A Principle Discussion of Edge Detection based on Gaussian Wavelet

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***Abstract*—Edge detection is an image processing technique for finding the boundaries of objects within images. It works by detecting discontinuities in brightness. In this paper, we develop the Canny edge detector, introduce wavelet basis functions of *n*th Gaussian derivative and utilize them to extract the edge. At first, the principle of the edge detection by the wavelet transform is given briefly and new basis of wavelet functions are introduced and admissibility conditions of them are discussed. Then, the theoretical edge detection analysis of three significant edge types (step, ramp and stage) via new wavelet basis functions is studied, and relevant formulas are derived. The theory of the step response in the x, y and arbitrary direction is given and the effect of smoothing filter is obtained. We show that all introduced wavelet functions can detect the breakpoint of the step function. A model of the ramp function is presented and an approximation of it is used to simplify the results.**

***Index Terms*— Edge Detection; Wavelet Transform; Gaussian Filter; Step Function; Wavelet Bases; Smooth Filter; Canny Edge Detector; Multiscale.**

I. INTRODUCTION

Edges as basic features have significant information to understand the content of images and videos. The rule of edge detection is to distinguish homogenous areas and identify the boundary in images and simplify the understanding of the images by the remaining meaningful data. Edge detection is used for image segmentation and data extraction in areas such as image processing and machine vision.

Edge detection, especially the step, ramp and stair edge detection has been widely applied in various different computer vision systems. It is an important technique to extract useful structural information from different vision objects and dramatically reduce the amount of data to be processed. The Canny edge detection algorithm is known to many as the optimal edge detector [20]. Canny's intentions were to enhance the many edge detectors already out at the time he started his work. He was very successful in achieving his goal and ideas. Canny has found that the requirements for the application of edge detection on diverse vision systems are relatively the same.

Multiresolution based on wavelet for the signal and image analysis has received attention in recent years like [1-7] or [8-10]. Multiresolution analysis is a technique that decomposes an image to multi levels to perform the selective edge detection and inhibit the noise [11]. By multiresolution analysis in the low scale, weak edges could be focused and in the higher scale, bold edges could be investigated [12].

Our paper is organized as follows: Section 2 is divided into two parts. The first part briefly discusses the principle of the edge detection by the wavelet transform. The second part introduces the new wavelet bases by developing Canny edge detector. Section 3 deals with the three main edge types (step, stair and ramp) and their responses. In this section, the three types of the edge are modeled and wavelet responses are investigated in the vertical, horizontal and the general direction and relevant formulas are derived.

II. WAVELET TRANSFORM AND EDGE DETECTION

A. Basic wavelet transform

By definition, $\theta(x)$ is considered as a smooth function, if its integral over R is one and tends to be zero in $\pm\infty$. Let $\psi(x)$ be a wavelet function and is defined as the first order derivation of the smooth function $\theta(x)$.

$$\psi(x) = \frac{\partial}{\partial x}\theta(x) \tag{1}$$

$\psi(x)$ would be a mother wavelet function if admissibility condition is satisfied.

$$\int_{-\infty}^{+\infty} |\Psi(\omega)|^2 |\omega|^{-1} d\omega < +\infty \tag{2}$$

where $\Psi(\omega)$ is the Fourier Transform of $\psi(x)$. In 2D dimensional space, we need 2 wavelet functions in the horizontal and vertical direction, which are identified as:

$$\psi_x(x,y) = \frac{\partial\theta(x,y)}{\partial x} \tag{3a}$$

$$\psi_y(x,y) = \frac{\partial\theta(x,y)}{\partial y} \tag{3b}$$

Note that:

$$\theta_s(x,y) = \frac{1}{s^2}\theta\left(\frac{x}{s},\frac{y}{s}\right) \tag{4}$$

Wavelet transforms of *f(x,y)* with respect to $\psi_{s,x}(x,y)$ and $\psi_{s,y}(x,y)$ are defined as:

$$W_{s,x}f(x,y) = f(x) * \psi_{s,x}(x,y) \tag{5a}$$

$$W_{s,y}f(x,y) = f(x) * \psi_{s,y}(x,y) \tag{5b}$$

which are gradient coefficients of smoothing function $\theta(x)$ in x and y directions. Then, the modulus and angle in the scale s are defined as:

$$M_s f(x,y) = \sqrt{|W_{s,x} f(x,y)|^2 + |W_{s,y} f(x,y)|^2} \tag{6a}$$

$$A_s f(x,y) = \operatorname{atan}\left(\frac{W_{s,y} f(x,y)}{W_{s,x} f(x,y)}\right) \tag{6b}$$

