Review of Modern Vehicle Modelling

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***Abstract*—Earlier models were a simplified form of the complex vehicle system, which was governed by the important parameters and states. These parameters and states were either obtained indirectly through some simplified equations or directly measured using instrumentation. In either case, the models were designed on the basis of parameters and state availability or importance, and not to have a complete model representation and maximum parameter utilization through estimation and instrumentation. Since a complete vehicle model can lead to a better controller. Without considering all the important parameters, a complete model is not obtained to design better controllers. All types of controllers, classical, optimal, nonlinear and linear controllers use the basic equations that govern the vehicle dynamics. This paper reviews all the previous models with greater insight into each system. The aim is to provide a better understanding of each model, its shortcomings and how it can represent the complex vehicle model.**

***Index Terms*—Parameter Estimation; Vehicle Models; Tire Modelling; Estimation.**

I. INTRODUCTION

The first cars used was a crude form of modern vehicles, having thin bicycle like tires, and a basic chassis with rigid suspension that resembled the horse driven carts [1]. The vehicle parts were a modified form of the bicycle technology [1]. Based on the model, the insight into the system helped the industry to modernize the design into modern cars. Since most of the design and control work was done by purely mechanical engineers, earlier cars were a marvel of mechanical design only [2-3]. As electronics made its way into highly complex system like airplanes and ships, their importance in automobiles was realized [3]. The first use of electronics in cars was in the firing of engine plugs to have smooth ride. At that time, passive electrical systems were introduced. The first active electronic system was in the design of antilock braking system [3]. Its utilization helped save the lives of many people. It also influenced the industry to have active electrical systems.

The following sections provide a brief overview of the historical achievements that has resulted into modern cars and has provided greater insight to build better ones in the future.

II. VEHICLE MODELLING

There are several types of models available, such as, bicycle model, half car model and full car model [1,3]. Following is a short description of some of the commonly used models.

A. Bicycle Model

In bicycle model the front wheel and the rear wheel of the automobile are considered as the two wheels of a bicycle. The distance between the wheels is considered to be "l" distance. The center of gravity of the bicycle is considered to lie between the front and the rear wheels at distance "a" from the front wheel and distance "b" from the rear wheel.

The net forces acting on the bicycle are resolved along the direction of motion (F_x) and perpendicular to the direction of motion (F_y) as shown in Figure. 1.

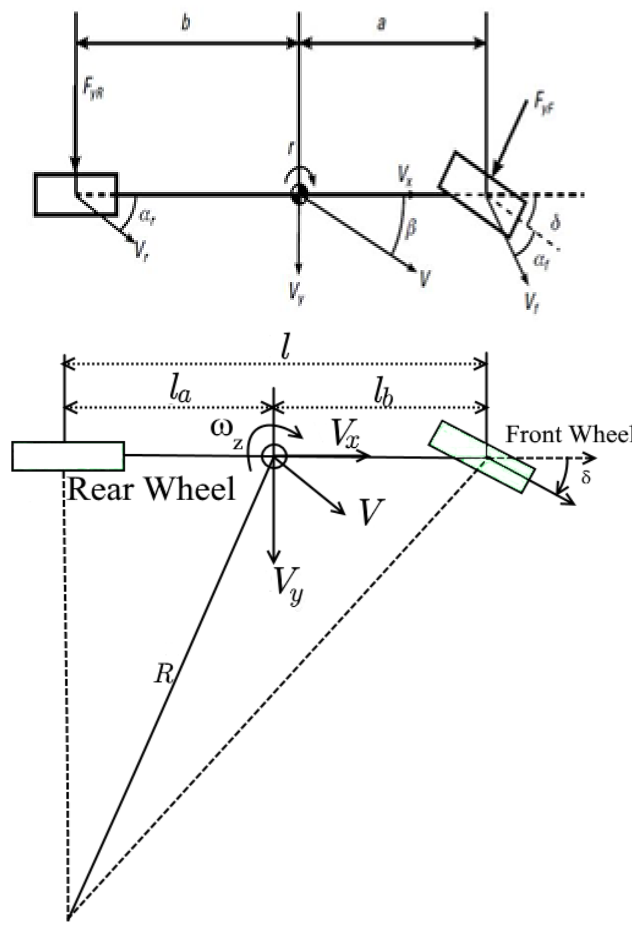

Figure 1: Force diagram of the bicycle model

In Figure. 1, the bicycle is moving with a velocity "V" and is turning with a steering angle (δ). Due to the motion, the center of gravity (c.g) is traveling with the velocity "V" at a net direction (β), caused by the interaction of the steering angle (δ) and the front and rear tire slip angles $\lfloor \alpha_x \; \alpha_y \rfloor$. The non-zero slip angle (β) causes the bicycle to turn at a turn rate of (ω_z) rad/s. The state space of the bicycle model is given in (1)[1-5]. Consider:

