Voltage Stability Assessment of Power System Network using QV and PV Modal Analysis

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***Abstract*—The analysis of voltage instability in electric power system is very crucial in order to maintain the equilibrium of the power system. This paper presents the analysis of voltage instability of electric power system by using reactive power-voltage (QV) and real power-voltage (PV) modal analysis. This research focuses on the voltage instability analysis by implementing the QV and PV modal analysis for mesh-type power system. IEEE 14-bus system has been chosen as the power system. Both QV and PV modal analysis will be run by using MATLAB application software.**

***Index Terms*— Modal Analysis; MATLAB; Voltage Instability.**

I. INTRODUCTION

Voltage stability can be describe as the ability of a power system to maintain the voltage at all buses in the system remain unchanged right after the system is being subjected to a disturbance. Voltage instability on the other hand is the opposite of voltage stability. Voltage instability occurs due to the failure of the power system to supply ample power to cover the increased demand of load [1]–[8]. Hence, the analysis of voltage instability should be implemented in order to make sure that the voltage level at all buses is at stable state.

The most common method used to analyze voltage instability is the PV and QV curves method. However, one of the drawback of this method is it is not suitable for large power system. This is due to the fact that this method is only related to a specific bus. It will be very time consuming to apply the PV and QV curves method to every bus in large power system [1]–[4], [6], [8]–[12].

The work presented in this paper used PV and QV modal analysis to analyze voltage stability in IEEE 14-bus system. Modal analysis was first been introduced by Gao, Morisson and Kundur in 1992 [9]. This method is basically depends on obtaining the values of eigenvalue and eigenvector of the reduced Jacobian matrix. These values are used to calculate the participation factor. The participation factor is very useful to predict which bus that has the highest tendency towards voltage instability. Most of the literature that used modal analysis to evaluate voltage instability only focus on QV modal analysis [5], [8]–[10], [12]. However, this paper will also focus on using PV modal analysis to analyze voltage instability.

II. METHODOLOGY

A. QV Modal Analysis

a. Reduced Jacobian Matrix (Jr)

In the Newton Raphson power flow method, there is a Jacobian matrix that represents the injected real power (P) and reactive power (Q) in buses as shown in Equation (1) [9], [13].

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_{P\delta} & J_{PV} \\ J_{Q\delta} & J_{QV} \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta V \end{bmatrix} \tag{1}$$

where: ΔP is the incremental change in bus real power
ΔQ is the incremental change in bus reactive power
$\Delta\delta$ is the incremental change in bus voltage angle
ΔV is the incremental change in bus voltage magnitude

The reduced Jacobian matrix (Jr) for QV modal analysis can be obtained by letting the value of ΔP equal to 0 as shown in Equation 2.

$$\begin{bmatrix} 0 \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_{P\delta} & J_{PV} \\ J_{Q\delta} & J_{QV} \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta V \end{bmatrix} \tag{2}$$

From Equation (2), the following Equation (3) and Equation (4) can be obtained.

$$\Delta\delta = - J_{P\delta}^{-1} J_{PV} \Delta V \tag{3}$$

$$\Delta Q = J_{Q\delta} \Delta\delta + J_{QV} \Delta V \tag{4}$$

Equation (5) below is formed by substituting Equation (3) into Equation (4).

$$\Delta Q = \Delta V[J_{QV} - J_{Q\delta} J_{P\delta}^{-1} J_{PV}] \text{ or } \Delta Q = Jr\,\Delta V \tag{5}$$

where $Jr = J_{QV} - J_{Q\delta} J_{P\delta}^{-1} J_{PV}$.

Rearranging Equation (5) will form:

$$\Delta V = Jr^{-1} \Delta Q \tag{6}$$

Equation (6) shows the relationship between the incremental changes of voltage and reactive power.

b. Determination of the Most Critical Mode

The eigenvalues and eigenvectors of Jr can be used to determine the modes of the power network. Equation 7 [9], [10], [12] depicts their relationship.

$$Jr = \xi \, \Delta \, \eta \tag{7}$$

where: ξ is the right eigenvector of Jr
Δ is the diagonal eigenvalue of Jr
η is the left eigenvector of Jr

According to [9], [11], [12], the lowest value of eigenvalue of Jr determines the most critical mode of the power system.

c. Bus Participation Factor

The bus participation factor is an indicator that shows the tendency of a particular bus towards voltage instability. It should be calculated at the bus that has the most critical mode. The bus participation factor can be calculated by using Equation (8) [9]–[12].

$$P_{ki} = \xi_i \, \eta_i \tag{8}$$

where: P_{ki} is the participation factor of bus k to mode i
ξ_i is the ith column right eigenvector of Jr
η_i is the ith row of left eigenvector of Jr

In addition, the bus participation factor obtained from Equation (8) is shown in matrix form. The row of the matrix indicates the number of the bus. The column of the matrix shows the mode of the power network.

