An Improved-Water Filling Algorithm Power Allocation for DFFR Network MIMO

Noor Shahida M. K[1,2], Rosdiadee Nordin[1], Mahamod Ismail[1]
[1]*Department of Electrical, Electronic and Systems Engineering, Faculty of Engineering and Built Environment, Universiti Kebangsaan Malaysia, 43600 UKM Bangi, Selangor, Malaysia.*
[2]*Faculty of Electronic and Computer Engineering, Universiti Teknikal Malaysia Melaka, Hang Tuah Jaya, 76100 Durian Tunggal, Melaka, Malaysia.*
noorshahida@utem.edu.my

***Abstract*—In wireless systems, interference is a major factor that limits the total network capacity. Power allocation is one of the effective techniques that has garnered interest in Network MIMO system and improved the efficiency of wireless systems. This study presents the development of a new power allocation algorithm based on water filling. This algorithm combines the Dynamic Fractional Frequency Reuse (DFFR) with a Network MIMO to maximise the performance of cell edge users. Simulation results show that the proposed algorithm provides more ergodic capacity, compared with the existing power allocation strategies. In addition, it improves the network throughput, while ensuring fairness for cell edge users in the LTE-A system. When the total transmit power is 100W, the proposed algorithm offers 50% capacity, 37.5% throughput and 38% fairness advantage over the conventional water-filling algorithm**.

***Index Terms*—Dynamic Fractional Frequency Reuse (DFFR); FFR; Network MIMO; Power Allocation; Water-Filling Algorithm .**

I. INTRODUCTION

The issue of inter-cell interference (ICI) mitigation has garnered attention in the next generation OFDMA networks, such as LTE-A. ICI avoidance is one of the preferred techniques for LTE-A downlink networks because of its merits. For instance, ICI mitigates interference of cell edge users and employs better service of users; therefore, this technique has attracted considerable attention among researchers.

Interference avoidance technique can be described as a redistribution of frequency (or power) among users in different cells. Besides, it can support the system and increase the SINR for cell edge users without disturbing the cell centre throughput. To improve the performance of throughput and capacity in LTE-A, several mitigation techniques have been discussed in [1] and [2]. Frequency Reuse (FR) is one of the targeted techniques that reuse the whole frequency for every cell. However, ICI becomes worse since FR only reuses the same frequency when the signals are transmitted from the other cells. Fractional Frequency Reuse (FFR) [4-10] and Soft Frequency Reuse (SFR) [3-4] are the most common approaches that have been used to solve the drawback of FR. FFR divides the available frequency resources into the cell centre users (CCUs) and cell edge users (CEUs).

The main shortcoming of FFR scheme occurs at the cell edge because users cannot use the whole spectrum; therefore, the frequency resource system is wasted and spectral efficiency decreases.

Contrary to the FFR, DFFR version is more flexible as it can allow the cell centres users to use the whole spectrum. On the other hand, [4] mentioned that the frequency resources partition has been fixed in the wireless channel; therefore, the performance of frequency selectivity is ineffective. ICI can restrain the performance of LTE and can be evaluated based on the Signal to Interference and Noise Ratio (SINR) of users. The power and frequency allocation scheme can be used to achieve an optimal trade-off within the achieved SINR and the resulting interference. To enhance the LTE downlink performance, power allocation has been integrated into the DFFR scheme and Network MIMO. The DFFR scheme has recently been proposed as an inter-cell interference coordination (ICIC) technique in Network MIMO [5], where the water-filling algorithm has been proposed.

A Dynamic FFR (DFFR) developed based on strict conventional FFR is illustrated in Figure 1. The DFFR can be defined as users and subcarriers that are divided into two groups: (i) the super groups cover the entire cell, and the subcarriers are allocated and given to any user in the cell; and (ii) regular groups is further partitioned into 3 sectors [6].

