform and result in error floor (i.e. a region where the BER performance curve flattens as the SNR increases) at high Signal-to-Noise Ratio (SNR). Therefore, MSD is proposed and analysed in Section IV.

IV. PROPOSED MULTIPLE SYMBOL DIFFERENTIAL DETECTION SCHEME BASED ON DOUBLE DIFFERENTIAL MODULATION

Although DD transmission is able to circumvent the degradation because of frequency offset as compared to single differential method, higher SNR power is required to achieve the similar performance for the coherent detection in terms of BER. Hence, MSDD detection decision rule is proposed in [26] to mitigate the SNR loss. The decision rule is determined by assuming all possible symbols received and detected at the destination.

The $N>2$ received symbols at the destination via relayed and directly transmitted link from the source referring to (4), (5) and (6) can be written as:

$$r_{s,d}=\sqrt{P_s}h_{s,d}e^{jw_{s,d}}z+e_{s,d} \tag{10}$$

$$r_{r,d}=G\sqrt{P_s}h_{r,d}h_{s,r}e^{jw}+e \tag{11}$$

where: $h_{s,d}=diag[h_{s,d}(1),..h_{s,d}(N)]^t$
$h_{r,d}=diag[h_{r,d}(1),..h_{r,d}(N)]^t$
$z=diag[z(1),..z(N)]^t$
$w=$ frequency offsets along relayed link
$w_{s,d}=$ frequency offsets along source-destination link

Non-coherent Maximum Likelihood (ML) detection is proposed due to its optimum performance without requiring the knowledge of channel. Thus, the optimal decision for the relayed and direct link transmission at the destination is expressed as:

$$\hat{z} = \arg\max f(r_{s,d} \mid z) \cdot \arg\max f(r_{r,d} \mid z, h_{r,d}) \qquad (12)$$

where $f(\cdot \mid \cdot)$ represents the conditioned PDF of the channel output.

The PDF of $r_{s,d}$ conditioned on z is written as [27]:

$$f(r_{s,d} \mid z) = \frac{\exp(-Tr\{r_{s,d}{}^{H} \psi_{r_{s,d}}^{-1} r_{s,d}\})}{\pi^{N} \det(\psi_{r_{s,d}})} \qquad (13)$$

and the PDF of $r_{r,d}$ conditioned on z and $h_{r,d}$ is:

$$f(r_{r,d} \mid z, h_{r,d}) = \frac{\exp(-Tr\{r_{r,d}{}^{H} \psi_{r_{r,d}}^{-1} r_{r,d}\})}{\pi^{N} \det(\psi_{r_{r,d}})} \qquad (14)$$

The conditional variance of $r_{s,d}$ and $r_{r,d}$ from (13) and (14) yield:

$$\begin{aligned} \psi_{r_{s,d}} &= E\{r_{s,d} r_{s,d}^{H} \mid z\} = P_s \cdot z \cdot \psi_{h_{r,d}} \cdot z^{*} + N_{s,d} I_N \\ &= z(P_s \cdot \psi_{h_{r,d}} + N_{s,d} \cdot I_N) z^{*} = z C_{s,d} z^{*} \end{aligned} \qquad (15)$$

and

$$\begin{aligned} \psi_{r_{r,d}} &= E\{r_{r,d} r_{r,d}^{H} \mid z, h_{r,d}\} \\ &= G^2 P_s \cdot z \cdot h_{r,d} \cdot \psi_{h_{s,r}} \cdot z^{*} + (1 + G^2 \cdot \sigma_{r,d}^2) N_{s,d} I_N \end{aligned} \qquad (16)$$

