Range and Bearing-based Simultaneous Localization and Mapping of Unmanned Ground Vehicle using Unscented Kalman Filter

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***Abstract*—This study deals with simultaneous localization and mapping problem by using unscented Kalman filter to compensate for observation outliers. In solving simultaneous localization and mapping problem using algorithms such as EKF or UKF, robot observations play a crucial part in determining its position estimation in any environment. If the robot observations obtained unexpected or fault values, the accuracy of the estimation will be deteriorated. In this research, an enhanced method based on UKF is developed to overcome the fault observation by assigning a weights to the observations. By comparing the observation values with its own estimate to detect the fault observations and then the weights of these observations are determined. Simulations were carried up to investigate the performance of the new method by comparing it with EKF-SLAM, UKF-SLAM and H∞ SLAM. The algorithms are compared in terms of parameters such as the RMSE and the runtime of the algorithm by using MATLAB. Results show that proposed method can performed better compared to other in dealing with observation outliers.**

***Index Terms*—SLAM; Kalman Filter; Particle Filter; H∞ Filter; Mobile Robot.**

I. INTRODUCTION

Simultaneous Localization and Mapping (SLAM) enable a mobile robot to operate in an unknown location in an unknown environment and incrementally build a map of the environment while using the map to determine its own location concurrently [1]. Smith et al. pioneered into this field by introducing the concept of a stochastic map where a mobile robot acquires information about its location and using sensor observations in different places at different times to organizes the environment [2].

Extended Kalman filter (EKF) [3] is one of the main approaches used in SLAM which consists of a state space model with additive Gaussian noise. However, due to the used of Taylor expansion in EKF which introduced the truncation errors, the accuracy of EKF-SLAM is very limited. Julier et al. proposed unscented Kalman filter (UKF) to overcome the problem caused by EKF. A set of chosen samples is used in UKF to represent the state distribution instead of using linearization in EKF [4]. Thus, the calculation of Jacobian and Hessian matrices can be avoided by using UKF-SLAM. Besides, higher approximation accuracy can be obtained with the unscented transformation.

Particle filter particularly the method known as FastSLAM proposed by Montemerlo and Thrun [5] in solving the SLAM problem using a set of state samples through expensive computational cost. The advantages of FastSLAM are it is more robust to data association error and nonlinear system can be better handled. The H∞ filter approach to solve SLAM problem has been proposed by Chandra, Gu and Postlethwait [6]. H∞ overcome the limitation of conventional Kalman filter which suffers from the assumption of statistical noises and estimation accuracy is improved under non-Gaussian noise distribution [7]. SLAM method, which used UKF for autonomous mobile robot is studied and being improved to deal with observation outliers in this study.

Autonomous mobile robot consists of several type such as unmanned aerial vehicle (UAV), unmanned ground vehicle (UGV) and unmanned underwater vehicle (UUV). Oguz and Temeltas conducted a comparison study on the filter's consistency between EKF and UKF of airborne SLAM for UAV navigation and concluded that UKF performs better than EKF in terms of filter consistency [8]. An improved adaptive UKF-SLAM algorithm for UUV with noise statistic estimator proposed by Wang et al. to solve the issue of declining accuracy and divergence occurring when the prior noise statistic is unknown [9]. Panah et al. proposed an optimized UKF-SLAM via Radial Basis Function on UGV to overcome the error inherited by its noise assumption and linearization process [10].

In this study, UGV is considered as the autonomous mobile robot for performing SLAM. Furthermore, such UGV usually equipped with range and bearing sensor such as laser range finder or sonar sensor for collecting its surrounding information. However, the range and bearing information from the sensor such as mounted on UGV frequently suffer from false recognition of landmark due to the mobile robot drifting behaviour, which eventually causes the divergence of the value estimated by any filtering method.

Therefore, a method is proposed in which a matrix that shows the reliability of the observation information is constituted by using difference between the observed and estimated values at each time and the weighting parameter related to the observed value is adjusted dynamically by using this matrix. Thus, the deterioration in the accuracy of estimation by the unscented Kalman filter can be prevented even though false recognition of landmarks occurred.

This paper is organized as follows: Section II describes SLAM models and proposed algorithm. Section III presents the simulation setup for comparison between different SLAM algorithms for unmanned grounded vehicle. Results and discussions in Section IV and conclusions are made in Section V.

II. SLAM MODELS

The position of the mobile robot and environment landmarks are normally stored in a single state vector estimated through a process of prediction and update which is done recursively. The mobile robot builds a complete map of landmarks through the observation of sensor model during motion process and uses it to estimate the position of mobile robot. Both position of mobile robot and position of landmarks can be estimated simultaneously via the relative distance between the landmarks and the mobile robot [1].

