2-DOF Lead-plus-PI Control Approach for Magnetic Levitation System

A. C. R. Soon[1], S. H. Chong[1], M. A. Said[2]
[1]Motion Control Research Laboratory, Faculty of Electrical Engineering, Universiti Teknikal Malaysia Melaka, 76100 Durian Tunggal, Melaka, Malaysia.
[2]Department of Electrical Engineering Technology, Faculty of Engineering Technology, Universiti Teknikal Malaysia Melaka, 76100 Durian Tunggal, Melaka, Malaysia.
horng@utem.edu.my

***Abstract*—This paper proposes Two-Degree of Freedom (2-DOF) Lead-plus-PI a classical linear control system for positioning control of a magnetic levitation (maglev) system. Maglev system has practical importance in many engineering system. However, maglev has inherently nonlinear and open loop unstable characteristics. Thus, it is a challenging task to control the maglev system. In this paper, the 2-DOF Lead-plus-PI controller is developed to control the positioning performance of the maglev system as it has simple control structure and straightforward design procedure that can be designed using root locus technique and Ziegler Nichols second method. The proposed controller can be easily implemented into the maglev system without require deep knowledge in control system. The effectiveness of the proposed controller is validated experimentally. Experimental results show the 2-DOF Lead-plus-PI controller has a better positioning accuracy and transient response in point-to-point motion, as compared to Lead-plus-PI controller. The proposed controller shows a position accuracy of 40 μm, which is around the vibration amplitude of the sensor output in open loop. It also takes less than 1 second to stabilize the ball within ± 200 μm and the steady state error has improved to around 45% in point-to-point positioning performance. Besides, the proposed controller also reduced the tracking error to about 48% as compared to Lead-plus-PI controller.**

***Index Terms*—Maglev; Classical Controller; Feedforward type; 2-DOF Controller; Positioning Performance.**

I. INTRODUCTION

Lately, the development of magnetic levitation (maglev) system has gained wide attention around the globe because of their practical importance in many engineering systems such as high speed maglev passenger trains, magnetic bearing, flywheel energy system, vibration isolation systems, photo-lithography, elevators models and the magnetic platform found in medical application [1]. Maglev system has been regularly used in the high-speed motion applications due to their contactless and frictionless characteristics. Besides, it can reduce the noise and components wears significantly. However, the electromagnetic maglev system is inherently open loop unstable. It can be described by highly nonlinear differential equation such as the relationship between electromagnetic force with current and electromagnetic force with levitated displacement which present additional difficulties in controlling the system [2]. Therefore, it is a challenging task to design a feedback controller for achieving good positioning performance of the maglev system.

Classical controller such as proportional-integral-derivative controller and lead-lag compensators are popular in many industries due to their simple structure and easy design procedure which can be designed by using Ziegler Nichols tuning method and root locus techniques. Besides, the classical controller only requires the position sensor when controlling the maglev system. Thus, it saves the cost when the classical controller is implemented into the system. Several types of classical controller such as PD controller, PI-plus-Lead controller and phase lead compensator were designed by using the root locus techniques to stabilize the maglev system [3].

Furthermore, nonlinear controller has been proposed to achieve better positioning performance of the maglev system. Feedback linearization technique was proposed by John in [4] to improve the maglev system robustness against payload variation. The input-output linearization and exact linearization technique were proposed by Ahmed in [5] and Barie in [6] respectively to enhance the positioning performance of the maglev system. Besides, Wai has proposed the backstepping design technique in [7] to improve the positioning performance of the maglev system by including the compensation for parametric uncertainty in the design procedure. However, the nonlinear controllers require the full-state information in the design procedure.

The advance controller such as sliding mode control and H_∞ control were designed by Dan in [8] and Shen in [9] for compensating the disturbances and nonlinearities occurrence to achieve better positioning performance in the maglev system. Furthermore, Al-Muthairi has modified the sliding mode control in [10] to reduce the chattering occurrence phenomenon in the conventional sliding mode control system. Lin in [11] has implemented the adaptive neural network in the robust sliding mode control to enhance the positioning performance of the maglev system by considering the uncertainties in the design procedure. The performance of sliding mode control is limited because it has chattering phenomena occurrence due to the switching condition. On the other hand, the adaptive control technique requires a complex computations and exhibits unsatisfactory transient performance which rely on the speed of the adaptive parameter estimation. Furthermore, the performance of the adaptive control technique is highly dependent on the system parameters accuracies.