$$C_0 = C_{af} + C_{ar}$$

$$C_1 = aC_{af} + bC_{ar}, C_2 = a^2 C_{af} + b^2 C_{ar}$$

$$\begin{bmatrix} \dot{\beta} \\ \dot{\omega}_z \end{bmatrix} = \begin{bmatrix} \frac{-C_0}{mV_x} & \frac{-C_1}{mV_x^2} - 1 \\ \frac{-C_1}{I_z} & \frac{-C_2}{I_z V_x} \end{bmatrix} \begin{bmatrix} \beta \\ \omega_z \end{bmatrix} + \begin{bmatrix} \frac{C_{\alpha f}}{mV_x} \\ \frac{aC_{\alpha f}}{I_z} \end{bmatrix} \delta \qquad (1)$$

The model given in (1) uses the tire coefficients of the front and rear wheels. It considers the automobile has full freedom in roll, to compensate the centripetal forces acting on the model during turns. This assumption is not true for automobiles, therefore, the model is not suitable for complete vehicle analysis.

B. Full Car Model

The full car model is more accurate than the bicycle model, since it considers the 4-wheel traction and braking forces when modeling. The forces acting on the vehicle are resolved along the body longitudinal, lateral and vertical axis. The angular velocity along the longitudinal axis ($\dot{\phi}$), lateral axis ($\dot{\theta}$) and vertical axis ($\dot{\psi}$) are also included in the analysis. A typical block diagram of a four-wheel car model is given in Figure. 2.

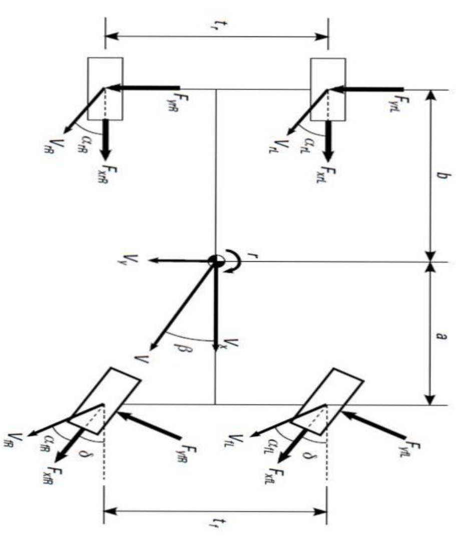

Figure 2: Force diagram of a full car model [1].

In Figure 2, the vehicle front wheels are turning with a steering angle (δ) causing lateral force components to act on each tire. This results into tire saturation causing slips in each wheel at a slip angle (α_i). The resulting motion of the vehicle causes it to turn with a yaw rate (ω_z). The net forces and moments acting in the longitudinal and lateral direction are given by (2) [1-7,33-40].

$$\sum F_x = (F_{flx} + F_{frx})\cos\delta + F_{rlx} + F_{rrx} - (F_{fly} + F_{fry})\sin\delta - A_p V_x^2 sign(Vx)$$

$$\sum F_y = (F_{flx} + F_{frx})\sin\delta + (F_{fly} + F_{fry})\cos\delta + F_{rly} + F_{rry} - A_p V_y^2 sign(V_y) \qquad (2)$$

$$I_{xx}\omega_z = a[F_{flx} + F_{frx}\sin\delta + F_{fly} + F_{fry}\cos\delta] - b(F_{rly} + F_{rry}) + e[(F_{flx}\cos\delta - F_{fly}\sin\delta) + (F_{fry}\sin\delta - F_{frx}\cos\delta)] + e(F_{rrx} - F_{rlx})$$

Similarly, the simplified roll model is represented in state space form in Equation (3) (assuming ϕ is small) [1], [4], [5], [7], [17].