It is considered that all the fading and noise are complex Gaussian with mean = 0. Therefore, the conditional variance received vector r, $\psi_{h_{s,d}}$ and $\psi_{h_{s,r}}$ are complex Gaussian vector with mean = 0 where $\psi_{h_{s,d}}$ represents the autocorrelation matrix of $h_{s,d}$ and its expectation can be written as:

$$E\{h_{s,d} h_{s,d}^{*}\} = toeplitz\{\varphi_{s,d}(0), \ldots, \varphi_{s,d}(N-1)\} \qquad (17)$$

where $\varphi(i) = E\{hh^{*}\} = J_0(2\pi f_d)$. Similarly, $\sum h_{s,r}$ denotes the autocorrelation matrix of $h_{s,r}$ and its expectation is expressed as:

$$E\{h_{s,r} h_{s,r}^{*}\} = toeplitz\{\varphi_{s,r}(0), \ldots, \varphi_{s,r}(N-1)\} \qquad (18)$$

By substituting (13) and (14) into (12), the ML detection approach is given by:

$$\begin{aligned} \hat{z}_{ML} &= \arg\max\{\frac{1}{\pi^{N} \det(\psi_{r_{s,d}})} \exp[Tr(-r_{s,d}^{H} \psi_{r_{s,d}}^{-1} r_{s,d})]\} \\ &\{E\{\frac{1}{\pi^{N} \det(\psi_{r_{r,d}})} \exp[Tr\{-r_{r,d}^{H} \psi_{r_{r,d}}^{-1} r_{r,d})]\}\} \end{aligned} \qquad (19)$$

It is noted that from (19), the ML detection requires the expectation for $h_{r,d}$, which does not result in closed-form expression. Therefore, the following modified decision metric is proposed:

$$\frac{1}{\pi^{N} \det(\psi_{r_{r,d}})} \exp[Tr(-r_{r,d}^{H} \overline{\psi}_{r_{r,d}}^{-1} r_{r,d})] \qquad (20)$$

where

$$\begin{aligned} \overline{\psi}_{r_{r,d}} &= E_{h_{r,d}}\{\psi_{r_{r,d}}\} \\ &= G^2 P_s \cdot z \cdot \psi_h \cdot z^{*} + (1 + G^2 \sigma_{r,d}^2) N_{s,d} I_N \\ &= z(G^2 P_s \cdot \psi_h + (1 + G^2 \sigma_{r,d}^2) N_{s,d} I_N) z^{*} = z C_{s,d} z^{*} \end{aligned}$$

where

$$\psi_{h_{r,d}} = E\{h_{r,d} \psi_{h_{s,d}} h_{r,d}^{*}\}$$

Since $\frac{1}{\pi^{N} \det(\psi_{r_{s,d}})}$ and $\frac{1}{\pi^{N} \det(\psi_{r_{r,d}})}$ is independent of z using the rule of $\det\{AB\} = \det\{BA\}$, (19) is further simplified as:

$$\begin{aligned} \bar{z}_{ML} &= \arg\max\{\exp[Tr(-r_{s,d}^{H} \psi_{r_{s,d}}^{-1} r_{s,d})]\} \\ &\{\exp[Tr(-r_{r,d}^{H} \overline{\psi}_{r_{r,d}}^{-1} r_{r,d})]\}\} \end{aligned} \qquad (21)$$

Thus, the Multiple Symbol Double Differential Sphere Decoding (MSDDSD) decision rule can be obtained by searching the sequence $\hat{z}$ that maximizes the conditional PDF. The concept is similar to finding the sequence $\hat{z}$ with the smallest sum of matrix, formulated as:

$$\begin{aligned} \hat{z}_{ML} &= \arg\min\{\sum r^{H} \psi^{-1}\} \\ &= \arg\min\{r_{s,d}^{*} z C_{s.d}^{-1} z^{*} r_{s,d} + r_{r,d}^{*} z C^{-1} z^{*} r_{r,d}\} \end{aligned} \qquad (22)$$