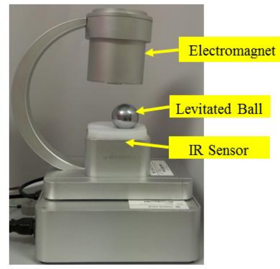
The intelligent controller such as Fuzzy Logic Control (FLC) has been implemented in the maglev system to stabilize the levitated ball at the desired position. The single input FLC was proposed by Kashif to reduce the rules and tuning parameters of the conventional FLC which greatly depended on the inference rules to provide an accurate control action [12]. FLC is extensively applied in the nonlinear system because it does not require any mathematical modeling in the design procedure.

Classical controller is chosen in this paper because it has simple control structure and straightforward design procedure. Despite the simplicity of design procedure of conventional classical control system, the control system can only achieve a good disturbance compensation or a rapid transient to a set point response, but not both at the same time. In [13]-[14], the researchers had proposed the 2-DOF control system to solve the limitation of the conventional classical control system in positioning and disturbance rejection performance. The performance is evaluated through simulation of typical industrial processes that can be represented by the transfer functions. Ghosh had designed the 2-DOF PID compensation using pole placement method to improve the transient performance and control effort responses of the maglev system in [15]. Therefore, the 2-DOF Lead-plus-PI controller which consists of proportional gain at the feedforward path is proposed in this paper to improve the system positioning performance. The positioning performance of the proposed controller is validated through point-to-point motions and tracking motions.

The rest of the paper is outlined as follow: Section II introduces the experimental setup and mathematical modeling of the maglev system. The controller design procedure is presented in Section III. The results are evaluated and discussed in Section IV. Lastly, conclusion is drawn in Section V.

II. EXPERIMENTAL SETUP

The maglev system in Figure 1(a) is used as a testbed to evaluate the usefulness of the 2-DOF Lead-plus-PI controller and Fig. 1 (b) illustrates the dynamic model of the maglev system. It is an upper drive coils one degree of freedom maglev system, where an electromagnet exerted a tractive force across an air gap to levitate a ferromagnetic ball. It is a voltage-controlled maglev system, which consists of current amplifier to actuate the electromagnet. The ball levitation height is changed by varying the controlled voltage of the electromagnet. The mechanism has a working range of $5\,\text{mm}$. To measure the displacement of the levitated ball, a laser position sensor (Panasonic laser distance sensor HG-C1050) with a resolution of $1.83\,\mu\text{m}$ is used. The controller is designed in MATLAB/SIMULINK 2014b environment and interfaced with the real plant by using the xPC Target at a sampling rate of $1\,\text{kHz}$.



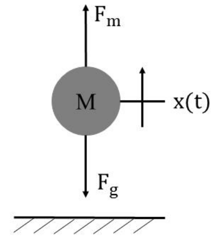


Figure 1: Magnetic levitation ball system and dynamic model of the system (a) Magnetic levitation system (b) Dynamic model of the maglev system

The equation of motion of the maglev system is defined as:

$$M\frac{d^2x}{dt^2} = F_m + F_g \tag{1}$$

The electromagnetic force, F_m is described by the following:

$$F_m = -K\frac{i^2}{x^2} \tag{2}$$

F_m is always negative indicating that it always working in opposite direction against gravitational force, F_g where:

$$F_g = Mg \tag{3}$$

By substituting (2) and (3) into (1), the nonlinear differential equation relating levitated ball position, x and electromagnet coil current, i is defined as:

$$\frac{d^2x}{dt^2} = -\frac{K}{M}\frac{i^2}{x^2} + g \tag{4}$$

Equation (2) shows the inherently nonlinear characteristic of F_m which can be linearized using Taylor Series approximation at the equilibrium position where:

$$F_m(i,x) = F_m\left(i_o, x_o\right) + \frac{\partial F_m(i,x)}{\partial i} i(t) + \frac{\partial F_m(i,x)}{\partial x} x(t) \tag{5}$$

During equilibrium position, the relationship between F_m and F_g is given by:

$$Mg = -F_m\left(i_o, x_o\right) \tag{6}$$

Substituting (5) and (6) into (1), and taking Laplace transform on (1), the linearized open loop transfer function is:

$$\frac{X(s)}{I(s)}=\frac{-\dfrac{2Ki_o}{Mx_o^{2}}}{\left[s^2-\dfrac{2Ki_o^{2}}{Mx_o^{3}}\right]} \tag{7}$$

