

# Walking Motion Trajectory of Hip Powered Orthotic Device using Quintic Polynomial Equation

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**Abstract**—In lower limb exoskeleton system walking motion profile generation, cubic polynomial is commonly used to generate smooth walking profile on flexion angle, velocity and acceleration data of three joint movements (ankle, knee and hip joints). However, cubic polynomial does not closely matched human motion. For this reason, a higher-order-polynomial i.e. quintic polynomial is proposed to generate walking motion profile. Error analysis was conducted to measure how closely quintic polynomial could represent human walking motion profile. Result shows that quintic polynomial could closely represent human walking trajectory with maximum RMS error of 0.2607rad occurred during mid-swing phase.

**Index Terms**—Trajectory Generation; Quintic Polynomial; Exoskeletons Device.

## I. INTRODUCTION

Many diseases cause gait disorder or movement disorder. This causes falling, imbalance walking pattern, and so on. In Malaysia, according to World Health Organization (WHO), 92 percent of Malaysian people who are above 60 years old suffer from disability motion [1]. Whereas, based on the annual report of the Malaysian Ministry of Health, about 7.2 percentages to 11 percentages of children between 0 to 18 years old are sustained with physical and cerebral palsy disabilities [1]. Due to technological advances, powered orthosis devices became a feasible solution to assist walking. However, the movement of the device must be lifelike, imitating exactly the wearer walking pattern. Generating trajectory that is accurately matched to real human motion could protect from discomfort and injuries. There is different methods which are used in order to overcome the difficulties imposed by the extraction of human gait features. Model-based and non-model-based methods are two approaches methods which are being used for human gait analysis [2].

Many methods have been conducted in term of generation trajectory motion and each method has concentrated on specific parameters. Also, each method has its own way to apply knowledge in order to get the expected trajectory.

Polynomial trajectory algorithm is a method that has been applied on a biped robot of 10 DOFs in [3] in order to generate walking trajectories for different topography condition by using Hermite Spline interpolation. Additionally, Cubic spline and Hermite polynomial interpolation are implemented in [4] to generate walking trajectory for 8 Degrees of freedom Biped Robot. To add to that, Task generation of human motion by phase sequences is one method that has been used in [5] to generate a

trajectory motion by using analysis human motion and divide it into various phase based on the phase sequence method which can easy transfer in the humanoid robot. Furthermore, hierarchical trajectory generation method is used and implemented in [6] to generate trajectory (more natural gait) with considering the mechanical effect.

The Ohishi's method is a 3-D dynamic gait control of walking biped robot. In this method, the motion has been described depend on the inverted pendulum relative to sagittal plane and lateral plane in Cartesian space by using the kinematics calculation of biped robot and the robust joint position controller of each joint [7]. Furthermore, Resolved Momentum Control method is used to generate motion the lower part of the body with given linear or angular momenta[8].

Related to the literature review, many researches study about the methods of trajectory generation motion, some of these researches used these methods either to generate motion in upper part (e.g. hands) or lower part of body (e.g. legs). Furthermore many researches have been focused on the lower limb motion such as hip, knee and ankle but each research has been applied a specific method to improve a certain parameters such as stability of movement and getting motion gait which is accurately matched to the real human motion.

Quintic polynomial has been used in another research to generate trajectory motion particularly on the hip joint[9]. However, the suitability in terms of accuracy between human and quintic hip motion profile has not been studied before. For this reason, this paper presents a study on the accuracy of employing quintic polynomial to generate hip motion trajectory.