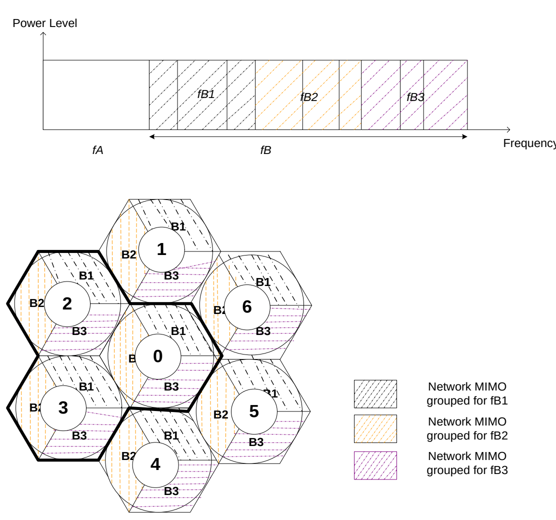

Figure 1: Dynamic Fractional Frequency Reuse model (DFFR)

DFFR scheme has been introduced in the Homogeneous Poisson Point Process (PPP) model [7]. This model is designed to capture the real network scenario, and the users are allocated randomly at the base stations. Using the PPP model in an actual network, the results were compared with the square grid model, and the authors found that the PPP model is pessimistic, while a square grid model is optimistic. However, the authors proposed irregular cell geometry only.

The water-filling algorithm has been widely applied in the area of power allocation in wireless networks due to its optimal performance and ease of analysis [5]. The water-filling algorithm also has a single water level and power constraints. Because of the power constraints, the subcarriers are deactivated throughout this algorithm; thus, degrading the system capacity and causing the algorithm to have a limited performance in low interference scenario. An improved water-filling algorithm (I-WFA) has been proposed to rectify this problem.

By combining Network MIMO and DFFR, the limitation of FFR can be resolved, where the FFR cannot accommodate all the cell edge users alone for a highly loaded network. Therefore, an algorithm of power allocation has been developed to increase the signal quality of intra-cell and inter-cell CEUs and to fulfil the intra-cell CCUs performance in each cell.

In this paper, we present Dynamic Fractional Frequency Reuse (DFFR) in Network MIMO systems. The primary contributions of this study can be given as follows:

i. We introduce a power allocation scheme that resembles the water-filling (WF) algorithm by modifying the current version of water filling algorithm to achieve higher capacity performance.
ii. This work aims to overcome the limitations of the WF algorithm, which allowed the water-filling level, λ to be found efficiently and the capacity to be maximised for multi-user communications.
iii. This work aims to show that the proposed power allocation algorithm achieves a substantial network throughput gain while ensuring fairness.

The remainder of this paper is organised as follows. In Section II, the related work is reviewed, and the contributions of the paper are summarised. In Section III, IV and V, the proposed method and the analytical framework for the theoretical calculations are described. Then, the simulation parameters are presented in Section VI. The simulation results and analysis have been compared with the different frequency reuse techniques and presented in Section VII. Lastly, Section VIII concludes the report.

II. RELATED WORKS AND CONTRIBUTION

The combination of Network MIMO and FFR, have been broadly introduced in the literature in the context of OFDMA networks [8-11], where cells are divided into sectors for efficiently managing radio resource and mitigating ICI. However, the cell sectorisation that can lead a certain impact on the system capacity have never been discussed.

In [8], the authors proposed a zero-forcing dirty paper coding (ZF-DPC) and FFR sectorisation techniques in Network MIMO to improve the SINR performance. However, it is not workable because the computational of the pre-coding matrix is complex and the instantaneous channel feedback is required. In [9], the authors introduced linear pre-coding methods for intra-cluster coordination and limited inter-cluster coordination to mitigate the inter-cluster interference for Network MIMO. Moreover, the idea from [9] has been extended in [10] to improve the interference coordination strategy. Block Diagonalisation (BD) pre-coding technique was used to mitigate intra-cluster interference, while a novel adaptive clustered FFR scheme has been proposed to cancel the inter-cluster interference. However, these approaches have limitations, such as higher CSI feedback, and signalling overhead is needed to mitigate the inter-cluster interference. In [11], a two-method resource allocation scheme is presented in a clustered Network MIMO, where the system is statically divided into a number of disjoint clusters. This scheme results in inter-cluster interference mitigation and maximisation of the sum utility function of all MSs in the cluster, under per-sector power constraints. However, limited network orientations were taken into account (i.e. reduced the number of active sectors and a limited number of subcarriers or active users per sector). Moreover, power and subcarrier allocation are treated as two separate problems.