Using Cholesky decomposition approach $C^{-1} = LL^{H}$ (22) becomes:

$$\begin{aligned} \hat{z}_{ML} &= \arg\min\{(r_{s,d} z^{*})^{H} L_{s,d} L_{s,d}^{H} r_{s,d} z^{*} \\ &+ (r_{r,d} z)^{H} LL^{H} r_{r,d} z^{*}\} \\ &= \arg\min\{|U_{s,d} z|^2 + |Uz|^2 \end{aligned} \qquad (23)$$

where $U = \{L^{H} r\}^{*}$ and $U_{s,d} = \{L_{s,d}^{H} r_{s,d}\}^{*}$ denotes the upper triangular matrix entry in row and column that further simplify (23) as:

$$\hat{z} = \arg\min\{|Us|^2 \qquad (24)$$

In order to reduce the exhaustive search of the transmitted signal sequence, sphere decoding by Lampe *et al.* in [17] is proposed to examine only the candidate vectors that are located within the sphere of radius R.

V. NUMERICAL ANALYSIS

A. Numerical Analysis of Double Differential Detection

In this section, performance of the average BER is numerically analysed for the DD transmission using PSK constellation in the AF cooperative wireless networks.

The PDFs of $\gamma_{s,d}$, $\gamma_{s,r}$ as well as $\gamma_{r,d}$ following the Rayleigh fading characteristic, is written as:

$$P_{\gamma_{s,d}}(\gamma_{s,d}) = \frac{1}{\bar{\gamma}_{s,d}} e^{\frac{-\gamma_{s,d}}{\bar{\gamma}_{s,d}}} \tag{25}$$

$$P_{\gamma_{s,r}}(\gamma_{s,r}) = \frac{1}{\bar{\gamma}_{s,r}} e^{\frac{-\gamma_{s,r}}{\bar{\gamma}_{s,r}}} \tag{26}$$

$$P_{\gamma_{r,d}}(\gamma_{r,d}) = \frac{1}{\bar{\gamma}_{r,d}} e^{\frac{-\gamma_{r,d}}{\bar{\gamma}_{r,d}}} \tag{27}$$

where $\bar{\gamma}_{s,d}$, $\bar{\gamma}_{s,r}$ and $\bar{\gamma}_{r,d}$ are the average SNR for each channel links. From (4), (5) and (6) the instantaneous SNR for the relayed link based on the DD modulation is written as:

$$\gamma_{s,r,d} = \frac{P_s P_R |h_{s,r}|^2 |h_{r,d}|^2}{(P_s\sigma_{s,r}^2 + \sigma^2 + P_R |h_{r,d}|^2)\sigma^2} = \frac{\gamma_{s,r}\gamma_{r,d}}{\gamma_{s,r} + \gamma_{r,d} + 1} \tag{28}$$

Note that the above expression can be seen from [2] Equation (8)][28][19] for a dual-hop transmission scheme employing a fixed-gain relay. Fixed-gain relay is preferably chosen for its benefit to eliminate the requirement of channel knowledge at the relay [29].

Theorem 1. The PDF of $\gamma_{s,r,d}$ can be expressed by:

$$\begin{aligned} p(\gamma_{s,r,d}) &= \frac{2\sigma^2 P_s\sigma_{s,r}^2 + 2\sigma^4}{(P_s P_R \sigma_{s,r}^2 \sigma_{r,d}^2} \exp(-\frac{\sigma^2\gamma}{P_s\sigma_{s,r}^2}) \times K_0(2\beta) \\ &+ \frac{2\sigma^4}{P_s\sigma_{s,r}^2\sigma_{r,d}^2} \times \sqrt{\frac{P_R\sigma_{r,d}^2(P_s\sigma_{s,r}^2 + \sigma^2)}{P_s\sigma_{s,r}^2\sigma^2}} \exp(-\frac{\sigma^2\gamma}{P_s\sigma_{s,r}^2}) \\ &\times K_1(2\beta) \end{aligned} \tag{29}$$

where $K_n(\cdot)$ denotes the modified Bessel function of the second kind with $n = 0$ represents zero order and $n = 1$ represents first order.