## II. QUINTIC POLYNOMIAL EQUATION FOR GENERATING WALKING MOTION TRAJECTORY

Quintic polynomial equation is a fifth degree polynomial equation which is formulated as shown in Equation (1) [10]. Quintic polynomial equation can be used to represent hip joint angular position profile as shown in Equation (1).

$$\theta(t) = a_0 + a_1t + a_2t^2 + a_3t^3 + a_4t^4 + a_5t^5 \quad (1)$$

By differentiation Equation (1), the velocity profile is obtained as shown in Equation (2).

$$\dot{\theta}(t) = a_1 + 2a_2t + 3a_3t^2 + 4a_4t^3 + 5a_5t^4 \quad (2)$$

In order to obtain the acceleration profile, we need to differentiate Equation (2) as shown in Equation (3).

$$\theta(t) = 2a_2 + 6a_3t + 12a_4t^2 + 20a_5t^3 \quad (3)$$

From Equation (1) there are six unknown constraints. To determine the values of each of unknown, we can use the Equations (4).

$$\begin{aligned} a_0 &= \theta_0 \\ a_1 &= \dot{\theta}_0 \\ a_2 &= \frac{\ddot{\theta}_0}{2} \\ a_3 &= \frac{[20\theta_f - 20\theta_0 - (8\dot{\theta}_f + 12\dot{\theta}_0)t_f - (3\ddot{\theta}_0 - \ddot{\theta}_f)t_f^2]}{2t_f^3} \\ a_4 &= \frac{[30\theta_0 - 30\theta_f - (14\dot{\theta}_f + 16\dot{\theta}_0)t_f - (3\ddot{\theta}_0 - 2\ddot{\theta}_f)t_f^2]}{2t_f^4} \\ a_5 &= \frac{[12\theta_f - 12\theta_0 - (6\dot{\theta}_f + 6\dot{\theta}_0)t_f - (\ddot{\theta}_0 - \ddot{\theta}_f)t_f^2]}{2t_f^5} \end{aligned} \quad (4)$$

Walking is a complex motion. To allow quantic to represent a complete walking motion, the motion needs to be separated into several phases. Each phase will be represented by a quantic Equation (see Figure 1).

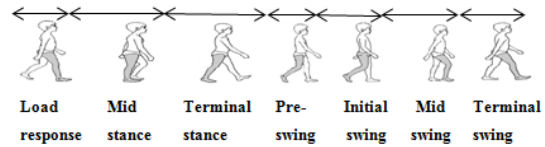


Figure 1: the phases per gait cycle.

To achieve smooth transition from one phase to another, the boundary conditions for initial and final angle (position), velocity and acceleration must be satisfied as shown in (5).

$$\begin{aligned} \theta(t_0) &= \theta_0 \\ \dot{\theta}(t_0) &= v_0 \\ \ddot{\theta}(t_0) &= a_0 \\ \theta(t_f) &= \theta_f \\ \dot{\theta}(t_f) &= v_f \\ \ddot{\theta}(t_f) &= a_f \end{aligned} \quad (5)$$

All the values of coefficient in Table 1 were obtained from the value of the velocity and acceleration of the human walking for different sub phase of one gait cycle which is shown in Table 2.

Table 1  
Values of Coefficients  $a_0, a_1, a_2, a_3, a_4,$  and  $a_5$  of the trajectory (flexion angle) polynomial for hip joint [9]

Gait sub phase	Time period	$a_0$	$a_1$	$a_2$	$a_3$	$a_4$	$a_5$
Loading response	0-10%	0.223402	-0.81	3.965	61.236	-198.017	-2098.176
Mid stance	10-30%	0.193987	-1.725	47.578	-412.456	1321.923	-1448.931
terminal stance	30-50%	-1.026	12.738	-58.895	133.676	-159.586	81.316
Pre swing	50-60%	182.637	-1663.42	6040.09	-10923.12	9823.767	-3508.813
Initial swing	60-73%	-638.331	4919.857	-15073.23	2928.693	-17310.604	5190.041
mid swing	73-87%	-688.374	4151.719	-10015.9	12077.426	7272.47	1747.812

Table 2  
Trajectory (flexion angle), angular velocity values and angular acceleration of human's hip walking for various gait cycle sub phases [9]

Gait cycle	Time Instant	Angle(rad)	Ang.Vel.(rad/s)	Ang.Acc. rad/s/s)
Initial Contact	0%	0.223402	-0.81	7.93
Opp. Toe Off	10%	0.202458	-0.05	-22.88
Heel Rise	30%	0.006981	-0.47	-5.26
Opp. I.C.	50%	-0.10647	-0.22	8.61
Toe Off	60%	-0.04363	2.08	38.01
Feet Adjacent	73%	0.329867	1.66	-32.81
Tibia Vertical	87%	0.376991	-0.9	-23.22
Initial Contact	100%	0.223402	-0.81	7.93

### III. RESULT AND DISCUSSION

Simulation using MATLAB was conducted to determine the accuracy of using Quintic in representing human hip motion walking profile. Figure 2 shows the angular position profile of the hip joint trajectory based on the quintic polynomial method.