A power allocation algorithm is presented in [12] and [13] to maximise the network capacity. Power allocation at the sub-band granularity level may lead to over allocating power to some of the resource blocks (RBs) if they are located in a sub-band that serves the edge users. Reference [12], proposed two steps to achieve the balancing system throughput between cell edge and cell centre users. The first step involved the interference coordination scheme, where the cell-edge users would only interfere together with the centre users of neighbouring cells. In the second step, the optimal power allocation problem for the cell-edge users that can be solved by using the Lagrangian method is introduced. However, spectrum efficiency can be reduced by allocating only cell edge users and neighbouring cell centre users that could select the same spectrum resource. In addition, this proposed scheme does not consider the required transmission rate on the control channel of Primary Component Carrier (PCC), where the rate should be assured for signal transmission.

In [13], the authors evaluated the effect of SFR on Spectral Effiency (SE) depending on power allocation schemes and the number of sub bands. In this case, ICI can be reduced because adjacent cell uses a different frequency band; thus, interference from neighbouring cells is not an issue. However, using FFR greater than one may decrease spatial spectrum efficiency of the cellular network.

A dynamic inter-cell coordination scheme based on WINNER system is studied in [14]. In this paper, the scheme works without the need of a central controller. Therefore, the centralised processing of Radio Resource Management (RRM) algorithms is not supported. The authors in [15] enhance and improve the scheme presented in [14] by using the dynamic inter-cell coordination through an X2 interface to suit the LTE downlink. A heuristic approach is used in the dynamic interference coordination scheme to handle inter-cell intra-eNB interferences. The scheme aims at maximising the throughput of the cell-edge users, by exchanging vital interference information among BSs over the backbone network connection. However, the scheme can become unstable due to the delay of the control signals between the BSs.

The authors in [17] address a resource and power allocation problem by jointly adjusting the subcarrier and power allocation. In the proposed algorithm, a joint subcarrier and power allocation optimisation were considered for the SFR scheme. Although the joint optimisation for the SFR scheme improves the system throughput performance, the data rate in the proposed method is unstable and not accurate for the cell centre and cell edge users. This problem is solved in [5] by using water filling algorithm, which in return gives higher accuracy of a given cell for the cell edge target data rate. For convenience, some notations and symbols used throughout the paper are listed in Table 1.

Table 1
Some notations and symbols used in this paper

Symbol	Description
w_k	precoding weighting vector
s_k	data symbol of the k-th (cooperatively served by the sector BSs)
$H_{m,k}$	channel matrix from the m-th BS and the k-th user
$\mathcal{H}_{n\rightarrow m,k}$	channel matrix from the n-th BS and the k-th user in the n-th BS
$n_{i,k}$	noise vector of the k-th user in the i-th BS
P_k	transmit power of the k-th antenna
$H_{t,r}$	channel transfer function between transmitter t and receiver r
X_σ	log-normal shadowing with standard deviation σ
L	path loss (dB)
$P_{s,k}$	power transmitted by the base station s for user k.
r	distance between the user and the base station
h	random variable that represents the exponential distribution with the cumulative effect of small scale fading and shadowing
α	path loss exponent
σ^2	noise power
g	statistical distribution that consists of shadowing, the value of fading and anything of random effect
R	distance from users to other stations
N	subcarriers
P_t	power budget
H	channel matrix
μ	power
g_k	channel realization
W	constant value for all the power P_k with a range from 0 to 1

III. SYSTEM MODEL

A. Dynamic Fractional Frequency Reuse on Network MIMO with three sector frequency partition

A network consists of seven cells base stations (BSs) in a hexagonal cell is shown in Figure 1, where each cell is allotted into four sectors, a circular area of radius R_{th} in the centre of the cell served by an omnidirectional antenna, and three 120° sectors at the cell-edge served by sectorial antennas. The four antennas are co-located and allocated a total power P_{sc} per subcarrier. Figure 1 shows the dynamic frequency allocation, where the users are uniformly distributed within the cell such that each sector has a different number of users. The required bandwidth can be higher if a higher number of users are allocated in the larger sector. For each sector, the spectrum is dynamically allocated based on the bandwidth required. The proposed dynamic frequency allocation method can be formulated as follows:

$$BW_{total} = BW_{center} + BW_{edge} \quad (1)$$

where BW_{total} is the total available bandwidth, which is the sum of the BW_{centre} bandwidth assigned to the cell centre zone and the BW_{edge} bandwidth assigned to the cell edge zone. The gain pattern used for each sector antenna is specified as [8],

$$A(\theta)_{dB} = \begin{cases} 1 & \text{omnidirectional} \\ -\min\left[12\left(\frac{\theta}{\theta_{3dB}}\right)^2, A_m\right] & \text{sectorial} \end{cases} \quad (2)$$

for directional antennas, where θ_{3dB} = 70° is the angle at which the antenna gain is 3 dB lower than the antenna gain at the main-beam direction, and the parameter A_m = 20 dB is the maximum attenuation measured at the sidelobe.