The voltage to current amplifier is implemented into the system which can be defined as,

$$G_a(s)=\frac{V_i(s)}{I(s)}=K_a \tag{8}$$

The output voltage across the sensor is directly proportional to the steel ball levitated position, which can be modelled as:

$$G_s(s)=\frac{V_s(s)}{X(s)}=K_s \tag{9}$$

From the equation above, the overall transfer function between the input voltage to the electromagnet and the output voltage of the sensor can be defined as:

$$G(s)=\frac{V_s(s)}{V_i(s)}=\frac{-K_s\dfrac{2Ki_o}{Mx_o^{2}}}{K_a\left[s^2-\dfrac{2Ki_o^{2}}{Mx_o^{3}}\right]} \tag{10}$$

Equation (10) shows the uncompensated system is unstable because one pole is located at the right-half plane of *s*-plane. The maglev system parameters value is shown in Table 1.

Table 1
Model parameters

Symbol	Description, unit	Value
M	Steel ball mass, Kg	9.400×10^{-2}
x_o	Nominal air gap, m	10.000×10^{-3}
i_o	Nominal current, A	3.943×10^{-1}
K	Electromagnetic constant, Nm^2/A^2	2.314×10^{-4}
K_a	Power amplifier, V/A	6.508×10^{0}
K_s	Sensor sensitivity, V/m	-1.667×10^{2}
g	Gravitational acceleration, m/s^2	9.807×10^{0}
F_m	Electromagnetic force, N	-3.598×10^{-1}
F_g	Gravitational force, N	9.219×10^{-1}

III. Controller Design

The 2-DOF Lead-plus-PI controller is designed using the root locus technique and Ziegler Nichols (Z-N) second method. The stabilization and positioning performance are controlled by the Lead-plus-PI controller which is designed with root locus method, followed by the design of feedforward proportional gain with Z-N 2nd method to improve the system transient response and positioning accuracy. The block diagram of the proposed controller is shown in Figure 2. Equation (11) shows the transfer function of the 2-DOF Lead-plus-PI controller where *C(s)* is the Lead-plus-PI controller and $C_f(s)$ is the feedforward proportional gain.

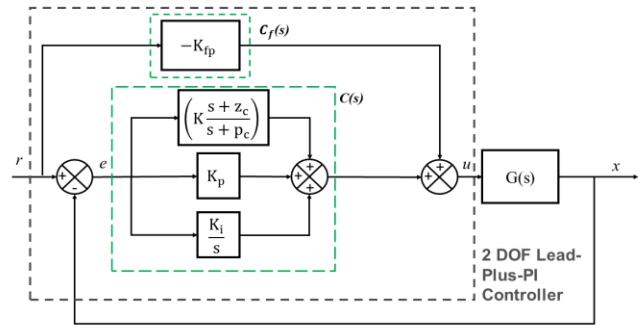


Figure 2: Block diagram of 2-DOF Lead-plus-PI controller

Lead-plus-PI controller:

$$C(s)=K\frac{s+z_c}{s+p_c}+K_p+\frac{K_i}{s} \tag{11a}$$

Feedforward proportional gain:

$$C_f(s)=-K_{fp} \tag{11b}$$

The transfer function between reference input, *R(s)* and actual output, *X(s)* of the 2-DOF Lead-plus-PI control system is:

$$\frac{X(s)}{R(s)}=\frac{G(s)C(s)-G(s)C_f(s)}{1+C(s)G(s)} \tag{12}$$

The reference input, *R(s)* and control signal, *U(s)* transfer function becomes:

$$\frac{U(s)}{R(s)}=\frac{C(s)-C_f(s)}{1+C(s)G(s)} \tag{13}$$

The Lead-plus-PI control system yields, *X(s)/R(s)* = *C(s)G(s)/1+C(s)G(s)* and *U(s)/R(s)* = *C(s)/1+C(s)G(s)*. Both of the control systems have an identical open loop transfer function where:

$$L(s)=C(s)G(s) \tag{14}$$

From the theoretical derivation, both controllers consist of identical loop robustness. However, the 2-DOF Lead-plus-PI control system, which consists of feedforward proportional gain that can drive the mechanism faster to improve the transient response and tracking motions.