A. Motion Model

The Ackerman vehicle model is used in which the control inputs are set by the robot velocity v_k , the steering angle ϕ_k , the distance between wheels L , and denotes the time interval from $k-1$ to k .

$$x_k = f(x_{k-1}, u_k) + e_{1k} \tag{1}$$

$$f(x_{k-1}, u_k) = \begin{bmatrix} x_k \\ y_k \\ \varphi_k \end{bmatrix} = \begin{bmatrix} x_{k-1} + \Delta t \cdot v_k \cdot \cos(\phi_{k-1} + \theta_k) \\ y_{k-1} + \Delta t \cdot v_k \cdot \sin(\phi_{k-1} + \theta_k) \\ \varphi_{k-1} + \dfrac{\Delta t \cdot v_k \cdot \sin(\theta_k)}{L} \end{bmatrix} \tag{2}$$

where x_k , y_k and φ_k is the x co-ordinate, y co-ordinate and heading of the mobile robot respectively.

B. Sensor Model

The mobile robot is equipped with range-bearing sensor which takes observations of the features. The current mobile robot position x_k , and the position of an observed feature m_k , the range and bearing can be modelled as

$$z_k = h(x_k) + e_{2k} \tag{3}$$

$$h(x_k) = \begin{bmatrix} r \\ \theta \end{bmatrix} = \begin{bmatrix} \sqrt{(m_{k,i} - x_k)^2 + (m_{k,i} - y_k)^2} \\ \arctan\left(\dfrac{m_{k,i} - y_k}{m_{k,i} - x_k}\right) - \phi_k \end{bmatrix} \tag{4}$$

where r and θ is the range and bearing of the mobile robot.

III. ENHANCED UKF-SLAM USING A WEIGHTING MATRIX METHOD

The divergence of the estimated values by using any filtering method is caused by the system's observational outliers [11]. Thus, a robust estimation method is proposed by constitute a matrix showing the reliability of the observation information by using the difference between the observed and estimated values at each time, and using the combination of this matrix and the unscented Kalman filter. First, a measure to determine the observational outliers is described. The Mahalanobis distance r_{ik} between observed values z_{ik} and observation predicted value $\hat{z}_{ik|k-1}$ as a measure for evaluating the reliability of the observations z_{ik} . The observation predicted value $\hat{z}_{k|k-1}$ is the value determined by the following equation, using the state transition model and the observation model equation using (1) and (3) respectively. Here, $\hat{x}_{k|k-1}$ indicates that the value is a pre-estimated value.

$$\hat{x}_{k|k-1} = f\left(x_{k-1}, u_{k-1}\right) \tag{5}$$

$$\hat{z}_{k|k-1} = h\left(\hat{x}_{k|k-1}\right) \tag{6}$$

The Mahalanobis distance is a multivariate standardized measure that takes into consideration the correlation between variables. The Mahalanobis distance between the observed value and the observation predicted value is expressed as follows, using the observation error covariance matrix.

$$S_k = \overline{Z}_k W_c^{[i]} \left[\overline{Z}_k\right]^T + R_k \tag{7}$$

$$r_{ik} = \sqrt{\left(z_{ik} - \hat{z}_{ik|k-1}\right)^T S_{ik}^{-1} \left(z_{ik} - \hat{z}_{ik|k-1}\right)} \tag{8}$$

The property of the Mahalanobis distance E_k is strengthened by squaring it, and the matrix for dynamically evaluating he reliability of each observation is composed as shown below. This matrix is a mechanism that increases the value of the element corresponding to a landmark when false recognition of the landmark occurs.

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$$E_k = diag\{\eta_{1k}, \eta_{2k}, ..., \eta_{Mk}\} \tag{9}$$

$$\eta_{ik} = \begin{bmatrix} r_{ik}^2 & 0 \\ 0 & r_{ik}^2 \end{bmatrix} \tag{10}$$

The recursive process of the estimation is as described as follows:

Compute the sigma points, χ_i and corresponding weights, $W^{[i]}$:

$$\chi_{k-1} = [\mu_{k-1} \quad \dots \quad \mu_{k-1}] + \left[0 \quad \sqrt{(n+\lambda)\Sigma_{k-1}} \quad -\sqrt{(n+\lambda)\Sigma_{k-1}}\right] \tag{11}$$

$$\begin{aligned} W_m^{[0]} &= \lambda/(n+\lambda) \\ W_c^{[0]} &= \lambda/(n+\lambda) + (1-\alpha^2+\beta) \\ W_m^{[i]} &= W_c^{[i]} = 1/\{2(n+\lambda)\} \end{aligned} \tag{12}$$

where $\lambda = \alpha^2(n+\kappa) - n$ is a scaling parameter, α determine the spread of the sigma points around μ_{k-1} and is set to a small positive value, κ is a secondary scaling parameter and is set to zero and β is used to incorporate prior knowledge of the distribution of x_k .