In this model, the center of gravity is considered to be (h_{cg}) high from the road floor with roll stiffness (K_ϕ) and roll damping coefficient (B_ϕ). The parameters and states used in the model are defined in Figure (3).

$$\begin{bmatrix} \ddot{\phi} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{K_\phi - mgh_{cg}}{I_x} & \frac{B_\phi}{I_x} \end{bmatrix} \begin{bmatrix} \phi \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{mh_{cg}}{I_x} \end{bmatrix} a_y \qquad (3)$$

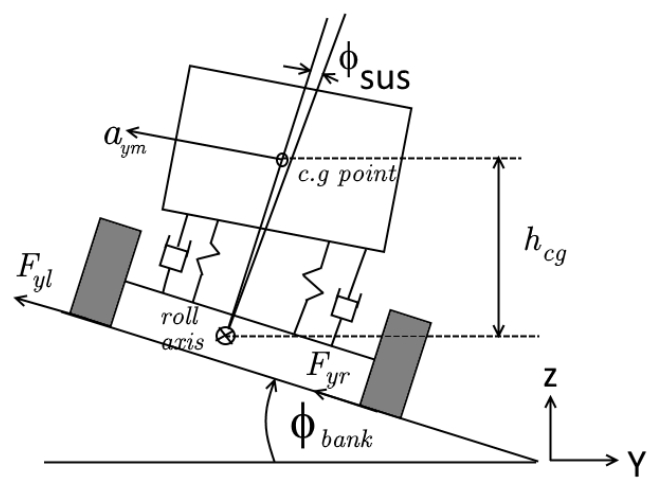

Figure 3: Vehicle Roll Model (ISO8855) [5].

Equation (3) is important for roll dynamics of the vehicle, which is related to suspension dynamics. In automobile, the purpose of the suspension system is to absorb vibrations caused by the road surface irregularities, engine vibrations, adjust terrain angles to ensure better ride comfort and improve the contact surface between tires to ensure stability and improved mileage [8-12].

Earlier, suspension models were usually obtained by considering the quarter car model (one wheel only). The equations for a quarter car model are given in Equation (4) [8-10].

$$m\ddot{x}_1 = -K(x_1 - x_5) - B(x_1 - x_5) - K_t(x_1 - x_t)$$
$$M\ddot{x}_5 = K(x_1 - x_5) + B(x_1 - x_5) \quad (4)$$

In Equation. 4, x_1 represents the vertical displacement of the suspension lower part and x_5 represents the vertical displacement of the suspension upper part. While x_t represents the disturbance input through the tire with spring constant K_t. The spring and damper coefficients K are B and respectively. A full car suspension system considers the suspension having four wheels. A detailed suspension system includes the 4-wheel suspension system model as given in Figure. 3

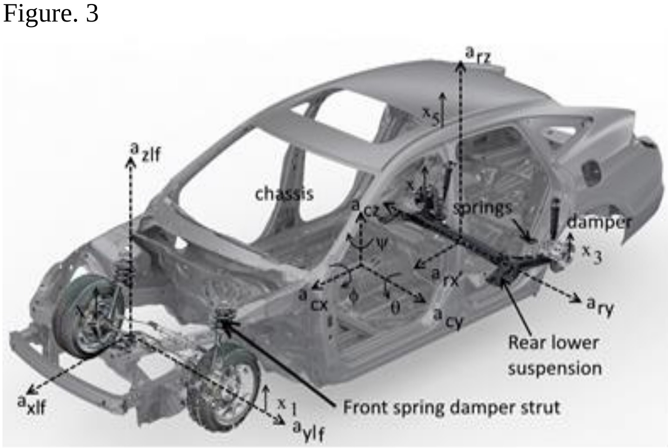

Figure 3: Full car model with states defined

In Figure. 3, the front lower suspension moves with the terrain with accelerations $\lfloor a_{fx}\ a_{fy}\ a_{fz} \rfloor$ along the vertical axis of the lower suspension. The rear suspension is also moving with the terrain with accelerations $\lfloor a_{rx}\ a_{ry}\ a_{rz} \rfloor$. The chassis has engine vibrations, terrain gradients and induced road grade vibrations via the lower suspensions with accelerations $\lfloor a_{cx}\ a_{cy}\ a_{cz} \rfloor$ [11-12].

In full car suspension model, the linear displacements $x = [x_1\ x_2\ x_3\ x_4\ x_5]$ represent the front left, front right, rear left and rear right lower suspension and chassis displacements respectively, while the angular displacements $[\phi\ \theta] = [x_6\ x_7]$ represent the roll and pitch of the chassis [28-30]. The set of spring constants $K = [k_1\ k_2\ k_3\ k_4]$ and $B = [b_1\ b_2\ b_3\ b_4]$ represent the individual spring constants and damping coefficients of each damper. The full car suspension system is given in (5) [1], [3], [10,13-15].