B. Network MIMO

Consider K users are located in the downlink DFFR OFDM area, with a Network MIMO system, where the B base stations (BSs) are at the cells' centres. As shown in Figure 2, each cell is served by a BS with two transmitting antennas N_t, while receiving antenna N_r with a single antenna for each user in the cell. The transmitted signal vector of the system can be stated as

$$x = Ws \begin{bmatrix} w_{1,1} & \cdots & w_{1,M} \\ \vdots & & \vdots \\ w_{M,1} & \cdots & w_{M,M} \end{bmatrix} \begin{bmatrix} s_1 \\ \vdots \\ s_M \end{bmatrix} \quad k = 1,\ldots,B \quad (3)$$

Thus, the received signal of the user k can be represented as:

$$y_{i,k} = H_{i,k}x_i + \sum_{j\neq i}^{M} H_{j\rightarrow i,k}x_j + n_{i,k} \quad (4)$$

In the downlink channel, the power for each antenna is limited by the maximum downlink transmit power: $P_k \leq P_{max,BS}, K = 1,2,3,\ldots, B$.

C. Channel Model

$G_{t,r}$ is the channel gain and split by the distance (d) between a transmitter t and receiver r as follows:

$$G_{t,r} = |H_{t,r}|^2 10^{\frac{-L(d)+X_\sigma}{10}} \quad (5)$$

where X_σ value is in dB and the path loss model is denoted as:

$$L(d) = 15.3 + 37.6\log_{10}(d) \quad [\text{dB}] \quad (6)$$

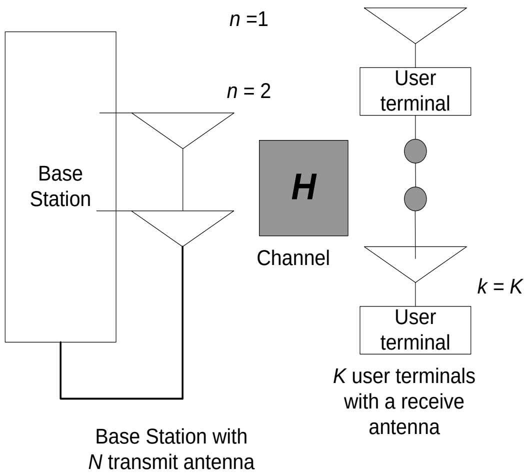

Figure 2: Block diagram of Network MIMO

IV. PROBLEM FORMULATION

A. Signal-to-Interference-and-noise-Ratio (SINR)

In the system model, the k user is served by a base station b. The users are uniformly distributed within the coverage area of a cell randomly, and each user is served by its nearest base station. The SINR of the k-th user can be calculated as:

$$SINR = \frac{P_{s,k} h r^{-\alpha}}{\sigma^2 + I_r} \tag{7}$$

The interference power is expressed as follows:

$$I_r = \sum_{k \in \phi / b_o} \left(g_k R_k^{-\alpha} \right) \tag{8}$$

B. Achievable Rate

Network sum-rate is defined as the total achievable rate over the network across the available subcarriers, N, expressed in bits per second (bps). It is formulated based on the well-known Shannon capacity theorem written as follows:

$$R_{total} = \sum_{k=1}^{K} R_k \tag{9}$$

where R_k the achievable rate is expressed as:

$$R_k = \log_2 \left(1 + \frac{P_{s,k} h r^{-\alpha}}{\sigma^2 + I_r} \right) \tag{10}$$

C. Fairness Index

Fairness is an important metric, in which a good resource allocation algorithm must be preserved. Jain's Fairness Index (JFI) is used in this work ranged from 0 to 1. When resources are equally partitioned, the index is 1. The cell-edge fairness is an important measure in Network MIMO. The cell-edge fairness FI_{CE} represents the fairness index achieved by the cell-edge users (CEUs) in different cells over the network as a whole. The cell-edge fairness can be written as follows:

$$FI = \frac{\left(\sum_{k=1}^{K} R_k \right)^2}{\sum_{k=1}^{K} (R_k)^2 * K} \tag{11}$$