B. New Bases Introduction

We used 2D Gaussian filter $g(x,y)$ as the smooth function with Fourier Transform $G(\omega_x, \omega_y)$:

$$g(x,y) = \frac{1}{2\pi\sigma_x\sigma_y} e^{-(\frac{x^2}{2\sigma_x^2}+\frac{y^2}{2\sigma_y^2})} \tag{7a}$$

$$G(\omega_x, \omega_y) = e^{-(\frac{\sigma_x^2\omega_x^2}{2}+\frac{\sigma_y^2\omega_y^2}{2})} \tag{7b}$$

Canny has used the first order derivative of the Gaussian filter as the wavelet function. He introduced three criteria for optimal edge detectors:

1. Better signal to noise ratio. It means that reducing and suppressing of the interfered noise will help to have a cleaner edge detection.
2. Better edge localization. According to this criterion, an edge must be detected as close as its real location.
3. Low fake response. There must be only one response to each real edge and the edge detector should not produce fake edges for a real edge.

We develop this idea to *n*th order derivative of the Gaussian filter. The wavelet functions is derived from $g(x,y)$ in the direction of x and y as:

$$\psi_x^n(x,y) = \frac{\partial^n g(x,y)}{\partial x^n} \tag{8a}$$

$$\psi_y^n(x,y) = \frac{\partial^n g(x,y)}{\partial y^n} \tag{8b}$$

These bases satisfy the admissibility condition and tend to 0 in $\pm\infty$. Fourier Transform of each wavelet function can be calculated as:

$$\Psi_x^n(\omega_x, \omega_y) = (j\omega_x)^n e^{-\left(\frac{\sigma_x^2\omega_x^2}{2}+\frac{\sigma_y^2\omega_y^2}{2}\right)} \tag{9a}$$

$$\Psi_y^n(\omega_x, \omega_y) = (j\omega_y)^n e^{-(\frac{\sigma_x^2\omega_x^2}{2}+\frac{\sigma_y^2\omega_y^2}{2})} \tag{9b}$$

Assume that $g_s(x,y)$ is the smoothing function at the scale s with Fourier Transform of $G_s(\omega_x, \omega_y)$:

$$g_s(x,y) = \frac{1}{2\pi s^2} e^{-\frac{1}{2s^2}(x^2+y^2)} \tag{10a}$$

$$G_s(\omega_x, \omega_y) = e^{-\frac{s^2}{2}(\omega_x^2+\omega_y^2)} \tag{10b}$$

Hence, scaled wavelet bases and their Fourier Transform are defined as:

$$\psi_{s,x}^n(x,y) = s^n \frac{\partial^n g_s(x,y)}{\partial x^n} \tag{11a}$$

$$\psi_{s,y}^n(x,y) = s^n \frac{\partial^n g_s(x,y)}{\partial y^n} \tag{11b}$$

$$\Psi_x^n(\omega_x, \omega_y) = s^n (j\omega_x)^n e^{-\frac{s^2}{2}(\omega_x^2+\omega_y^2)} \tag{12a}$$

$$\Psi_y^n(\omega_x, \omega_y) = s^n (j\omega_y)^n e^{-\frac{s^2}{2}(\omega_x^2+\omega_y^2)} \tag{12b}$$

For example, the traditional wavelet function could be derived as the first derivation of Gaussian smoothing function (i.e. $n = 1$)

$$\psi_{s,x}^1(x,y) = s\frac{\partial g_s(x,y)}{\partial x} = -\frac{x}{2\pi s^3} e^{-\frac{1}{2s^2}(x^2+y^2)} \tag{13a}$$

$$\psi_{s,y}^1(x,y) = s\frac{\partial g_s(x,y)}{\partial y} = -\frac{y}{2\pi s^3} e^{-\frac{1}{2s^2}(x^2+y^2)} \tag{13b}$$

III. PRINCIPLE EDGE RESPONSE

In the previous section, we introduced the developed *n*th order derivative of the Gaussian function as wavelet bases and determined relative formulas in the spatial and frequency domain. We need more information about the behavior of proposed basis functions in different edge types. The next step is the study of the basic edges' detection via wavelet functions. In this section, three significant edges, step line and ramp, which occur frequently in the image will be studied.