The 2-DOF Lead-plus-PI control design procedure is presented as follow:

Step 1: Lead-plus-PI controller design

a) Lead compensator design
   i) *A zero, zc is added to the system:*
      A zero is added in the left half-plane of *s*-plane which is near to the open loop pole location to move the root locus into the left half-place.
   ii) *A pole, pc is added to the system:*

A pole is placed at a location that 7 times farther than the z_c to improve the system transient response. If the pole is placed too close to z_c, the root locus moves back towards it uncompensated shape. On the other hand, there is no significant improvement at transient response when the pole is located more than 7 times further than the z_c.

iii) Gain adjustment, K:

The gain, K is calculated to meet the desired specification. The gain is then fine-tuned experimentally.

* The steady state error stays around 30% of 6 mm, 5 mm and 4 mm jump heights. Therefore, PI controller is designed to improve the steady state error by 0.3 factors (30% / 100%).

b) PI compensator design

Assume: Velocity error constant, $K_v = 0.3$

i) Calculating the integral gain, K_i:

The K_i is calculated using the velocity error constant equation which is defined as:

$$K_v = \lim_{s\to 0} sG_{PI}(s)G_{Lead}(s)G(s) \qquad (15)$$

From the calculation, K_i is obtained as 0.27.

ii) Calculating the proportional gain, K_p:

The K_p is obtained by setting the asymptote location, σ_A further than the dominant pole location to guarantee the two branches of the loci bend into the desired region. The asymptote location, σ_A is defined as:

$$\sigma_A = \frac{\sum(-p_i) - \sum(-z_i)}{n_p - n_z} \qquad (16)$$

Based on the calculation, the K_p must be greater than 0.002 to achieve the desired performance. The K_p and K_i are then fine-tuned experimentally.

Step 2: Feedforward proportional gain design

Determination of feedforward proportional gain, K_{fp}:

The feedforward gain, K_f is increased until the occurrence of excessive overshoot. Then, K_{fp} is calculated as:

$$K_{fp} = 0.5K_f \qquad (17)$$

IV. Results

The effectiveness of the 2-DOF Lead-plus-PI controller is evaluated through experiments. The positioning performance of the proposed controller is compared with Lead-plus-PI controller. The controller parameters are shown in Table 2. In the result, the zero (0) mm indicates the initial position for the experiments.

Table 2
Controller Parameters

Controller	K	K_p	K_i	K_{fp}
Lead-plus-PI	3.000	0.010	0.300	-
2-DOF Lead-plus-PI	3.000	0.010	0.300	0.105

a) Positioning performance

Figure 3 shows the experimental positioning responses of 2 controllers: Lead-plus-PI and 2-DOF Lead-plus-PI controller to 6 mm, 5 mm and 4 mm jump heights respectively. In 6 mm jump height (see Figure 5 (a)), there are significant vibration as compared to 5 mm and 4 mm jump heights. This phenomenon is caused by the nonlinearities of the electromagnet as the ball is levitated closer to it.

The settling time, T_s is determined within ± 2% of the final position. From the experimental results, the 2-DOF Lead-plus-PI controller is able to settle faster within ± 200 µm at 6 mm, 5 mm and 4 mm jump heights as compared with the Lead-plus-PI controller. In Figure 5, the 2-DOF Lead-plus-PI controller shows better transient response with shorter settling time and greater positioning accuracy than the Lead-plus-PI controller. The quantitative results of ten (10) times repeatability in Table III shows that the 2-DOF Lead-plus-PI controller improves the settling time and positioning accuracy by 61% and 45% respectively as compared with the Lead-plus-PI controller.

Table 3
Average of ten (10) Experiments for positioning performance – the 2-DOF Lead-Plus-PI and the Lead-plus-PI Controllers

Jump height	Performance index	Lead-plus-PI	2-DOF Lead-plus-PI
6 mm	T_s, s	1.996×10^{0}	7.780×10^{-1}
	OS, %	0.000×10^{0}	5.478×10^{0}
	e_{ss}, mm	8.807×10^{-3}	4.856×10^{-3}
5 mm	T_s, s	1.688×10^{0}	4.450×10^{-1}
	OS, %	0.000×10^{0}	0.000×10^{0}
	e_{ss}, mm	1.107×10^{-2}	7.856×10^{-3}
4 mm	T_s, s	1.556×10^{0}	8.230×10^{-1}
	OS, %	0.000×10^{0}	2.924×10^{0}
	e_{ss}, mm	1.004×10^{-2}	3.959×10^{-3}

b) Tracking performance

The sinusoidal trajectory is chosen to examine the controllers' trajectory tracking capabilities. Therefore, sinusoidal reference inputs with two different amplitudes and frequencies are applied to the maglev system for tracking motion. The maximal tracking error is stated as $\max|x_r - x|$ where x_r is the reference input and the x denotes the displacement of the levitated ball. In addition, the root mean square (RMS) error (e_{RMS}) is calculated using (18) where N is the number of data sample and e is the tracking error.