V. POWER ALLOCATION

Equal power is a simple power allocation scheme, where the equal power is optimal when the channel state information (CSI) is not recognised at the transmitter. Therefore, equal power allocation can be written as:

$$P_k = P_{total} / N \tag{12}$$

A. Water-Filling (WF) Algorithm

In the network, there are various channels being used and in order to allocate powers to each of the channels, WF power allocation is used and the best way to achieve similar levels of energy is by having input power to spread to the transmitter as one of the solutions of unequal power gain. According to [5], the system capacity can be maximised when the power allocated is different for each antenna at the eNB. The channel quality is good for users if more power is transmitted, while the channel quality is bad if less power is transmitted. As a result, system data rate is improved significantly assuming higher transmitted power. Based on (7), the design of power allocation scheme for the multi-objective optimisation problem is summarised as:

$$\max \quad \sum_{k=1}^{K} log_2(1 + g_k P_k) \tag{13}$$

$$subject\ to \quad C1: P_k \le P_{max,BS} \tag{14}$$

$$C2: \sum_{k=1}^{K} log_2(1 + g_k P_k) \ge R_{min} \tag{15}$$

The power allocated is shown as follows:

$$Power\ allocated = \frac{P_t + \sum \frac{1}{H_n}}{\sum channels} - \frac{1}{H_n} \tag{16}$$

Hence, the capacity of a MIMO system is the algebraic sum of the capacities of all channels and given by:

$$Capacity = \sum_{k=1}^{K} log_2(1 + Power\ allocated^* H_n) \tag{17}$$

B. Improved Water-Filling Algorithm (I-WFA)

In the proposed algorithm, subchannels with higher SINR is allocated with more power to maximise the network throughput. This implies that a poor channel must not be used for transmitting data. From equation (17), we formulate the following optimisation problem, aiming to maximise the overall capacity with constraint (14). The maximum of total squared weights among all the BSs is defined as below:

$$K_k = \max_{s=1,\ldots,M}\left(\left|w_k^s\right|^2\right) \tag{18}$$

Therefore,

$$\max\left\{\sum_{k=1}^{K}\upsilon_k *\left(\log_2\left(1+g_kP_k\right)\right)\right\}$$
$$s.t \tag{19}$$
$$\sum_{k=1}^{K}P_kK_k \le P_{\max,\,BS}$$

To reach the solution of the problem defined in (21), the Lagrangian that entails one vector of Lagrange multipliers μ is defined as below:

$$\sum_{k=1}^{K}\upsilon_k *\log_2\left(1+g_kP_k\right)+\mu\left(\sum_{k=1}^{K}P_k\mathrm{K}_k - P_{\max,\,BS}\right) \tag{20}$$

$\upsilon_k \in [0,1]$ represents each user priority. In the case of equal priority, $\upsilon_k = 1/K$, for all k. This corresponds to an I-WFA distribution with variable water levels that can be changed only by the user priorities. The set of equations can be written as:

$$\frac{\upsilon_k}{2\ln(2)}\frac{g_k}{1+g_kP_k}+\mu\mathrm{K}_k = 0 \tag{21}$$

Hence, the new power allocated expression from (21) can be written as follows:

$$P_k = \left[W\frac{\upsilon_k}{\mathrm{K}_k}-\frac{1}{g_k}\right]_0^{P_{\max,BS}} \tag{22}$$

where $[-]_0^{P_{max,BS}}$ represents the maximum between zero and the $P_{max,BS}$. According to (7), the SINR of user can be obtained as:

$$SINR_k = \frac{g_kP_k}{\sigma^2} \tag{23}$$

Hence, the channel capacity can be defined as follows:

$$Capacity = \sum_{k=1}^{K}\upsilon_k *\log_2(1+SINR_k) \tag{24}$$

Finally, the pseudo-code for I-WFA algorithm is shown in Figure 3.