Compute the predicted state mean, $\bar{\mu}_k$ and predicted covariance, $\bar{\Sigma}_k$ with motion noise Q_k

$$\bar{\chi}^*_{k-1} = f(u_k, \chi_{k-1}) \tag{13}$$

$$\bar{\mu}_k = W_m^{[i]} \bar{\chi}^*_{k-1} \tag{14}$$

$$\bar{\Sigma}_k = \bar{\chi}^*_{k-1} W_c^{[i]} \left[\bar{\chi}^*_{k-1}\right]^T + Q_k \tag{15}$$

Compute the predicted state and covariance of the measurement, $\bar{y}_k$ and $\bar{S}_k$ respectively

$$\bar{\chi}_k = [\bar{\mu}_k \quad \dots \quad \bar{\mu}_k] + \left[0 \quad \sqrt{(n+\lambda)\bar{\Sigma}_k} \quad -\sqrt{(n+\lambda)\bar{\Sigma}_k}\right] \tag{16}$$

$$\bar{Z}_k = h(\bar{\chi}_k) \tag{17}$$

$$\bar{y}_k = W_m^{[i]} \bar{\Upsilon}_k \tag{18}$$

Compute the weighting matrix, E_k :

$$r_{ik} = \sqrt{(z_{ik} - \hat{z}_{ik|k-1})^T S_{ik}^{-1} (z_{ik} - \hat{z}_{ik|k-1})} \tag{19}$$

$$S_{ik}^{-1} = \bar{Z}_k W_c^{[i]} \left[\bar{Z}_k\right]^T \tag{20}$$

$$\eta_{ik} = \begin{bmatrix} r_{ik}^2 & 0 \\ 0 & r_{ik}^2 \end{bmatrix} \tag{21}$$

$$E_k = diag\{\eta_{1k}, \eta_{2k}, \dots, \eta_{Mk}\} \tag{22}$$

$$\bar{S}_k = \bar{Z}_k W_c^{[i]} \left[\bar{Z}_k\right]^T + E_k R_k \tag{23}$$

Compute the cross-covariance of the state and measurement $\bar{\Sigma}_k^{x,y}$, Kalman gain K_k and updated state mean μ_k and covariance Σ_k :

$$\bar{\Sigma}_k^{x,y} = \bar{\chi}^*_{k-1} W_c^{[i]} \left[\bar{Z}_k\right]^T \tag{24}$$

$$K_k = \bar{\Sigma}_k^{x,y} \bar{S}_k^{-1} \tag{25}$$

$$\mu_k = \bar{\mu}_k + K_k (z_k - \bar{z}_k) \tag{26}$$

$$\Sigma_k = \bar{\Sigma}_k + K_k S_k K_k^T \tag{27}$$

In Step 2 of this recursive process, the square of the Mahalanobis distance between the observation information y_{ik} of each characteristic point and the observation predicted value $\hat{z}_{ik}$, i.e., r_{ik}^2 is stored in the matrix η_{ik}. The matrix E_k for dynamically assessing the reliability of each observation is created by combining with this matrix. The property of matrix E_k is that if the observation information that has been acquired includes outliers, the values of the elements corresponding to these outlier observations take on a very large numerical value compared to other elements.

In step 3, by multiplying this output evaluation matrix E_k and the matrix R_k representing the observation error, a state estimation weighted with reliable observational values is obtained.

IV. Simulation Setup

The simulator is developed based on T. Bailey's work [1] with MATLAB 2014b platform (Windows 10 system with 2.40GHz and Intel Core i7-4700MQ CPU). The mobile robot proceeds with initial state $x_{v,0} = [0 \quad 0 \quad 0]$ and then travels along the path. The simulation parameters and environments are shown in Table 1 and Figure 1 respectively. When the simulation reached the calculation step of 1000, the mobile robot is assigned the false landmark recognition to the observation to create the effect of drifting.