Equation (5) shows the performance of the suspension depends upon the performance of the dampers. In dampers, the significant source of the disturbance is due to the road surface, which has a spectral content in the 1 Hz to 12 Hz range, making it difficult to reduce using fixed dampers [9], since they are tuned to reduce only a limited range of disturbance frequencies [9-15].

An effective method to reduce the road surface induced disturbance is to use semi-active dampers in comparison to other semi-active and fully active or passive suspensions [9-15], due to their failsafe and lower power requirements [10]. The suspension system is nonlinear [16-18], so suspension dynamics must be considered in the suspension dynamic model. A completely nonlinear model is suitable for proper control of the suspension [18]. Since suspensions are in contact with the terrain through tires, the performance of the suspension and automobile dynamics depends upon the tire models.

$$m_1\ddot{x}_1 = -k_1(x_1 - x_5 - r_{11}\phi - r_{12}\theta) - b_1(\dot{x}_1 - \dot{x}_5 - r_{11}\dot{\phi} - r_{12}\dot{\theta}) - k_{t1}(x_1 - x_{t1})$$
$$m_2\ddot{x}_2 = -k_2(x_2 - x_5 - r_{21}\phi + r_{22}\theta) - b_2(\dot{x}_2 - \dot{x}_5 - r_{21}\dot{\phi} + r_{22}\dot{\theta}) - k_{t2}(x_2 - x_{t2})$$
$$m_3\ddot{x}_3 = -k_3(x_3 - x_5 + r_{31}\phi - r_{32}\theta) - b_3(\dot{x}_3 - \dot{x}_5 + r_{31}\dot{\phi} - r_{32}\dot{\theta}) - k_{t3}(x_3 - x_{t3})$$
$$m_4\ddot{x}_4 = -k_4(x_4 - x_5 + r_{41}\phi + r_{42}\theta) - b_4(\dot{x}_4 - \dot{x}_5 + r_{41}\dot{\phi} + r_{42}\dot{\theta}) - k_{t4}(x_4 - x_{t4})$$
$$M\ddot{x}_5 = k_1(x_1 - x_5 - r_{11}\phi - r_{12}\theta) + b_1(\dot{x}_1 - \dot{x}_5 - r_{11}\dot{\phi} - r_{12}\dot{\theta}) + k_2(x_2 - x_5 - r_{21}\phi + r_{22}\theta) + b_2(\dot{x}_2 - \dot{x}_5 - r_{21}\dot{\phi} + r_{22}\dot{\theta}) + k_3(x_3 - x_5 + r_{31}\phi - r_{32}\theta) + b_3(\dot{x}_3 - \dot{x}_5 + r_{31}\dot{\phi} - r_{32}\dot{\theta}) + k_4(x_4 - x_5 + r_{41}\phi + r_{42}\theta) + b4(\dot{x}_4 - \dot{x}_5 + r_{41}\dot{\phi} + r_{42}\dot{\theta}). \quad (5)$$
$$I_{xx}\ddot{\phi} = k_1 r_{11}(x_1 - x_5 - r_{11}\phi - r_{12}\theta) + b_1 r_{11}(\dot{x}_1 - \dot{x}_5 - r_{11}\dot{\phi} - r_{12}\dot{\theta}) - k_2 r_{21}(x_2 - x_5 - r_{21}\phi + r_{22}\theta) - b_2 r_{21}(\dot{x}_2 - \dot{x}_5 - r_{21}\dot{\phi} + r_{22}\dot{\theta}) + k_3 r_{31}(x_3 - x_5 + r_{31}\phi - r_{32}\theta) + b_3 r_{31}(\dot{x}_3 - \dot{x}_5 + r_{31}\dot{\phi} - r_{32}\dot{\theta}) - k_4 r_{41}(x_4 - x_5 + r_{41}\phi + r_{42}\theta) + b_4 r_{41}(\dot{x}_4 - \dot{x}_5 + r_{41}\dot{\phi} + r_{42}\dot{\theta}).$$
$$I_{zz}\ddot{\theta} = k_1 r_{12}(x_1 - x_5 - r_{11}\phi - r_{12}\theta) + b_1 r_{12}(\dot{x}_1 - \dot{x}_5 - r_{11}\dot{\phi} - r_{12}\dot{\theta}) + k_2 r_{22}(x_2 - x_5 - r_{21}\phi + r_{22}\theta) + b_2 r_{22}(\dot{x}_2 - \dot{x}_5 - r_{21}\dot{\phi} + r_{22}\dot{\theta}) - k_3 r_{32}(x_3 - x_5 + r_{31}\phi - r_{32}\theta) - b_3 r_{32}(\dot{x}_3 - \dot{x}_5 + r_{31}\dot{\phi} - r_{32}\dot{\theta}) - k_4 r_{42}(x_4 - x_5 + r_{41}\phi + r_{42}\theta) - b_4 r_{42}(\dot{x}_4 - \dot{x}_5 + r_{41}\dot{\phi} + r_{42}\dot{\theta}).$$