B. PV Modal Analysis

In order to find Jr for PV modal analysis, the value of ΔQ in Equation (1) will be set to 0 as shown in the following equation.

$$\begin{bmatrix}\Delta P\\ 0\end{bmatrix} = \begin{bmatrix}J_{P\delta} & J_{PV}\\ J_{Q\delta} & J_{QV}\end{bmatrix}\begin{bmatrix}\Delta\delta\\ \Delta V\end{bmatrix} \tag{9}$$

From Equation (9), the following Equation (10) and Equation (11) can be obtained.

$$\Delta P = J_{P\delta}\,\Delta\delta + J_{PV}\,\Delta V \tag{10}$$

$$\Delta V = -\, J_{QV}^{\,-1}\, J_{Q\delta}\, \Delta\, \delta \tag{11}$$

Equation (12) below is formed by substituting Equation (11) into Equation (10).

$$\Delta P = \Delta\delta\,[J_{P\delta} - J_{PV}\ J_{QV}^{\,-1} J_{Q\delta}] \text{ or } \Delta P = Jr\,\Delta\delta \tag{12}$$

where $Jr = J_{P\delta} - J_{PV}\, J_{QV}^{\,-1} J_{Q\delta}$.

Rearranging Equation (12) will form:

$$\Delta\delta = Jr^{-1}\,\Delta P \tag{13}$$

Equation (13) shows the relationship between the incremental changes of voltage angle and real power.

The equations and methods used for the determination of the most critical mode and bus participation factor for the PV modal analysis are similar to the QV modal analysis as explained in Section 2.1.2 and Section 2.1.3.

C. IEEE 14-bus System

Both QV and PV modal analysis methods explained in Section 2.1 and Section 2.2 are applied to IEEE 14-bus system as shown in Figure 1 [14]. This system consists of one slack bus (Bus 1), four PV buses (Bus 2, Bus 3, Bus 6 and Bus 8) and nine load buses (Bus 4, Bus 5, Bus 7 and Bus 9 until Bus 14). A Matlab program based on [8], [15] has been developed in order to simulate both QV and PV modal analysis onto the IEEE 14-bus system.

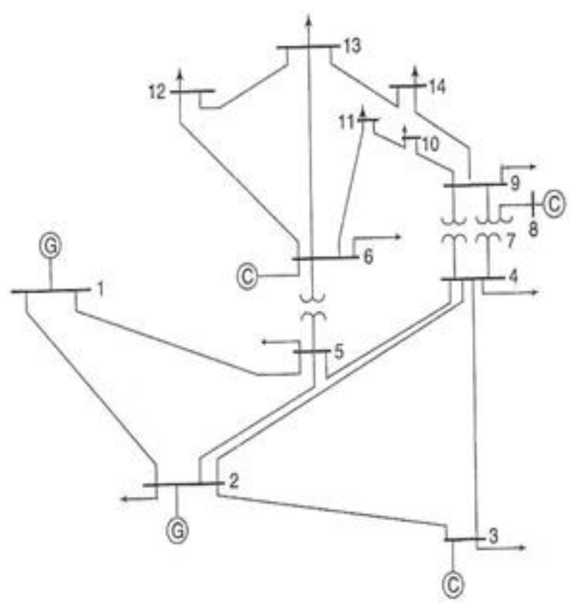

Figure 1: IEEE 14-bus system

III. Results And Discussion

A. QV Modal Analysis

As explained in Section 2.1, QV modal analysis shows the relationship between incremental changes of voltage and reactive power. Figure 2 shows the voltage values at all buses obtained from the load flow analysis. It can be seen in Figure 1 that all buses operate within the acceptable values of voltage which is between 0.95 until 1.05 per unit. Matlab application software is used to obtain the eigenvalues of the Jr. Table 1 shows the eigenvalues.