Additionally, Figure 3 shows the hip joint angular velocity profile where the angular velocity is produced by taking the differentiation of the angular position (first derivative of quintic polynomial) with same value of the coefficient constant of (t) variables. Whereas Figure 4 shows the hip joint angular acceleration profile which is produced by taking the differentiation of the angular velocity (second derivative of

quintic polynomial) with same value of the coefficient constant of (t) variables.

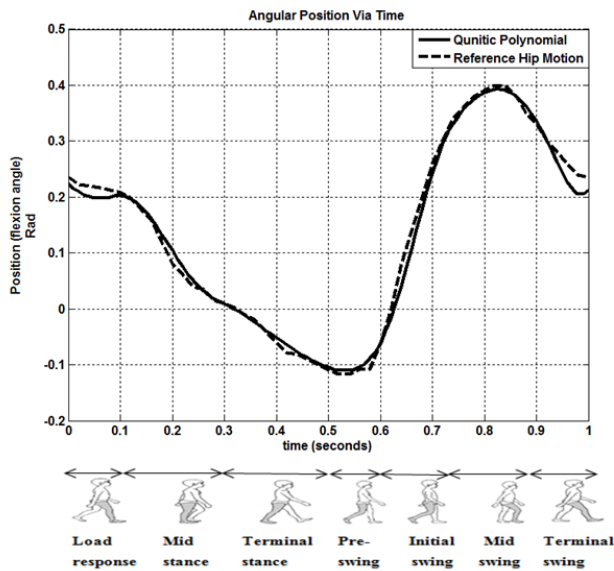


Figure 2: Comparison between reference hip motion and quintic polynomial in term of angular position.

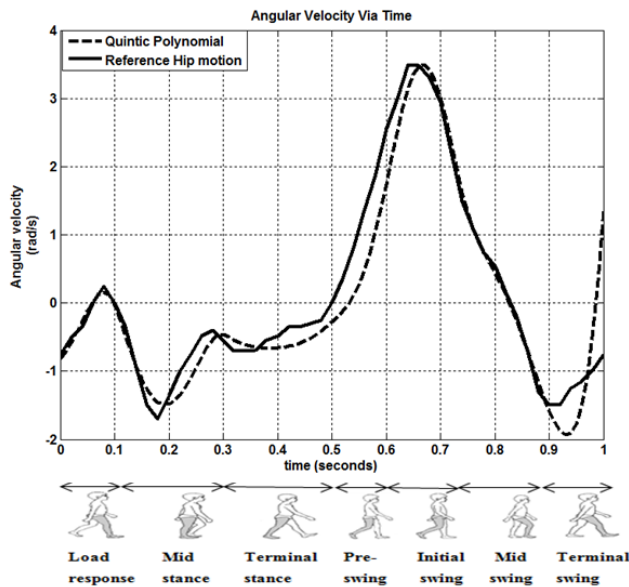


Figure 3: Comparison between reference hip motion and quintic polynomial in term of angular velocity.