Proof. Theorem 1 is proven with aid of results given in [28] Section III]

According to [30], the conditional BER of the differential transmission is defined as:

$$P_b(\gamma_T) = \frac{1}{2^{2L-1}} e^{-\gamma T} \sum_{n=0}^{L-1} C_n \gamma_T^n \tag{30}$$

where γ_T is the instantaneous total SNR at the destination and C_n is given by:

$$C_n = \frac{1}{n!} \sum_{k=0}^{L-1-n} \binom{2L-1}{k} \tag{31}$$

With the independent 2-channel reception, the differential PSK conditional BER based on (30) can be expressed as [23], Equation (12.1-13)]:

$$P_b(\gamma) = \frac{1}{8}(4 + \gamma_T)e^{-\gamma T} \tag{32}$$

where γ_T is the total instantaneous SNR at the destination is defined as:

$$\gamma_T = \gamma_{s,d} + \gamma_{s,r,d} \tag{33}$$

The BER of DDAF system with binary PSK modulation across the joint PDF of $\gamma_{s,d}$ and $\gamma_{s,r,d}$ is averaged over the channels and is expressed as:

$$\begin{aligned} P_b &= \frac{1}{8}\int_0^\infty\int_0^\infty (4 + \gamma_{s,d} + \gamma_{s,r,d}) e^{-(\gamma_{s,d}+\gamma_{s,r,d})} \times \rho_{s,d}(\gamma_{s,d}) \\ &\times \rho_{s,r,d}(\gamma_{s,r,d}) d\gamma_{s,d} d\gamma_{s,r,d} \end{aligned} \tag{34}$$

The approximate BER of cooperative diversity with DD modulation is given as:

$$\begin{aligned} P_b &= \frac{\exp(0.5)}{16} \times \{ \frac{4\sigma^6 + 5P_s\sigma_{s,d}^2\sigma^4}{(2\sigma^2 + P_s\sigma_{s,d}^2)^2(2\sigma^2 + P_s\sigma_{s,r}^2)} \times W_{-1.0,0.5}(\beta) \\ &+ \frac{\sqrt{2(\sigma^2 + P_s\sigma_{s,r}^2)}(4\sigma^5 + 5P_s\sigma_{s,d}^2\sigma^3)}{\sqrt{P_R(2\sigma^2 + P_s\sigma_{s,r}^2)}\sigma_{r,d}(2\sigma^2 + P_s\sigma_{s,d}^2)^2} \times W_{-0.5,0}(\beta) \\ &+ \frac{16P_s\sigma_{s,r}^2\sigma^4}{(2\sigma^2 + P_s\sigma_{s,r}^2)^2(2\sigma^2 + P_s\sigma_{s,d}^2)} \times W_{-2,0.5}(\beta) \\ &+ \frac{8\sqrt{2(\sigma^2 + P_s\sigma_{s,r}^2)}P_s\sigma_{s,r}^2\sigma^4}{\sqrt{P_R\sigma_{r,d}(2\sigma^2 + P_s\sigma_{s,r}^2)^3(2\sigma^3 + P_s\sigma_{s,d}^2\sigma)}} \times W_{-1.5,0}(\beta) \end{aligned} \tag{35}$$

where

$$\beta = \frac{\sigma^2}{P_R\sigma_{r,d}^2} \tag{36}$$

with the assumption that $\sigma^2 = 1$ and symmetric case where $P_s = P_R = 0.5$.