A. Step response

Step shape is a basic form, which is considered in many edge detectors [5,13-15] such as [16] which has proposed an optimal edge detector based on the infinitive symmetric exponential filter (ISEF) or Gabor odd edge detector using canny measured criteria [17]. We studied the step response in three cases:

a. Step response in the x direction

Assume the step function lays in the x direction, and we have not done any changes in the y direction. So, in this case, the step function denoted by $u_{-1}(x,y)$ will be:

$$u_{-1}(x,y) = \begin{cases} 0 & x < 0 \\ 1 & x > 0 \end{cases} \tag{14}$$

A general step response will be driven by the convolution of the Heaviside function and wavelet function at an arbitrary scale s, which is denoted by $W_{s,x} f(x)$:

$$W_{s,x} f(x) = f(x) * \psi_{s,x}(x) \tag{15}$$

According to proposed wavelet bases, we have *n*th order derivative Gaussian functions. For every wavelet basis response, which is denoted as $W_{s,x}^n f(x,y)$, we have:

$$\begin{aligned} W_{s,x}^n f(x,y) &= f(x,y) * \psi_{s,x}^n(x,y) \\ &= u_{-1}(x,y) * s^n \frac{\partial^n g_s(x,y)}{\partial x^n} \\ &= s^n \delta(x) * \frac{\partial^{n-1} g_s(x,y)}{\partial x^{n-1}} \end{aligned} \quad (16)$$

For example, the wavelet step response of basis function *n=1* at the scale *s* (traditional Gaussian wavelet step detector), will be calculated as:

$$W_{s,x}^1 f(x,y) = \frac{e^{-\frac{x^2}{2s^2}}}{\sqrt{2\pi}} \quad (17)$$

Because the step function is oriented in the *x* coordination, it has no variation in the *y* direction. Thus:

$$\begin{aligned} W_{s,y}^n f(x,y) &= f(x,y) * \psi_{s,y}^n(x,y) \\ &= u_{-1}(x,y) * s^n \frac{\partial^n g_s(x,y)}{\partial y^n} = 0 \end{aligned} \quad (18)$$

Figure 1(a) shows $W_{s,x}^1 f(x,y)$ response where the step is in the *x* direction in scale *s=1*.

b. Step response in the y direction

In this case, the step function is defined in the y direction as:

$$u_{-1}(x,y) = \begin{cases} 0 & y < 0 \\ 1 & y > 0 \end{cases} \quad (19)$$

And the general step response will be:

$$W_{s,y} f(x) = f(x) * \psi_{s,y}(x) \quad (20)$$

For each wavelet basis, which is denoted by $\psi_s^n(x,y)$, the step edge response will be obtained:

$$\begin{aligned} W_{s,y}^n f(x,y) &= f(x,y) * \psi_{s,y}^n(x,y) \\ &= u_{-1}(x,y) * s^n \frac{\partial^n g_s(x,y)}{\partial y^n} \\ &= s^n \delta(y) * \frac{\partial^{n-1} g_s(x,y)}{\partial y^{n-1}} \end{aligned} \quad (21)$$

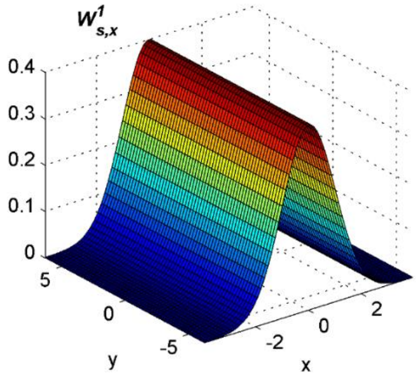
Similar to the *x* direction response, for n=1 we have:

$$W_{s,y}^1 f(x,y) = \frac{e^{-\frac{y^2}{2s^2}}}{\sqrt{2\pi}} \quad (22)$$

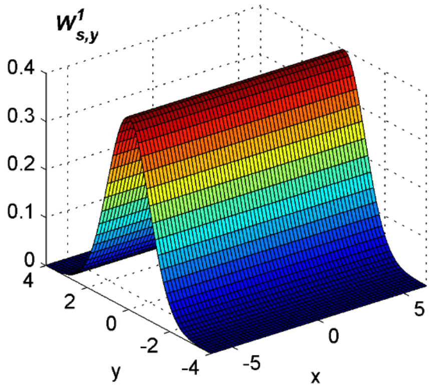
When a step edge is oriented in the *y* coordinate, there is no change in the *x* direction. Hence:

$$\begin{aligned} W_{s,x}^n f(x,y) &= f(x,y) * \psi_{s,x}^n(x,y) \\ &= u_{-1}(x,y) * s^n \frac{\partial^n g_s(x,y)}{\partial x^n} = 0 \end{aligned} \quad (23)$$