III. TIRE MODELS

Tires provide the necessary friction forces required by the vehicle against externally applied forces of traction, braking and lateral forces. When externally applied forces become excessive, the tires can no longer provide sufficient friction forces to overcome their effect, so they start slipping [19]. The tire slips are different when the vehicle is accelerating, when it is braking or moving along a curved path as given by Equation 6 [1,3,19]. Slipping in tires causes the vehicle to move at slower speeds and in arbitrary directions (tire slip angle). The tire force-slip angle relationship is anti-symmetric with hysteresis [1,3,19].

There are several models available for tires. The first tire model was the Fiala model presented in 1954 [20], in which the tire stiffness was assumed to be uniform along the tire slip angle C_α and tire center line C_σ.

The Fiala tire model is important because it describes the relationship of the reaction force μF_z on tire adhesion, measured in terms of stiffness coefficients along the longitudinal and lateral direction. It does not provide the relationship between tire and slip [1-3,20].

$$\lambda_x = \begin{cases} \dfrac{R\Omega_w - V_x}{R\Omega_w} & \textit{when accelerating} \\ \dfrac{V_x - R\Omega_w}{V_x} & \textit{when braking} \\ \dfrac{V_x}{R\Omega_w}\tan\alpha & \textit{when turning} \end{cases} \tag{6}$$

$$\textit{while } \mu F_z = 3\theta\sigma - \frac{1}{3}\theta\lambda^2 + \frac{1}{27}\left(3\theta\lambda^3\right) + \ldots$$

The Dug off tire was the first model that provided the analytical relationship of slip with force as given in Equation (7).

$$F_{xi} = \begin{cases} J_i \lambda_i & when \quad \lambda \geq 0 \\ J_i \lambda_i (\lambda_i + 1) & otherwise \end{cases},$$

$$F_{yi} = \begin{cases} u_i \tan(\alpha_i) \\ u_i(\lambda_i + 1)\tan(\alpha_i) \end{cases}, \tag{7}$$

$$where \quad J_i = \frac{C_{xi}\lambda_i}{1+\lambda_i} f(\gamma_i), \quad u_i = \frac{C_{yi}\tan(\alpha_i)}{1+\lambda_i} f(\gamma_i)$$

$$\gamma_i = \frac{\mu F_z (1+\lambda_i)}{2\sqrt{(C_{xi}\lambda_i)^2 + C_{yi}\tan\alpha_i}}, f(\gamma_i) = \begin{cases} 1 & if \quad \gamma \geq 1 \\ (2-\gamma_i)\gamma_i & otherwise \end{cases}$$

This model provides a comprehensive analytical model for tires. It does not however, consider the direction of motion effect on the performance of tires. The geometrical properties of the tires are also not sufficiently explained for obtaining the tires stiffness.

A more thorough model was the magic formula model presented in delft, October 1991 [21], which was later modified in 1997 [22]. According to the magic tire model, the forces exerted in the longitudinal (F_x) and lateral (F_y) direction are different and related with the tire shape, curvature, longitudinal slip angle and lateral slip angle as given by Equation (8)[1, 3, 21-22].

$$x(k+S_H) = \eta\sin[\upsilon \tan^{-1} \varsigma(k+S_H)] - E\{\varsigma(k+S_H) - \tan^{-1}\varsigma(k+S_H)\}]; \quad F_x = x(k+S_H)+S_H$$

$$y(\alpha+S_H) = \eta\sin[\upsilon \tan^{-1} \varsigma(\alpha+S_H)] - E\{\varsigma(k+S_H) - \tan^{-1}\varsigma(\alpha+S_H)\}]; \quad F_y = y(\alpha+S_H)+S_H \tag{8}$$

$$C_k = \frac{V_x}{J} \quad C_\alpha = \frac{V_x}{J}, \quad where \quad J = \max(V, \varepsilon_y)$$

The output behavior of this equation is modified with the parameters [υ, ς, k, E, S_V, S_H]. The various shape para meters used in the magic formula model are defined in Figure 4.