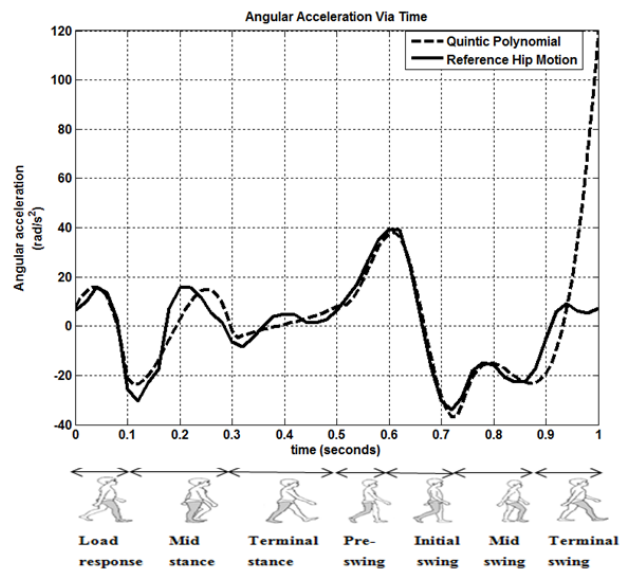


Figure 4: Comparison between reference hip motion and quintic polynomial in term of angular acceleration.

A. Accuracy of Quintic Polynomial

Figure 5 shows the comparison between human trajectory data and trajectory that is generated by quintic polynomial trajectory.

Based on the data in Table 1 and 2, both graphs have been formed in term of the angular position. The graphs have been drawn based on the data of each gait cycle phases which are 8 phases starting at 0 s and ending at 1 s.

During the stance phases, although the trajectory of quintic polynomial trajectories has less error during terminal stance phase but the error increases during the phases (load response and mid stance) compared to reference human hip trajectory.

During the swing phases, the trajectory of quintic polynomial trajectories has less error during pre-swing and mid-swing phases but the error increases during the phases (initial and terminal swing) compared to reference human hip trajectory.

The quintic polynomials method has the ability to generate a motion in four phases (mid stance, terminal stance, pre-swing and mid swing) with less error compared to the real human hip motion as shown in Figure 4. In contrast, the error between the human hip and quintic polynomial trajectories increases during (load response, initial swing and terminal swing).

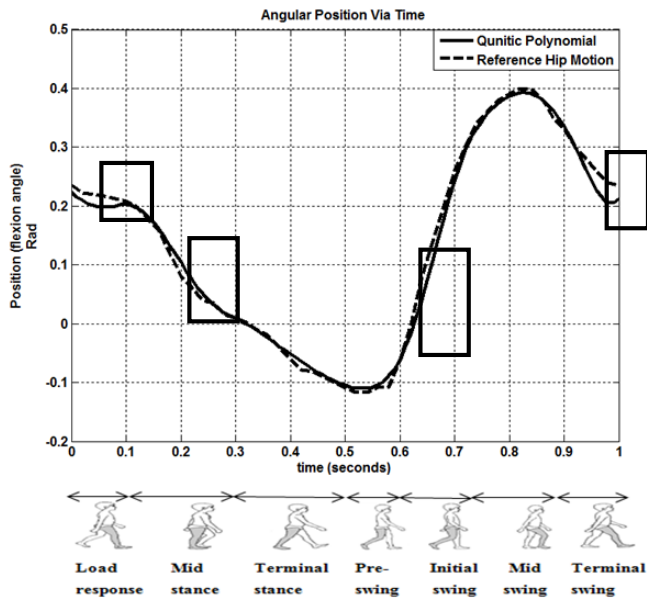


Figure 5: Comparison between human trajectory data and Quintic polynomial trajectory

Table 3 shows the error that occurred in sub phases between human hip trajectory and trajectory (that is generated by quintic polynomial).

Table 3: RMS error between Quintic Polynomial and Human Trajectories based on angular position.

Time period(s)	RMS Error(rad)
0 - 0.1 (loading response)	0.01424
0.1 - 0.3 (mid stance)	0.01047
0.3 - 0.5 (terminal stance)	0.00575
0.5 - 0.6 (pre-swing)	0.00915
0.6 - 0.72 (initial swing)	0.03015
0.72 - 0.88 (mid swing)	0.007274
0.88 - 1 (terminal swing)	0.021533

According to the data in Table 4, the rms error differs from on phase to other, which means that error increases in some phases and decreases in other by comparison between quintic polynomial and reference hip trajectory.