Figure 1(b) shows $W_{s,y}^1 f(x,y)$ response to this case in minimal scale s=1.



a)



b)

Figure 1: First order Gaussian wavelet step response where the step is in the x direction (a) and y direction (b) in scale *s=1*

c. General step edge response

In this part, we studied a general form of the step edge, which involves two previous cases. Assume a step function in the Euclidean space, which changes in a line with slope of *m* and passes through the origin:

$$u(x,y) = \begin{cases} 0 & y - mx < 0 \\ 1 & y - mx > 0 \end{cases} \quad (24)$$

Derivation of the general form for the parametric wavelet function n is complicated. For first wavelet step edge response, we have:

$$\begin{aligned} W_{s,x}^1 f(x,y) &= f(x,y) * \psi_{s,y}^1(x,y) \\ &= \frac{-m}{\sqrt{2\pi(m^2+1)}} e^{-\frac{(y-mx)^2}{2(m^2+1)s^2}} \end{aligned} \quad (25)$$

$W_{s,x}^1 f(x,y)$ is related to *m* and *s*. when *m* tends to be zero, the problem changes to the case *a* and when *m* tends to be negative infinity, the problem changes to the case *b*. i.e.:

$$\lim_{m \to -\infty} W_{s,x}^1 f(x,y) = \frac{e^{-\frac{x^2}{2s^2}}}{\sqrt{2\pi}} \quad (26a)$$

$$\lim_{m \to 0} W_{s,x}^1 f(x,y) = 0 \quad (26b)$$

$W_{s,y}^1 f(x,y)$ will be determined as:

$$W_{s,y}^1 f(x,y) = f(x,y) * \psi_{s,y}^1(x,y) = \frac{e^{-\frac{(y-mx)^2}{2(m^2+1)s^2}}}{\sqrt{2\pi(m^2+1)}} \quad (27)$$

When *m* tends to zero, the study case changes to the case *b* and when the step function break line has the slope m=-∞, the problem changes to the case *a*:

$$\lim_{m\to 0} W_{s,y}^1 f(x,y) = \frac{e^{-\frac{y^2}{2s^2}}}{\sqrt{2\pi}} \quad (28a)$$

$$\lim_{m\to -\infty} W_{s,y}^1 f(x,y) = 0 \quad (28b)$$

The absolute wavelet coefficient will be calculated as:

$$W_s^1 f(x,y) = \sqrt{{W_{s,x}^1}^2 f(x,y) + {W_{s,y}^1}^2 f(x,y)} = \frac{e^{-\frac{(y-mx)^2}{2(m^2+1)s^2}}}{\sqrt{2\pi}} \quad (29)$$

$W_s^1 f(x,y)$ value in origin (0,0) where the step function changes, is not dependent on the slope of the line $y = mx$. Figure 2 shows that $W_{s,x}^1 f(x,y)$ and $W_{s,y}^1 f(x,y)$ change reversely at the origin point. Maximum of one leads to the minimum of another. But the absolute value $W_{s,y}^1 f(0,0)$ is independent of *m* and equal to $\frac{1}{\sqrt{2\pi}}$.

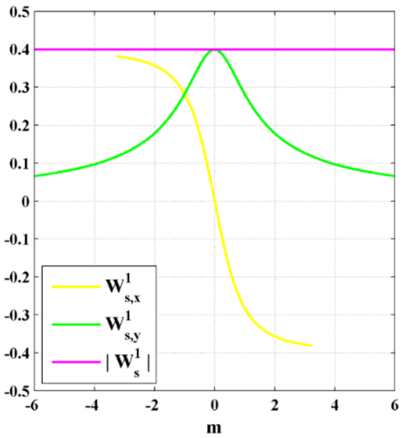


Figure 2: Wavelet coefficients of step edge which are separated with y=mx, in the x direction $W_{s,x}^1 f(x,y)$, y direction $W_{s,y}^1 f(x,y)$ and the absolute value $W_s^1 f(x,y)$ in scale s=1 at origin across the slope m.

d. Smooth step

Smoothing is a technique which is used for the noise removal. Smoothing is a procedure which a signal or image convolves with a Gaussian filter. The degree of the smoothing has a reverse relationship with the qualification. Assume a step edge blurred with a Gaussian filter with variance σ^2:

$$g_\sigma(x,y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}} \quad (30)$$