$$e_{RMS} = \sqrt{1/N \sum_{k=1}^{N} e^2} \qquad (18)$$

The 2-DOF Lead-plus-PI controller shows a better tracking performance than the Lead-plus-PI controller (see Figure 4 to 7). The 2-DOF Lead-plus-PI controller reduces the tracking error amplitude by 48% and e_{RMS} is 1.9 times smaller than the Lead-plus-PI controller. Thus, the 2-DOF Lead-plus-PI controller has shown a better motion accuracy (smaller maximal tracking error) than the Lead-plus-PI controller. The 2-DOF Lead-plus-PI controller consists of feedforward proportional gain that drives the mechanism faster than the Lead-plus-PI controller to reduce the tracking error. The quantitative comparison of ten (10) repeatability tests are shown in Table 4.

Table 4
Average of Ten (10) Experiments for tracking Performance – the 2-DOF Lead-plus-PI and the Lead-Plus-PI Controllers

Reference input	Controller	$\mathbf{max}\lvert x_r - x\rvert$ *Average, mm*	e_{RMS} *Average, mm*
sinusoidal 0.5 mm, 0.5 Hz	Lead-plus-PI	9.031×10^{-1}	5.703×10^{-1}
	2-DOF Lead-plus-PI	3.921×10^{-1}	2.413×10^{-1}
sinusoidal 1.0 mm, 0.5 Hz	Lead-plus-PI	1.930×10^{0}	1.104×10^{0}
	2-DOF Lead-plus-PI	9.303×10^{-1}	5.477×10^{-1}
sinusoidal 0.5 mm, 1.0 Hz	Lead-plus-PI	8.881×10^{-1}	5.633×10^{-1}
	2-DOF Lead-plus-PI	5.089×10^{-1}	3.193×10^{-1}
sinusoidal 1.0 mm, 1.0 Hz	Lead-plus-PI	2.029×10^{0}	1.194×10^{0}
	2-DOF Lead-plus-PI	1.131×10^{0}	6.590×10^{-1}