To add to that, from observation based on Table 3, the highest error between quintic polynomial and reference hip trajectory occurs when the leg is at initial swing phase and terminal swing (0.03015 and 0.021533 rad respectively). On the other hand, the smallest error is noticed when the leg is at terminal stance and mid swing (about 0.00575 and 0.007274 rad respectively).

Figure 5 shows a graphical analysis error versus time based on the data in Table 4. From Figure 5, it is easy to notice that the highest error occurs when the hip in the swing phase especially in the mid-swing. Whereas, according to the stance phase the highest error is recorded during terminal stance.

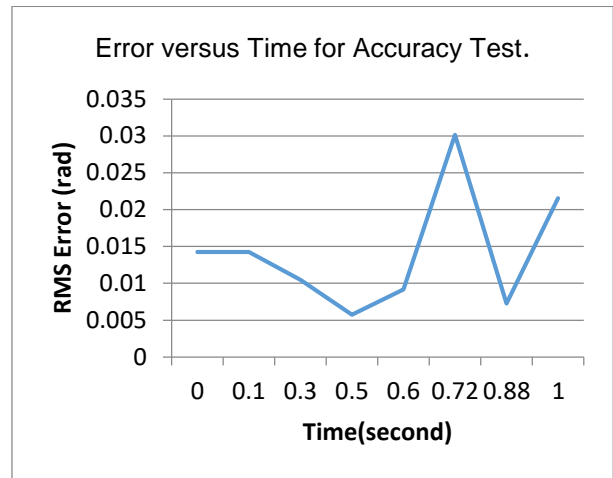


Figure 6: Error versus time for each interval phase during stance and swing for accuracy test.

At the end, the Table 3 and Figure 6 show that the quintic polynomial generate motion which is quite accurately matched to human motion during (mid stance, terminal stance, pre-swing and mid swing) but it is not accurately matched to the human hip during (load response, initial swing and terminal swing).

### B. Dynamic Error

Dynamic error is the difference between the true value with changing time and the measured value regardless of the static error.

Dynamic error of the acceleration profile between quintic and human trajectories is determined in order to figure out how the quintic polynomial has ability to follow the line of the reference hip motion.

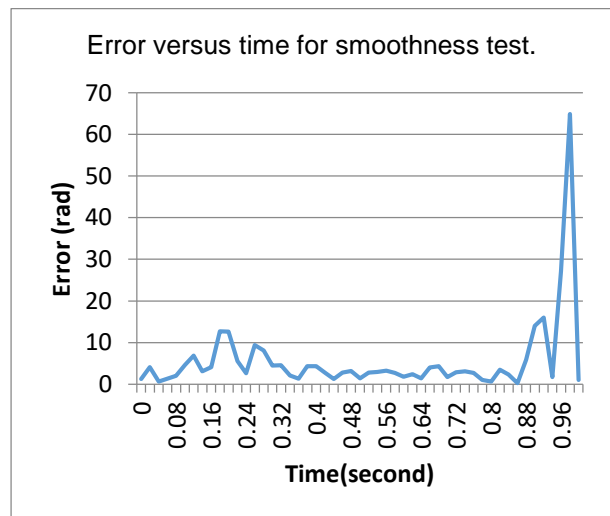


Figure 7: Error versus time for each interval time of 0.02 second for smoothness test.

Figures 7 and 8 show that the smoothness is accurately matched to the real hip motion during the load response pre-swing, initial swing and mid-swing phase Whereas, the smoothness is reduced and became less accurately matched to

the real hip motion during the mid-stance, terminal stance and terminal swing.