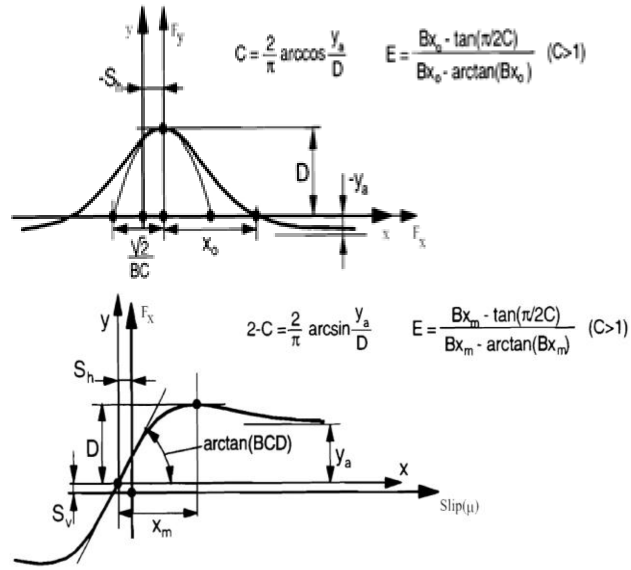

Figure 4: Definition of various parameters of Magic Formula Model [21]

Since, tires are responsible for providing the necessary traction and braking forces required by the automobile to be controllable [1,3]. They also help in overcoming the excessive lateral forces exerted on the automobile during heavy turns [23-27]. A better tire model can help in designing a controller that can help in better yaw control. The magic tire model is a promising model [45].

IV. STATE SPACE REPRESENTATION OF FULL CAR MODEL

The full car model can be linearized and represented in the state space representation form. The process of linearization starts with calculation of the wheel slip angle [28-30,43,44]. The 4-wheel slip angles are resolved using the steering angle inputs and velocity components. The electronic steering system used to model the steering input is also included. Similarly, the slips in each wheel are modeled using the wheel traction equation for angular velocity [28].

The space matrix representation of the full car model is given in Equation (9)[28-30].

$$a_{11} = \frac{A_p|V_x|}{M};$$

$$a_{12} = \omega_z + \frac{K_1\sin\delta}{M(V_x+e\omega_z)} + \frac{K_2\sin\delta}{M(V_x-e\omega_z)};$$

$$a_{13} = \frac{aK_1\sin\delta}{M(V_x+e\omega_z)} + \frac{aK_2\sin\delta}{M(V_x+e\omega_z)};$$

$$a_{15} = \frac{J_1\cos\delta}{M}; \tag{9}$$

$$a_{14} = \frac{(K_1+K_2)\sin\delta}{M};$$

$$a_{23} = \frac{aK_1\cos\delta}{M(V_x+e\omega_z)} + \frac{aK_2\cos\delta}{M(V_x-e\omega_z)} - \frac{bK_3}{M(V_x-e\omega_z)} - \frac{bK_4}{M(V_x+e\omega_z)};$$

$$a_{16} = \frac{J_2\cos\delta}{M}; \quad a_{17} = \frac{J_3}{M}; \quad a_{18} = \frac{J_4}{M}; \quad a_{21} = -\omega_z;$$

$$a_{37} = -e\frac{J_3}{I_z}, \quad a_{38} = e\frac{J_4}{I_z}, \quad a_{68} = -\frac{(1+\lambda_2)}{V_x}a_{18}$$

$$a_{83} = -\frac{(1+\lambda_4)}{V_x}a_{14}, \quad a_{44} = -\frac{1}{\tau}, \quad a_{63} = -\frac{(1+\lambda_2)}{V_x}a_{14}$$

$$a_{22} = \frac{A_p\left|V_y\right|}{M} + \frac{K_1\cos\delta}{M(V_x+e\omega_z)} + \frac{K_2\cos\delta}{M(V_x-e\omega_z)} + \frac{K_3}{M(V_x-e\omega_z)} + \frac{K_4}{M(V_x+e\omega_z)},$$