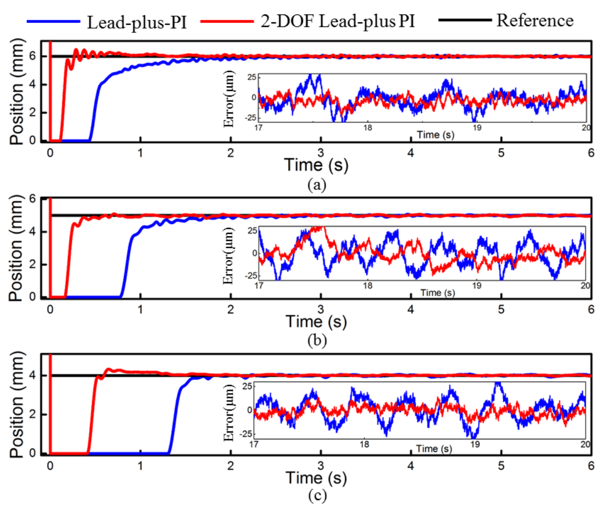


Figure 3: Step response to (a) 6 mm (b) 5 mm and (c) 4 mm jump height

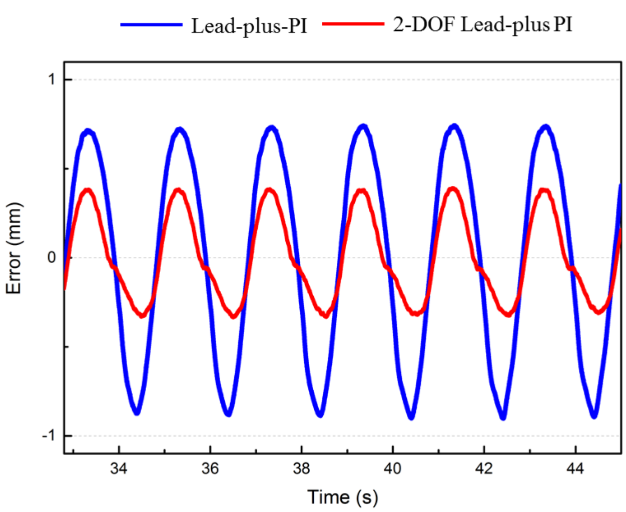


Figure 4: Tracking motion for Lead-plus-PI and 2-DOF Lead-plus-PI with frequency 0.5 Hz and amplitude 0.5 mm

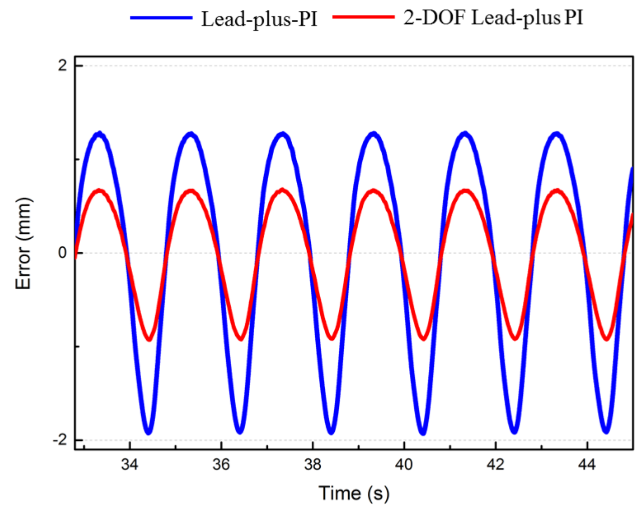
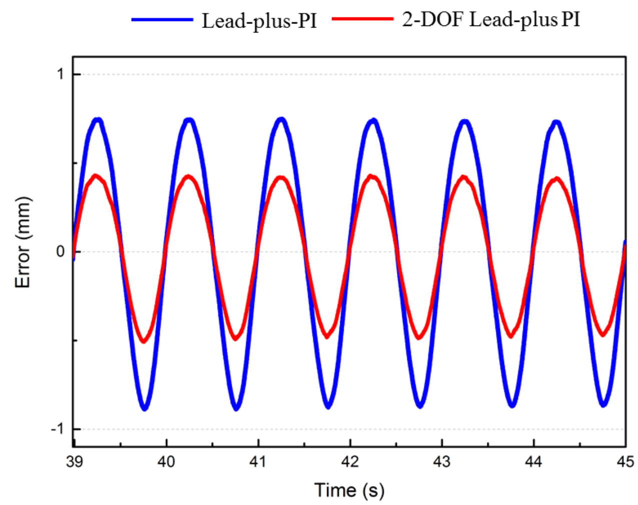


Figure 5: Tracking motion for Lead-plus-PI and 2-DOF Lead-plus-PI with frequency 0.5 Hz and amplitude 1.0 mm



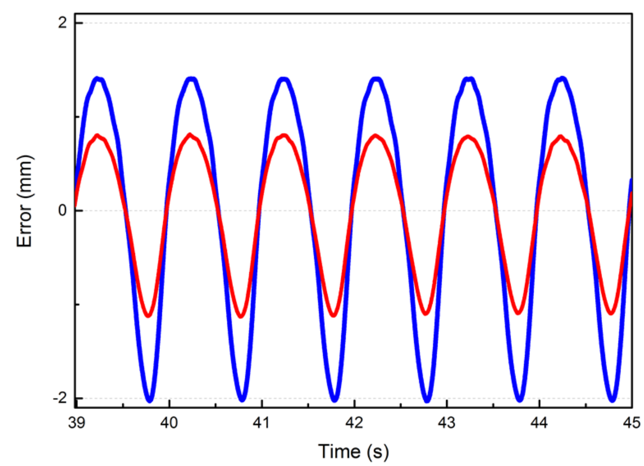


Figure 7: Tracking motion for Lead-plus-PI and 2-DOF Lead-plus-PI with frequency 1.0 Hz and amplitude 1.0 mm

V. CONCLUSIONS

This paper presented the 2-DOF Lead-plus-PI controller for positioning control of the maglev system. The experimental results proved that, the 2-DOF Lead-plus-PI controller is capable in reducing the settling time and motion error of the levitated ball. It has shown a better performance in positioning and tracking control than the Lead-plus-PI controller. Although the 2-DOF Lead-plus-PI controller has better positioning performance than Lead-plus-PI controller, overshoot reduction for improving the transient response of point-to-point positioning will be done in the future work. Besides, the disturbance rejection performance of the 2-DOF Lead-plus-PI controller will also be included in the future work.