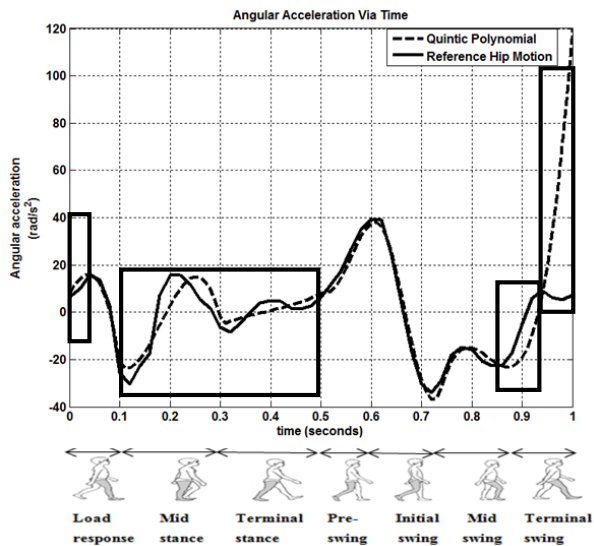


Figure 8: Comparison between reference hip motion and quintic polynomial in term of angular acceleration.

#### IV. CONCLUSION

To summarize that, the generating trajectory for hip is not easy task, which means many methods have been conducted in order to generate that trajectory motion. Quintic polynomial method is used in this project because it generated a smooth and accurate motion like human. Also, it is used to avoid the jerks which occurred during using cubic and spline polynomial. Thus, the trajectory generation is generated by using quintic polynomial. Moreover, the RMS error is highest at mid swing about 0.2607rad where the lowest RMS error is at load response about 0.0130rad. The comparison was conducted based on the RMS error between the trajectory that is generated by quintic polynomial and real human hip trajectory. Also, doing the comparison in term of RMS error can ease evaluation and determining the differences between the two graphs easily. The result shows that the proposed

quintic polynomial can be used to generate a motion that is accurately matched to real human hip motion.

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#### REFERENCES

- [1] Z. Taha, A. P. P. A. Majeed, M. Yashim, W. Paul, A. Ghaffar, and A. Rahman, "Preliminary Investigation on the Development of a Lower Extremity Exoskeleton for Gait Rehabilitation: A Clinical Consideration," vol. 4, no. 1, pp. 1–6, 2015.
- [2] B. Zhang, S. Jiang, K. Yan, and D. Wei, "Human Walking Analysis, Evaluation and Classification Based on Motion Capture System," *Heal. Manag. – Differ. Approaches Solut.*, pp. 361–398, 2011.
- [3] E. Cuevas, D. Zaldivar, M. Pérez-Cisneros, and M. Ramírez-Ortegón, "Polynomial trajectory algorithm for a biped robot," *Int. J. Robot. Autom.*, vol. 25, no. 4, pp. 294–303, 2010.
- [4] H. R. E. Haghghi and M. a. Nekoui, "Inverse Kinematic for an 8 Degrees of Freedom Biped Robot Based on Cubic Polynomial Trajectory Generation," pp. 935–940, 2011.
- [5] S.-H. Kim and Y. Sankai, "Task generation for humanoid robot walking using human motion by phase sequence," no. 3, pp. 524–530, 2005.
- [6] T. Arakawa and T. Fukuda, "Natural motion generation of biped locomotion robot using hierarchical trajectory generation method consisting of GA, EP layers," *Proc. Int. Conf. Robot. Autom.*, vol. 1, no. April, pp. 1–6, 1997.
- [7] K. Ohishi, K. Majima, and T. Fukunaga, "Gait control of biped robot based on kinematics and motion description in cartesian space," *Electron. Control*, no. 1, pp. 1317–1322, 1997.
- [8] S. Kajita, F. Kanehiro, K. Kaneko, K. Fujiwara, K. Harada, K. Yokoi, and H. Hirukawa, "Resolved momentum control: humanoid motion planning based on the linear and angular momentum," *Intell. Robot. Syst. 2003. (IROS 2003). Proceedings. 2003 IEEE/RSJ Int. Conf.*, vol. 2, pp. 1644–1650 vol.2, 2003.
- [9] Jayendra Kumar Rai and Ravi Tewari "quintic polynomial trajectory of biped robot for human-like walking" Electronics and Communication Engineering Department Amity School of Engineering and Technology, AUUP, Noida, India Applied Mechanics Department," pp. 2325–2328, 2014.
- [10] J. J. Craig, "Introduction to Robotics: Mechanics and Control 3rd," *Prentice Hall*, vol. 1, no. 3, p. 408, 2004.