Algorithm: I-WFA

Input: Set of $\{g_k\}$, υ_k, W and $P_{max,BS}$ $k = 1,\ldots,K$

Output: $\{P_k\}$and *Capacity*

1. Initialisation: Set $P_{total} = P_{max,BS}$
2. Calculate power allocated:

Sort g_k in decreasing order
$(g_k \ge g_k + 1)$
for given g_k, obtain the solution of P_k according to (22)
if $\mu = 0$ and $\lambda \ge P_{total}$
$P_k = 0$
else if $\mu > 0$ and $\lambda \le P_{total}$
then, $P_k = W\frac{v_k}{K_k} - \frac{1}{g_k}$

3. Calculate Capacity:

$$Capacity = \sum_{k=1}^{K} v_k\, log_2\,(1+SINR_k)$$

Figure 3: Pseudo-code for the improved water-filling algorithm

VI. Simulation Settings

We compare the performance of our proposed approach with the previous work done by authors in [5] and [9] using MATLAB software. Consider a 7-cell OFDMA system shown in Figure 4, where users are uniformly distributed in each cell. Table 2 summarises the simulation parameters that were cited from [5] and [18].

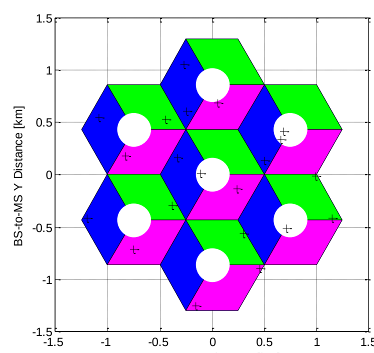

Figure 4: Hexagonal cell

Table 2
System Parameters for Urban Evaluation Environments [18]

Cell Parameters	
Grid Layout	3-sectored hexagonal 7 cells
Cell Radius	1 km
Distribution of Users	Uniformly distributed 15 to 20 [7]
Channel Model	
Channel Bandwidth	10MHz
Carrier frequency	2GHz
Number of Subcarriers	1200
Frequency Spacing	15kHz
Path loss exponent	4 (urban area)
Noise Density	-174dBm
Power Allocation	
Total Power Constraint at each BS	43dBm
Algorithm	Water Filling algorithm and Improved Water Filling (I-WFA)
R_{min} / R_{th}	2 Mbps [5]
$P_{max,BS}$	1.5 mW

VII. Results

Figure 5 represents the probability of acceptable rate by varying the rate threshold. It is obvious that the DFFR scheme improves the system throughput remarkably compared to FFR scheme. It should be noted that the DFFR scheme allowed cell-edge users to achieve higher rate than the FFR scheme since they are less prone to interference due to the higher frequency reuse factor of three. This indicates that the proposed power allocation algorithm could improve the cell-edge throughput, as well as the overall network throughput by combining the DFFR scheme and Network MIMO. Additionally, a comparison with existing studies was performed to validate the performance of the proposed technique. The proposed method showed a considerable improvement than [5] of 37.5% at a rate of 2 Mbps. The plotted graph indicates that power is a better approach for performing network operations, which can be observed by comparing Figure 5 and Figure 6.