The Whittaker function can be written as:

$$W_{k,\mu}(z) = \exp\left(-\frac{z}{2}\right) z^{\left(\mu+\frac{1}{2}\right)} U\left(\mu - k + \frac{1}{2}, 1 + 2\mu; z\right) \tag{37}$$

where $U(a,b,z)$ represents the confluent hypergeometric

function and can be expressed as [31]:

$$\begin{aligned} U(a,b,z) &= \frac{\Gamma(1-b)}{\Gamma(1-b+1)} M(1,b,z) \\ &+ \frac{\Gamma(b-1)}{\Gamma(a)} z^{1-b} M(a-b+1, 2-b, z) \end{aligned} \tag{38}$$

B. Numerical Analysis of Multiple Symbol Double Differential Detection

Assume that the data sequence is transmitted at the source. The erroneously decoded data instead of the originally transmitted s is represented as. In order to determine the manner when, Pairwise Error Probability

(PEP) is expressed as [32]:

$$P_{em}(z \to \hat{z} \mid h_m) = Q(\frac{d_m(z,\hat{z})}{2}),\ m = 1,\ldots,N \tag{39}$$

where m represents the number of relays (i.e. $m = 1$ corresponds to a single relay and $m = N$ indicates multiple relays) in the system. $d_m(z,\hat{z})$ is the Hamming distance between the z and $\hat{z}$ at the destination. It is assumed that single relay is used in the system and $Q(x)$ function is given as [33] :

$$Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} e^{-\frac{x^2}{2}}dx = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \exp(-\frac{x^2}{2\sin^2\theta})d\theta \tag{40}$$

Based on the decision rule, the distance between the z and $\hat{z}$ can be written as:

$$\begin{aligned} d_1^2(z,\hat{z}) &= \rho r^H diag\{\hat{z}^* - z^*\}C^{-1} r diag\{\hat{z} - z\} \\ &= \rho r^H Q r \end{aligned} \tag{41}$$

where $\rho = \frac{P_s}{\sigma^2}$ is the instantaneous SNR and $Q = diag\{\hat{z}^* - z^*\}C^{-1} diag\{\hat{z} - z\}$

Using the above equations, the average PEP can be calculated by taking the expectation of (38).

$$\begin{aligned} P_{e1}(z \to \hat{z}) &= E_{h_1}[Q(\frac{d_1^2(z,\hat{z})}{2})] \\ &= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} Eh_1[\exp(-\rho\frac{r^H Q r}{8\sin^2\theta})]d\theta \end{aligned} \tag{42}$$

Thus, the average PEP for single relay network at high SNR is given as:

$$\begin{aligned} P_{e1}(z \to \hat{z}) &= (\frac{6}{\det(diag\{\hat{z}^* - z^*\}diag\{\hat{z} - z\})}) \\ &(\rho^{-2}\ln\rho) + O(\frac{|\ln\det(diag\{\hat{z}^* - z^*\}diag\{\hat{z} - z\}|}{\rho^2}) \end{aligned} \tag{43}$$

According to Divsalar and Simon in [34], upon obtaining the PEP of the multiple symbol sequence detection, the BER over the set of errors can be determined by the union bound of the PEP and is derived as:

$$P_b \approx \frac{d}{\log_2(M)(N-1)} P(z \to \hat{z}) \tag{44}$$

VI. Simulation And Numerical Analysis Performance Of The DD Relayed

MATLAB is used as the simulation tool to examine the performance of the proposed ML-DDAF for cooperative diversity networks. Firstly, a comparison of performance for the proposed MSDD scheme, conventional DD scheme and coherent scheme in AF relayed system is examined. Figure 3 demonstrates the performance comparison of the proposed MSDD detection scheme, the conventional DD scheme [12] and coherent scheme in Rayleigh fading environment using BPSK constellation for window size length $N = 10$. In the simulation, the channels are affected by the frequency offsets distributed uniformly in $(-\pi,\pi)$ [11,21] and the fading gains are equal (i.e. $\sigma_{s,r}^2 = \sigma_{r,d}^2 = \sigma_{s,d}^2 = 1$). Also, the power transmitted at the source P_s and relay P_R are considered to be equally divided. It is observed in Figure 3 that, as expected, the coherent detection yields a better performance compare to the DD based scheme. This is because in the coherent scheme, the channel knowledge is assumed to be perfectly estimated. However, it is a complicated and difficult task to estimate the channels' properties accurately, particularly in a time-varying channel. Additionally, the coherent scheme does not consider the unknown frequency offsets effect that occurs in a fast fading environment. Hence, the coherent scheme is plotted in order to serve as the benchmark for the relayed system.