Table 1
Simulation Parameters

Simulation Parameter	Values
Mobile robot velocity	3 m/s
Velocity error	0.25 m/s
Maximum steering angle	±30˚
Maximum steering angle velocity	±20˚/s
Angular error	3˚
Scan area of laser range finder	0˚-180˚
Maximum scanning distance	30m
Distance error	0.1m
Bearing error	1˚
Control frequency	40Hz
Observation frequency	5Hz

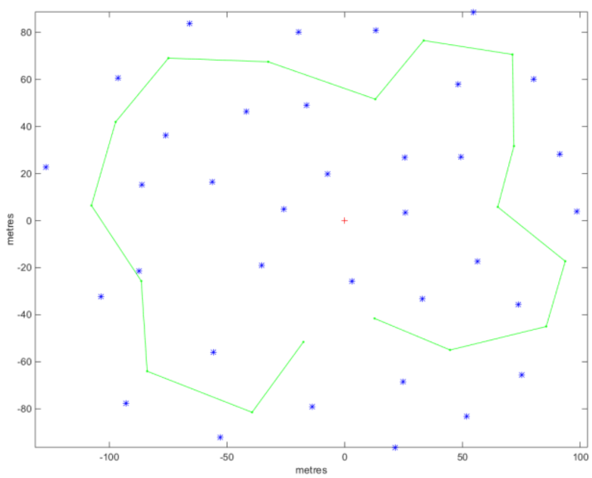

Figure 1: Simulation environment

V. Results and Discussions

Using the conditions above, thirty Monte Carlo simulations were conducted for three types of SLAM algorithms which consists of EKF-SLAM, UKF-SLAM and H∞ SLAM, to compare with the proposed method and the results are shown in Figure 2.

For fair and comprehensive comparison, EKF-SLAM, UKF-SLAM and H∞ SLAM are compared with the same simulation parameters in Table 1 and simulation environment as Figure 1. The mean of RMSE for the mobile robot position is used as the evaluation criterion. The RMSE for a simulation run is calculated as

$$RMSE(s) = \frac{\sum_{k=0}^{N} \left| s_k^{true} - \hat{s}_k \right|}{N} \quad (28)$$

where s_k^{true} is the true state of mobile robot and $\hat{s}_k$ is the estimated state of the robot, N is the number of discrete sampling points in one simulation run.

From the simulation result, the proposed method which used the Mahalanobis distance to compute the weighting matrix produced a better result compared to UKF-SLAM, EKF-SLAM and H∞ SLAM in term of accuracy as it has the lowest position RMSE as shown in Table 2. However, it takes a much longer time to complete the same task as compared to the other in term of computational complexity because it need to do extra task to compute the weighting matrix.

Table 2 shows the results of simulation in term of position RMSE and computational complexity.

Table 2:
Position RMSE and Computational Complexity

SLAM algorithm	Position RMSE	Average runtimes
EKF-SLAM	5.0094	148.3s
UKF-SLAM	4.6202	148.6s
Proposed Method	4.4246	158.7s
H∞ SLAM	8.6773	145.9s

Although H∞ SLAM has the less computational complexity, the estimation done by it is much more less accuracy compared to the other algorithm in comparison.

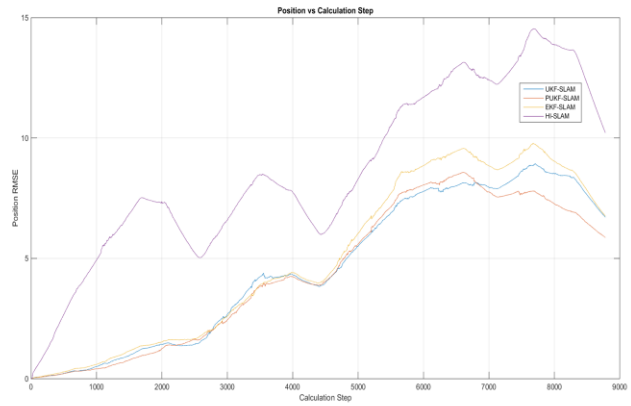

Figure 2: Mobile robot positing RMSE

VI. Conclusion

In this paper, the simultaneous localization and mapping of a range and bearing-based UGV is studied. The information observed from the range and bearing sensor is used to do the position estimation and mapping of UGV using UKF. However, the information obtained by the sensor normally came with some unwanted noise such as drifting of mobile robot. This affects the estimation of UGV as it will cause divergence. Thus, weighting matrix which representing the reliability of the predicted observation value of the range and bearing information is proposed to overcome it. The simulation result showed that the position estimation done on the UGV can be obtained accurately using the proposed method as the weighting matrix prevent the deterioration of estimation accuracy when false landmarks are detected.

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