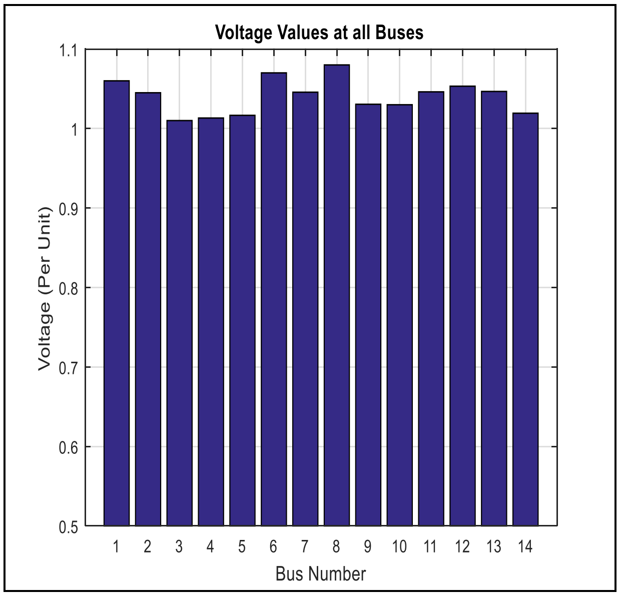

Figure 2: Voltage values at all buses

Table 1
Eigenvalues of QV modal analysis Jr

Eigenvalue	Mode of power system
65.0941	1
39.2032	2
21.6936	3
18.8471	4
16.2991	5
11.2138	6
2.6943	7
5.5227	8
7.5960	9

It can be seen from Table 1 that all of the eigenvalues are positive. Positive eigenvalues indicate that the power system is stable. Table 1 also tells that the 7th eigenvalue is the lowest which is 2.6943. Hence, mode 7 is the most critical mode for this power system.

The bus participation factor for mode 7 is calculated. The results are shown in Figure 3. Figure 3 conveys that Bus 14, Bus 10 and Bus 9 have high participation factor with Bus 14 is the highest which is 0.319. This means that Bus 14 has the highest contribution towards voltage instability even though the voltage magnitude of Bus 14 shown in Figure 2 is good (1.02 per unit).

It is also noticeable from Figure 3 that only the load buses that possess the participation factor. This is because the QV modal analysis method focuses on the relationship between the incremental changes of voltage and reactive power as stated in Equation 6. Since the voltages of the slack and PV buses are fixed prior to the load flow analysis, no participation factor is considered on these buses.

For the purpose of comparison QV curve method, the magnitude of voltage plays important role. The voltage stability margin (VSM) can be obtained from QV curve method. It is defined in Equation (14). The lower value of VSM indicates the closer of the bus towards voltage instability and vice versa [16]. Table 2 shows the VSM for all of the load busses.

$$VSM = \frac{V_{initial} - V_{critial}}{V_{critical}} \tag{14}$$

where: $V_{initial}$ is the bus voltage at normal operating condition
$V_{critical}$ is the bus voltage at voltage collapse point

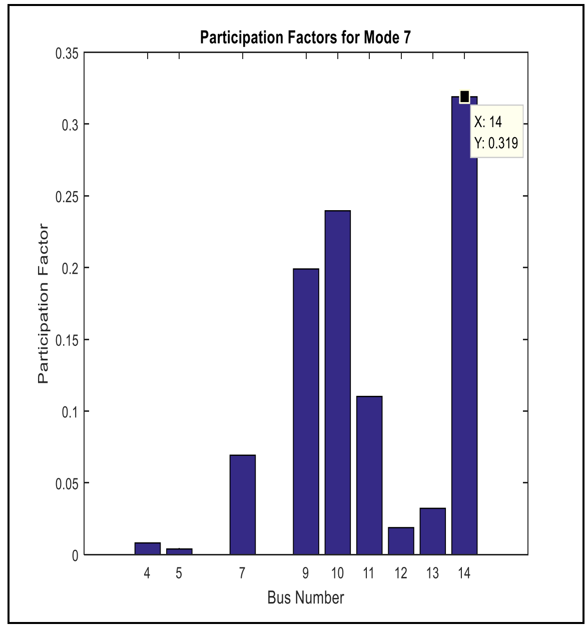

Figure 3: The bus participation factor for mode 7

Table 2
VSM for all of the load busses

BUS	VSM
4	0.9930
5	1.0311
7	0.9754
9	0.9084
10	0.8841
11	0.8039
12	0.9404
13	0.8011
14	0.7954

It can be seen from Table 2 that bus 14 has the smallest value of VSM. Hence, Bus 14 is prone to voltage instability.

B. PV Modal Analysis

It is stated in Section 2.2 that PV modal analysis shows the relationship between incremental changes of voltage angle and real power. Figure 4 depicts the voltage angle values at all buses obtained from the load flow analysis.

The eigenvalues of the Jr by using PV modal analysis are presented in Table 3.

Table 3 shows that the system is also stable since all of the eigenvalues are positive. Table 2 also tells that for this PV modal analysis method, the 9th eigenvalue is the lowest which is 0.5431. Hence, mode 9 is the most critical mode for this power system.