On the contrary, the DD based transmission allows the detector to omit the requirement of channel knowledge and the frequency offsets estimation, particularly in fast fading environment. However, from the plots in Figure 3, it is apparent that the conventional DD scheme fails to perform well at high SNRs due to the high mobility and symbol based transmission. Hence a multiple symbol based DD scheme is proposed and it is shown from the plots that the proposed scheme significantly improves at 92.96% than the symbol based transmission at SNR 30dB. Additionally, numerical expression for the conventional DD scheme and MSDD has been performed to verify the simulated result. As observed from Figure 3, the simulated results matches well with the analytical numerical expression.

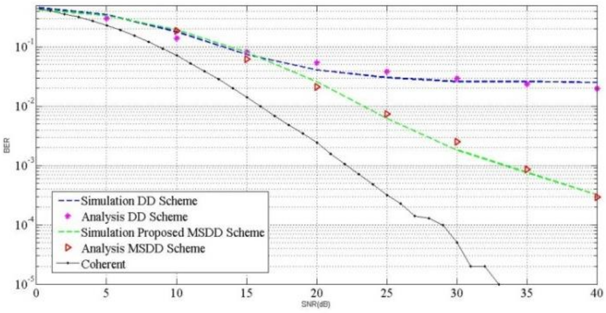


Figure 3: Comparison of BER versus SNR performance of MSDD scheme, conventional DD scheme and Coherent scheme when $\sigma_{s,r}^2 = 1; \sigma_{r,d}^2 = 10; \sigma_{s,d}^2 = 1$ and $P_S = P_R = 0.5P$

Subsequently, it is also observed that the performance of the MSDD cooperative system is dependent on the length of received symbol sequence N for data transmission. The results from Figure 4 show the strong impact between N and the performance enhancement. It can be shown from the trend of the curve that, the performance of BER significantly improves when N increases. As seen from the figure, the performance increases significantly when N is increased from 2 to 4. The BER slowly decreases with increasing N. The curve demonstrated specifically, that at BER of 10^{-3}, the performance improves at about SNR gain of 5dB as the symbol sequence length increases from $N = 6$ to $N = 10$. From the observation, it is suggested that the small value of N can be implemented to reduce complexity, albeit to some performance consequence.

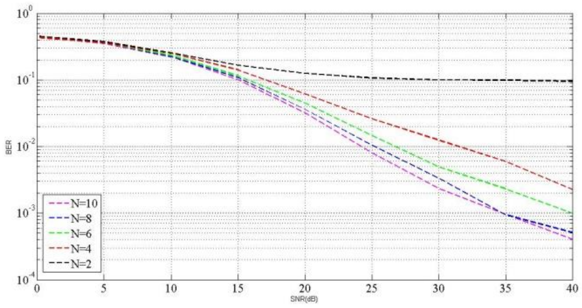


Figure 4: Performance of BER for the proposed MSDD scheme with different length received symbol sequence N when $\sigma^2_{s,r} = 1; \sigma^2_{r,d} = 10; \sigma^2_{s,d} = 1$ and $P_S = P_R = 0.5P$

Next, the equal distributed power and optimized power allocation for the proposed method is compared. In order to simplify the performance analysis of the DD based cooperative diversity system, the power is often equally divided and allocated between the source and relay. Additionally, it can be observed from (35) that the BER is subjected to P_S and P_R. Hence, the BER performance of the proposed MSDD scheme can be further enhanced by applying a more efficient use of distributing power resources among the source and relay nodes. P_S and P_R can be obtained by minimising the BER formulation in (35) using the power constraint, $P_T = P_S + P_R$ for the system. Figure 5 depicts the Power Allocation Factor (PAF) for different channel variances scenarios.