$$a_{86} = -\frac{(1+\lambda_4)}{V_x}a_{16}$$

$$a_{87} = -\frac{(1+\lambda_4)}{V_x}a_{17}, \quad a_{24} = \frac{(K_1+K_2)}{M}, \quad a_{25} = \frac{J_1\sin\delta}{M};$$

$$a_{26} = \frac{J_2\cos\delta}{M}, \quad a_{25} = \frac{J_1\sin\delta}{M}, \quad a_{26} = \frac{J_2\cos\delta}{M};$$

$$a_{88} = -\frac{(1+\lambda_4)}{V_x}a_{18} - \frac{R_{w4}^2J_4}{I_wV_x}, \quad a_{32} = \frac{K_1(a\cos\delta - e\sin\delta)}{I_z(V_x+e\omega_z)} + \frac{K_2(a\cos\delta+e\sin\delta)}{I_z(V_x-e\omega_z)} - \frac{bK_3}{I_z(V_x-e\omega_z)} - \frac{bK_4}{I_z(V_x+e\omega_z)};$$

$$a_{81} = -\frac{(1+\lambda_4)}{V_x}a_{11},$$

$$a_{33} = \frac{K_1a(a\cos\delta - e\sin\delta)}{I_z(V_x+e\omega_z)} + \frac{K_2a(a\cos\delta+e\sin\delta)}{I_z(V_x-e\omega_z)} - \frac{b^2K_3}{I_z(V_x-e\omega_z)} - \frac{b^2K_4}{I_z(V_x+e\omega_z)};$$

$$a_{78} = -\frac{(1+\lambda_3)}{V_x}a_{18}, \quad a_{15} - \frac{R_{w1}^2J_1}{I_wV_x}, \quad a_{55} = -\frac{(1+\lambda_1)}{V_x}$$

$$a_{34} = \frac{K(a\cos\delta - e\sin\delta) + K(a\cos\delta+e\sin\delta)}{I_z},$$

$$a_{35} = \frac{J_1(a\sin\delta+e\cos\delta)}{I_z}, \quad a_{36} = \frac{J_2(a\sin\delta - e\cos\delta)}{I_z};$$

$$a_{51} = -\frac{(1+\lambda_1)}{V_x}a_{11}, \quad a_{52} = -\frac{(1+\lambda_1)}{V_x}a_{12}, \quad a_{53} = -\frac{(1+\lambda_1)}{V_x}a_{13},$$

$$a_{54} = -\frac{(1+\lambda_1)}{V_x}a_{14}, \quad a_{84} = -\frac{(1+\lambda_4)}{V_x}a_{14}, \quad a_{85} = -\frac{(1+\lambda_4)}{V_x}a_{15},$$

$$a_{56} = -\frac{(1+\lambda_1)}{V_x}a_{16}, a_{57} = -\frac{(1+\lambda_1)}{V_x}a_{17}, a_{58} = -\frac{(1+\lambda_1)}{V_x}a_{18},$$

$$a_{61} = -\frac{(1+\lambda_2)}{V_x}a_{11}, \; a_{62} = -\frac{(1+\lambda_2)}{V_x}a_{12}, a_{64} = -\frac{(1+\lambda_2)}{V_x}a_{14},$$

$$a_{65} = -\frac{(1+\lambda_2)}{V_x}a_{15}, a_{66} = -\frac{(1+\lambda_2)}{V_x}a_{16} - \frac{R^2{}_{w2}J_2}{I_wV_x},$$

$$a_{67} = -\frac{(1+\lambda_2)}{V_x}a_{17}, a_{76} = -\frac{(1+\lambda_3)}{V_x}a_{16}, a_{71} = -\frac{(1+\lambda_3)}{V_x}a_{11},$$

$$a_{72} = -\frac{(1+\lambda_3)}{V_x}a_{12}, a_{73} = -\frac{(1+\lambda_3)}{V_x}a_{14}, \; a_{74} = -\frac{(1+\lambda_3)}{V_x}a_{14},$$

$$a_{75} = -\frac{(1+\lambda_3)}{V_x}a_{15}, \quad a_{77} = -\frac{(1+\lambda_3)}{V_x}a_{17} - \frac{R_{w3}^2J_3}{I_wV_x}$$