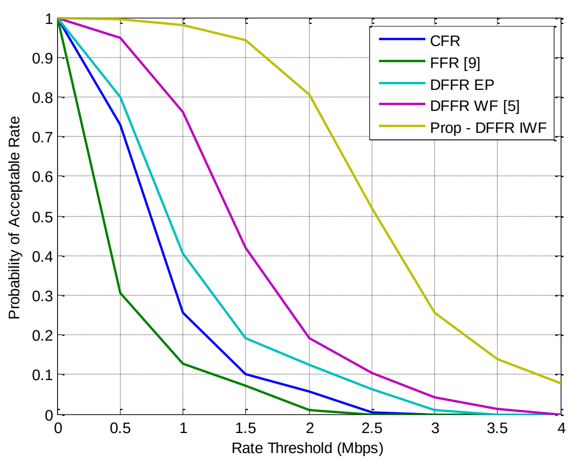

Figure 5: Probability of acceptable rate

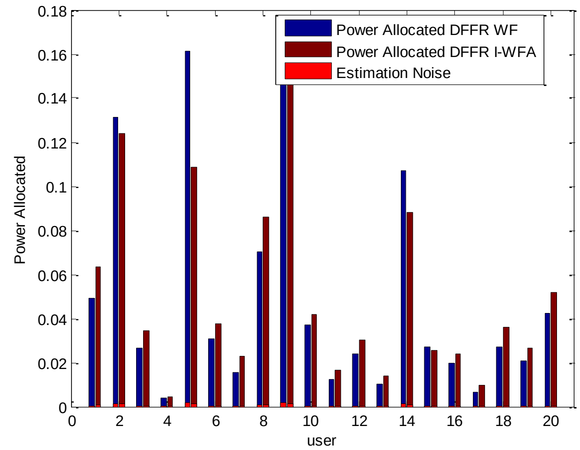

Figure 6: Amount of power allocated to each user

Figure 6 presents the power allocated when the number of users increases. This result shows the relationship between power allocation and system performance. It should be noted that the following remarks could be concluded from Figure 5 and Figure 6:

i. The DFFR IWF method performs better than the DFFR WF method in terms of capacity and power allocation algorithm.
ii. It can be observed that when the estimated sub-channel response is worse, the amount of power allocated to that sub-channel is less. The power allocation strategy is inclined to allocate the power among the data sub-channels equally.

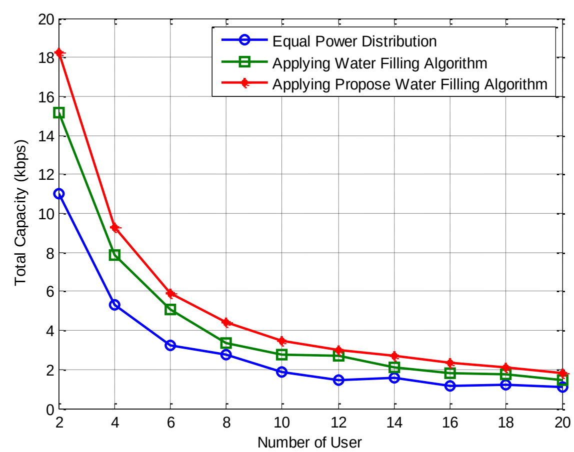

Figure 7: Power allocation based on the channel capacity

Figure 7 indicates the total capacity based on the different number of users. The number of users was represented by low (two users) to high density (20 users) [4]. Although, all the methods indicated a decrease in the high-density network, the DFFR IWF method provided a considerable increase for the cell edge users. Thus, it can be concluded that the IWF algorithm, in combination with the DFFR method, contributes significantly towards the overall performance in the considered LTE-A scenario.

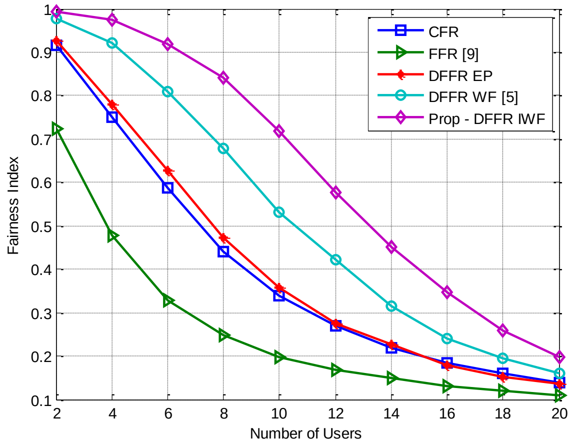

Figure 8: Fairness Index versus number of users

Lastly, the fairness performance, expressed in Jain's Fairness Index metric of the investigated algorithms was evaluated. The results are presented in Figure 8. The DFFR IWF achieved the best user fairness index by contrast to the algorithm proposed in [5] and [9]. The fairness enhancements of the DFFR IWF were achieved in [5] by 38% and [9] by 50%. This is because the utility function employed in I-WFA algorithm gives good user fairness, particularly for the users located at the cell edge.

VIII. Conclusion

In this paper, the proposed algorithm adaptively allocates the combination of DFFR and Network MIMO. The proposed algorithm maximizes the capacity of performance to mitigate the inter-cell interference problem in LTE-A downlink system. From the results, it can be concluded that the proposed algorithm that improved water filling is very important to achieve proper power allocating. Further it coordinates the interference so that the capacity, throughput and fairness in the network could be improved.

Acknowledgment

Authors acknowledged financial contribution from Ministry of Higher Education Malaysia, under grant ref no: FRGS/1/2015/ICT04/UKM/02/2, Universiti Teknikal Malaysia Melaka, and the Ministry of Higher Education for their moral, operational, and financial support.