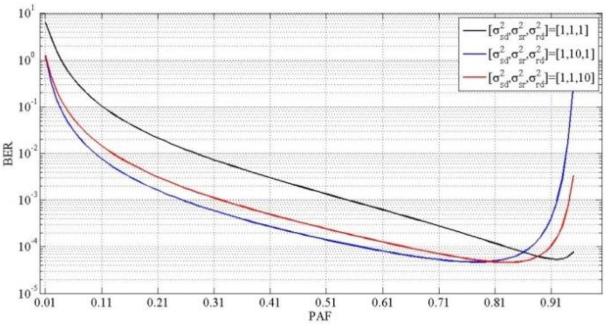


Figure 5: Power Allocation Factor (PAF) for different channel variances scenarios

The obtained optimum power allocation for SNR = 20dB corresponding to different channel variances is further tabulated in Table 1. For symmetric, strong SD, strong SR and strong RD links, more power is required to be allocated to source than relay. When the relayed link becomes stronger than the SD link, the system will perform better with equally distributed power allocation. Also, it is observed from Figure 6 that the optimized power allocation (i.e. $P_S : P_R = 0.92 : 0.18$) achieve better results as compare to the case when the power is equally divided between the source and relay under symmetry channels. For instance, at SNR = 20dB, the simulated BER of the equally distributed power equal to 2×10^2, while the BER from the power allocation analytical approach equal to 2.8×10^2.

Table 1
Optimum Power Allocation Based on Different Channel Variances

Scenarios	Channel Variances			Power Allocation	
	SD	SR	RD	P_S	P_R
Symmetric	1	1	1	0.92	0.08
Strong SD	10	1	1	0.92	0.08
Strong SR	1	10	1	0.78	0.22
Strong RD	1	1	10	0.83	0.17
Strong SR and RD	1	10	10	0.59	0.41

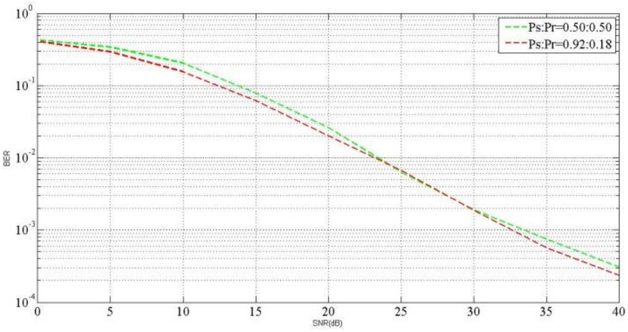


Figure 6: Comparison of the BER performance for the proposed MSDD scheme with symmetric power allocation and optimized power allocation when $\sigma^2_{s,r} = 1; \sigma^2_{r,d} = 1; \sigma^2_{s,d} = 1$

The proposed MSDD scheme is further simulated in various channel qualities based on the channel variances. The channel variances depends on the distances among source, relay and destination where symmetric channels with $\sigma^2_{s,r} = 1; \sigma^2_{r,d} = 1; \sigma^2_{s,d} = 1$ and non-symmetric channels with $\sigma^2_{s,r} = 1; \sigma^2_{r,d} = 1; \sigma^2_{s,d} = 10$ indicates strong SD link while $\sigma^2_{s,r} = 1; \sigma^2_{r,d} = 10; \sigma^2_{s,d} = 1$, represents strong SR link. Referred to the calculated optimum power allocation values in Table 1 and various cases of channel conditions (i.e. slow fading and fast fading) in Table 2, the BER using the conventional DD and proposed MSDD are simulated. The simulation results are plotted in Figure 7 and Figure 8 for different channel variances utilizing DDBPSK and DDQPSK modulations. The results revealed that in conventional DD scheme for case I, the BER is decreasing with SNR and the error floor does not take place. However for case II and case III which involve fast fading channels, error floor occurs between 10^{-2} and 10^{-1} at high SNR. Similar curves are observed in the use of DDQPSK as well. So as to mitigate the error floor experienced in fast fading channels, MSDD scheme is proposed and applied. It is observed that the proposed MSDD (colored solid line with points) is able to mitigate the error floor exhibited and increase the BER performance in fast fading channels. Clearly, it is shown in Figure 7 that the proposed MSDD scheme is capable of achieving 93.5% and 92.3% BER performance increment under case II and case III, respectively over its counterpart DD scheme at SNR 30dB. From Figure 8, it is also observed that the proposed MSDD scheme significantly outperform the DD scheme when modulation level (M=4) is utilized. Specifically, at SNR 30dB, the proposed MSDD scheme show a significant enhancement of 90.0% and 92.4% BER for Case II and Case III, respectively as compared to the DD scheme.