$$a_{82} = -\frac{(1+\lambda_4)}{V_x}a_{12},$$

$$\begin{bmatrix}\dot{x}_1\\\dot{x}_2\\\dot{x}_3\\\dot{x}_4\\\dot{x}_5\\\dot{x}_6\\\dot{x}_7\\\dot{x}_8\end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} & a_{17} & a_{18}\\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} & 0 & 0\\ 0 & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} & a_{37} & a_{38}\\ 0 & 0 & 0 & a_{44} & 0 & 0 & 0 & 0\\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} & 0 & 0\\ a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & a_{66} & a_{67} & 0\\ a_{71} & a_{72} & a_{73} & a_{74} & a_{75} & a_{76} & a_{77} & a_{78}\\ a_{81} & a_{82} & a_{83} & a_{84} & a_{85} & a_{86} & a_{87} & a_{88}\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\\x_5\\x_6\\x_7\\x_8\end{bmatrix} + \begin{bmatrix}0\\0\\0\\b_{14}\\0\\0\\0\\0\end{bmatrix}u$$

The model given by (9), considers the automobile traction system, the electronic steering system response, and the tire dynamics. It also considers the mass distribution of the automobile. The model therefore, is more accurate. Since the model includes several sub-models, traction and braking model, tire model, full car model, the model consists of a large number of states and parameters. The proper use of this model is possible if the required parameters and states are available. Since many of the parameters are not measurable [1,3], estimators of various types are suggested.

V. State and Parameter Observer

State and parameter estimators are used to estimate unknown states using either automobile kinematics equations, called "Kinematics based Estimation", or automobile model equations, called “Model based Estimation”. Tables 1 and 2, list both types of observers used for the estimation of various parameters and states.

Table 1
List of Some Kinematics Based Observers

Variables Measured	Variables Estimated	Method Used
Ω_w, a_y, δ	$\dot{\phi}$, $\dot{\beta}$	Nonlinear Observer [4], [31], [37], [42]
a_x, a_y, a_z	V_x, V_y	Kalman Filter [2], [26]
V_{GPS}, p_{GPS}	V_x V_y M θ ϕ	Kalman Filter [35], [42]
T_s ω_x ω_y ω_z	M, μ	Recursive least square [11], [33]
A_x,A_y,V	M_z	Fuzzy Logic [25]
V_{GPS} ,ϕ_{GPS} θ_{GPS}	V,ψ	SO3 Filter [32]
Received Mobile Signals	V	Triangulation [34]
ψ_{GPS} ,θ_{GPS} Image	$p(x)$,V	Particle Filter [36]

Table 2
List of Parameters And States Estimated Using Model Based Estimation

Variables Measured	Variables Estimated	Model Used	Method Used
$\delta\ \omega_z\ \psi_{GPS}$	$V_x\ V_y\ \omega_{bx}$ $\alpha_f\ \alpha_r$	Bicycle	Kalman Filter [5]
$a_x,\ a_y,\ \psi$	$V_x,\ V_y,$ $\theta,\ \mu$	Full Car	Nonlinear Observer [6], [12], [41]
$\delta,\ \psi,\ a_y$	$\beta, h_{cg}\ CG$	Bicycle	Multiple Models with switch [38]
$\delta,\ a_y\ V_x\ M$	β	Full Car	DHME [39]
$a_x, a_y, a_z,$ $\omega_x, \omega_y, \omega_z$	V_x, V_y, V_z ϕ, θ	Bicycle	Kalman Filter [40]
ω_z, δ, β	M_z	Full Car	Fuzzy Logic [25]
Ω_w, a_x, a_z ω_x, ω_z	$\mu,\ F_z$	Full Car	Extended Kalman Filter [42]

In either form of estimation, various sensors are used with complex equations. The performance of the estimators depends upon the performance, speed and cost of the algorithm and on the accuracy of the sensors [1, 3, 24, 46]. The above tables list the number of sensors used in respective observers. Among the estimation schemes, the most suitable observer is using SO (3) based observer, since its computational complexity is significantly less than the kalman based and non-linear observer based schemes [32, 46].

VI. Conclusion

This paper presents a conclusive review of the different vehicle models used for the control and analysis of automobiles. The bicycle model presented in the simplest model but it does not include the interaction of the four wheels in obtaining the slips. The full car model is a complete representation of the vehicle chassis motion but it is incomplete until the suspension dynamics and tire dynamics are included, making the model nonlinear. Linear methods, therefore, become in adequate for controller design and estimation. Nonlinear schemes like sliding mode [43-44] optimal control [28-30] and intelligent control [24-25] are possible candidates.

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