References

[1] S.Yoon,Y.Cho, C-B. Chae and H. Lee, "System Level Performance of OFDMA Systems," in Proc. *PIMRC 2004, Bacelona, Spain,* pp.1384-1388, Sept 2004.

[2] Young-June Choi, Cheol Seung Kim, and Saewoong Bahk,"Flexible Design of Frequency Reuse Factor in OFDMA Cellular Network," in Proc. *IEEE International Conference on Communications, Istanbul, Turkey*, June 11-15 2006.

[3] Further Analysis of Soft Frequency Reuse Scheme, 3GPP Project Document R1-050 841, Sept. 2005. [Online]. Available: http://www.3gpp.org.

[4] M. Rahman and H. Yanikomeroglu, "Enhancing cell-edge performance: a downlink dynamic interference avoidance scheme with inter-cell coordination," *IEEE Transactions on Wireless Communications,* vol. 9, no. 4, pp. 1414-1425, 2010.

[5] Noor Shahida M.K, Rosdiadee Nordin, Mahamod Ismail, "Power Allocation for Dynamic Fractional Frequency Reuse (DFFR) in Downlink LTE-A System," In Proceedings *Asia-Pacific Conference on Communications (APCC), Kyoto, Japan*, Oct 2015.

[6] Syed Hussain Ali and Victor C. M. Leung. "Dynamic Frequency Allocation in Fractional Frequency Reused OFDMA Networks," *IEEE Transactions On Wireless Communications,* vol. 8, No. 8, *2009.*

[7] Thomas David Novlan, Radha Krishna Ganti, Arunabha Ghosh and Jeffrey G. Andrews. "Analytical Evaluation of Fractional Frequency Reuse for Heterogeneous Cellular Networks," *IEEE Transactions On Communications,* vol. 60, No. 7, *2012.*

[8] C. Wang and C.-J. Yeh, "3-cell network MIMO architectures with sectorization and fractional frequency reuse," *IEEE Journal on Selected Areas in Communications,* vol. 29, no. 6, pp. 1185–1199, 2011.

[9] J. Zhang, R. Chen, J. G. Andrews, A. Ghosh, and J. Robert W. Heath, "Networked MIMO with Clustered Linear Precoding," *IEEE Transactions on Wireless Communications,* vol. 8, no. 4, pp. 1910–1921, 2009.

[10] A. Thampi, S. Armour, Z. Fan, and D. Kaleshi, "Clustered Network MIMO and Fractional Frequency Reuse for the Downlink in LTE-A Systems," *European Wireless.* pp. 802–807, 2014.

[11] Jingya Li, Carmen Botella and Tommy Svensson. Resource allocation for clustered network MIMO OFDMA systems. *EURASIP Journal on Wireless Communications and Networking*, pp. 1-15, 2012.

[12] Y. Yu, E. Dutkiewicz, X. Huang, and M. Mueck, "A resource allocation scheme for balanced performance improvement in LTE networks with inter-cell interference," in *Wireless Communications and Networking Conference (WCNC), 2012 IEEE*, pp. 1630–1635, 2012

[13] D. Kim, J. Y. Ahn, and H. Kim, "Downlink Transmit Power Allocation in Soft Fractional Frequency Reuse Systems," ETRI Journal, vol. 33, no. 1, pp. 1-5, Feb. 2011.

[14] M. Rahman and H. Yanikomeroglu, "Interference avoidance through dynamic downlink OFDMA subchannel allocation using intercell coordination," in Proc. IEEE VTC Spring 2008, pp. 1630-1635, May 2008.

[15] M. Rahman, H. Yanikomeroglu, and W. Wong, "Interference avoidance with dynamic inter-cell coordination for downlink LTE system," in Proc. IEEE Wireless Commun. Netw. Conf. (WCNC), Apr. 2009.

[16] M.Qian, W.Hardjawana, Y.Li, B.Vucetic, J.Shi, and X.Yang, "Inter-cell Interference Coordination Through Adaptive Soft Frequency Reuse in LTE networks", IEEE wireless Communications and Networking Conference, 2012.

[17] 3GPP TR 36.897 V13.0.0, "Study on elevation beamforming / Full-Dimension (FD) Multiple Input Multiple Output (MIMO) for LTE," (Release 13), 2015.