Table 2
Fading Case Based on Normalized Frequency

Case	Normalized frequency [f_{sd}; f_{sr}; f_{rd}]	Channel condition
I	0.001, 0.001, 0.001	SD, SR, RD slow fading
II	0.01, 0.01, 0.001	SD fast fading; SR, RD slow fading
III	0.01, 0.01, 0.01	SD, SR, RD fast fading

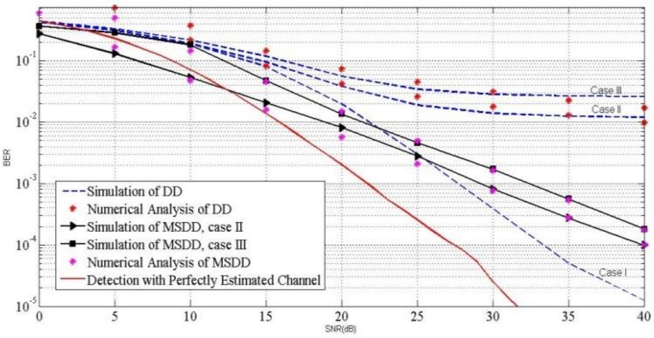


Figure 7: Comparison of the BER performance for the conventional DD, proposed MSDD scheme with optimized power allocation when $\sigma^2_{s,r} = 1; \sigma^2_{r,d} = 1; \sigma^2_{s,d} = 10$ for DDBPSK

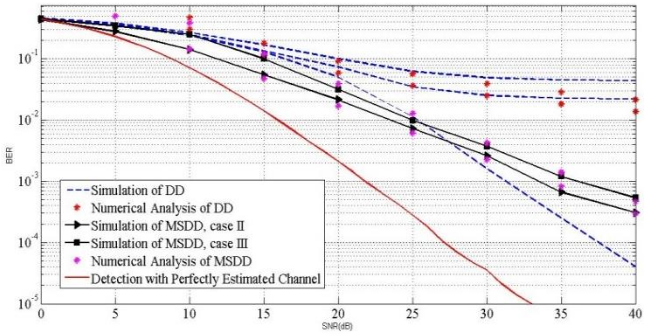


Figure 8: Comparison of the BER performance for the conventional DD, proposed MSDD scheme with optimized power allocation when $\sigma^2_{s,r} = 1; \sigma^2_{r,d} = 1; \sigma^2_{s,d} = 10$ for DDQPSK.

VII. CONCLUSION

This paper has presented the double differentially modulated based multiple symbols transmission for AF relay system over time varying Rayleigh fading channels. MSDD relaying transmission is proposed with the aim to encounter the error floor, which occurred for conventional DD scheme at high SNR in fast fading environment. The performance of the system has been analysed based on the BER analysis through simulation and approximate numerical analysis. The plots showed that the simulation results of the proposed multiple symbol based DD scheme is significantly improved as compared to the conventional DD transmission. Also, to further increase the performance of the relayed network, power allocation problem has been investigated based on the derived analytical expressions. The results of the simulation and numerical analysis indicate that the derived power allocation presents better performance enhancement over the equally distributed power between the source and relay.

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