ACKNOWLEDGMENT

The authors would like to be obliged to Motion Control Research Laboratory, Universiti Teknikal Malaysia Melaka for providing the laboratory facilities and equipment support. The Fundamental Research Grant Scheme (FRGS/2/2014/TK03/FKE/02/F00240) and scholarship MyBrain UTeM from UTeM is gratefully acknowledged. Besides, the authors would also like to thank all his laboratory members for providing ideas and comments in this paper.

REFERENCES

[1] M. Lashin, A. T. Elgammal, A. Ramadan, A. A. Abouelsoud, and S. F. M. Assal, “Fuzzy-based Gain Scheduling of Exact FeedForward Linearization Control and Sliding Mode Control for Magnetic Ball Levitation System : A Comparative Study,” in *2014 IEEE International Conference on Automation, Quality and Testing, Robotics*, 2014, pp. 1–6.

[2] P. K. Sinha, *Electromagnetic Suspension Dynamic & Control*. 1987.

[3] N. Shawki and S. Alam, “Design and Implementation of a Magnetic Levitation System Using Phase Lead Compensation Technique,” in *International Forum on Strategic Technology*, 2014, pp. 294–299.

[4] J. Y. Hung, “Nonlinear Control of a Magnetic Levitation System,” in *Industrial Electronics, Control and Instrumentation*, 1991, pp. 268–273.

[5] A. El Hajjaji and M. Ouladsine, “Modeling and Nonlinear Control of Magnetic Levitation Systems,” *IEEE Trans. Ind. Electron.*, vol. 48, no. 4, pp. 831–838, 2001.

[6] W. Barie and J. Chiasson, “Linear and nonlinear state-space controllers for magnetic levitation,” *Int. J. Syst. Sci.*, vol. 27, no. 11, pp. 1153–1163, 1996.

[7] R. Wai and J. Lee, “Development of Levitation Control for Linear Maglev Rail System via Backstepping Design Technique,” in *IEEE Conference on Robotics, Automation and Mechatronics*, 2006, pp. 1–6.

[8] D. Cho, Y. Kato, and S. Darin, “Sliding Mode and Classical Control Magnetic Levitation System,” *IEEE Control Syst.*, vol. 13, no. 1, pp. 42–48, 1993.

[9] Ji.-C. Shen, “H ∞ Control and Sliding mode control of magnetic levitation system,” *Asian Journal of Control*, vol. 4, no. 3. pp. 333–340, 2002.

[10] N. F. Al-Muthairi and M. Zribi, “Sliding Mode Control of a Magnetic Levitation System,” *Math. Probl. Eng.*, vol. 2, no. October 2003, pp. 93–107, 2004.

[11] F.-J. Lin, S.-Y. Chen, and K.-K. Shyu, “Robust dynamic sliding-mode control using adaptive RENN for magnetic levitation system.,” *IEEE Trans. neural networks*, vol. 20, no. 6, pp. 938–951, 2009.

[12] K. Ishaque, Y. Saleem, S. S. Abdullah, M. Amjad, M. Rashid, and S. Kazi, “Single Input Fuzzy Logic Controller for Magnetic Levitation System,” in *2011 4th International Conference on Modeling, Simulation and Applied Optimization*, 2011, pp. 1–6.

[13] M. Araki and H. Taguchi, “Two-Degree-of-Freedom PID Controllers,” *Int. J. Control. Autom. Syst.*, vol. 1, no. 4, pp. 401–411, 2003.

[14] R. Gorez, “New design relations for 2-DOF PID-like control systems,” *Automatica*, vol. 39, pp. 901–908, 2003.

[15] A. Ghosh, T. R. Krishnan, P. Tejaswy, A. Mandal, J. K. Pradhan, and S. Ranasingh, “Design and implementation of a 2-DOF PID compensation for magnetic levitation systems,” *ISA Trans.*, vol. 53, no. 4, pp. 1216–1222, 2014.