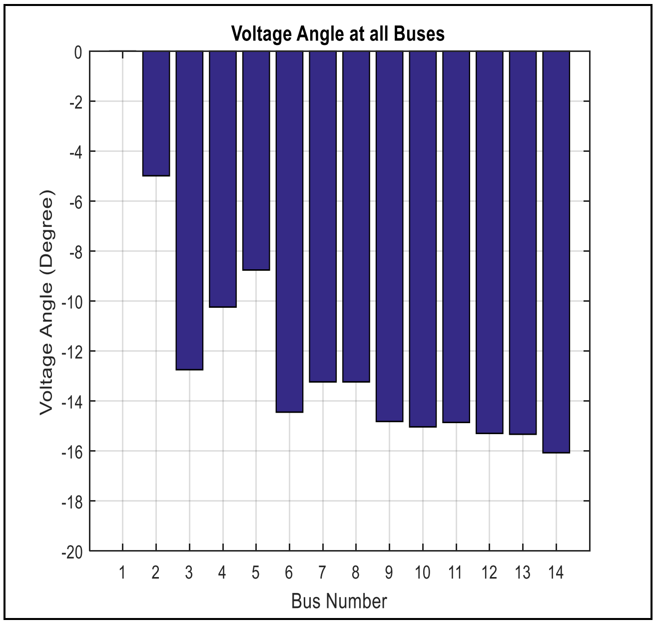

Figure 4: Voltage angle values at all buses

Table 3
Eigenvalues of PV modal analysis Jr

Eigenvalue	Mode of power system
66.4389	1
40.8794	2
37.1669	3
30.4163	4
23.2941	5
18.6321	6
15.4874	7
12.5672	8
0.5431	9
2.8241	10
6.6803	11
5.5849	12
5.8137	13

The bus participation factor for mode 9 is calculated. The results are shown in Figure 5. Figure 5 depicts that Bus 12, Bus 13 and Bus 14 have high participation factor with Bus 14 being the highest which is 0.1235. This delineates that bus 14 is closest towards voltage instability compared to other buses. On the other hand, the Figure 4 indicates that Bus 14 has the highest voltage angle followed by Bus 13 and Bus 12.

All buses shown in Figure 5 have participation factor except for Bus 1 (slack bus). This is because the PV modal analysis method considers the relationship between the incremental changes of voltage angle and real power as shown in Equation 13. Since the voltage angle of the slack bus are fixed prior to the load flow analysis, no participation factor is considered on Bus 1 for PV modal analysis method. However, modal analysis based on P produces many busses with high participation factor compared to the QV modal analysis. Most researchers [5], [8]–[10], [12] use Q based modal analysis.

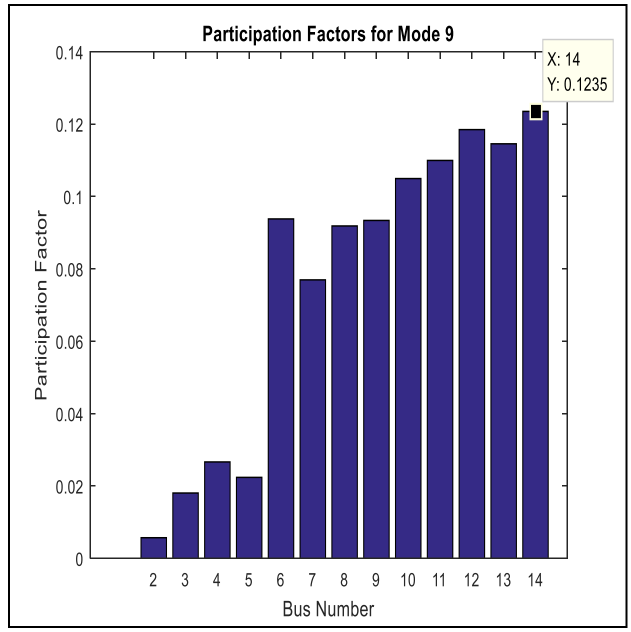

Figure 5: The bus participation factor for mode 9

IV. Conclusion

Voltage instability analysis is an important parameter for monitoring the bus voltage in the electrical power system. The study conducted in this paper show that both QV and PV modal analysis are very useful in order to determine which bus with the highest tendency towards voltage instability. Both of these analyses convey that Bus 14 of the IEEE 14-bus system has the highest bus participation factor compare to other buses. The studies also show that the changes in bus voltages are more sensitive than the changes in bus voltage angles. In addition, it is clear that voltage instability can occur even though the initial values of the load buses voltages are within acceptable range. Therefore, more attention need to be given to Bus 14 in order to avoid voltage instability from occurring in the power system. Q based modal analysis produces more accurate result for participation factors because Q is related directly to the magnitude of voltage. Again, the voltage magnitude plays significant role in voltage stability.

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