We have a wavelet coefficient in the *x* direction as:

$$W_{s,x}^n f(x,y) = f(x,y) * g_\sigma(x,y) * \psi_{s,x}^n(x,y) = s^n f(x,y) * \frac{\partial^n}{\partial x^n} g_{\sqrt{s^2+\sigma^2}}(x,y) \quad (31a)$$

Where $f(x,y)$ is the edge function and $\psi_{s,x}^n(x,y)$ is the nth order derivative Gaussian wavelet function. A general form of the blurred edge response in the *y* direction could be calculated as:

$$W_{s,y}^n f(x,y) = f(x,y) * g_\sigma(x,y) * \psi_{s,y}^n(x,y) = s^n f(x,y) * \frac{\partial^n}{\partial y^n} g_{\sqrt{s^2+\sigma^2}}(x,y) \quad (31b)$$

Now we focus on the step response. Assume the step is laid in the *x* direction. The wavelet coefficient of blurred step in the *x* direction is derived as:

$$W_{s,x}^1 f(x,y) = u_{-1}(x,y) * g_\sigma(x,y) * \psi_{s,x}^1(x,y) = s u_{-1}(x,y) * \frac{\partial}{\partial x} g_{\sqrt{s^2+\sigma^2}}(x,y) = s\delta(x) * g_{\sqrt{s^2+\sigma^2}}(x,y) = \frac{e^{-\frac{x^2}{2s^2+2\sigma^2}}}{\sqrt{2\pi}} \quad (32a)$$

Gaussian filter as a pre-processing stage does not change the wavelet coefficient in the *y* direction, and we have:

$$W_{s,y}^n f(x,y) = 0 \quad (32b)$$

If the edge changes in the *y* direction, the coefficients will change conversely:

$$W_{s,y}^1 f(x,y) = \frac{e^{-\frac{y^2}{2s^2+2\sigma^2}}}{\sqrt{2\pi}} \quad (33a)$$

$$W_{s,x}^n f(x,y) = 0 \quad (33b)$$

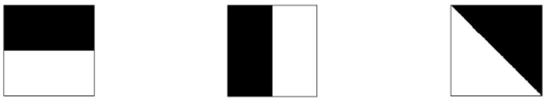
We investigated a general form for $W_s^1 f(x,y)$ where blurred step changes in the y=mx line. Wavelet coefficient in the *x* direction is:

$$W_{s,x}^1 f(x,y) = u(x,y) * g_\sigma(x,y) * \psi_{s,x}^1(x,y) = s u(x,y) * \frac{\partial}{\partial x} g_{\sqrt{s^2+\sigma^2}}(x,y) = \frac{-m}{\sqrt{2\pi(m^2+1)}} e^{-\frac{(y-mx)^2}{2(m^2+1)(s^2+\sigma^2)}} \quad (34a)$$

And in the y direction, Wavelet coefficient is calculated as:

$$W_{s,y}^1 f(x,y) = \frac{1}{\sqrt{2\pi(m^2+1)}} e^{-\frac{(y-mx)^2}{2(m^2+1)(s^2+\sigma^2)}} \quad (34b)$$

If we concentrate on the answers, we will find out the pure and blurred steps have a similar response. Indeed the act of σ of smoothing function on the edge detection is similar to the scale. Figure 3 ilustrates the horizonal, vertical and diagonal step images with wavelet responses using Otsu threshold in scale *s=1*.



a) Directional step edges

b) First order wavelet responses

Figure 3: Horizonal, vertical and diagonal step images with wavelet responses

B. Stair edge response

Stair edge detection is used in many applications in the image processing field, such as road detection. Figure 4 shows a schematic of the stair edge with a height of h_1 and h_2 and the weight of d and the center of x_0. If $h_2 = 0$, the stair will change to a line, and the line edge would be a case of the stair edge.

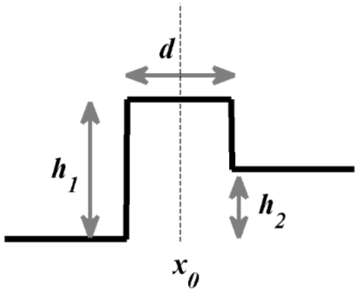


Figure 4: A basic stair edge shape

We discuss the stair edge response in the x direction. Assume the stair edge is denoted by $str(x,y)$ and defined as:

$$str(x,y) = h_1 u_{-1}\left(x + \frac{d}{2}, y\right) - (h_1 - h_2) u_{-1}\left(x - \frac{d}{2}, y\right) \quad (35)$$

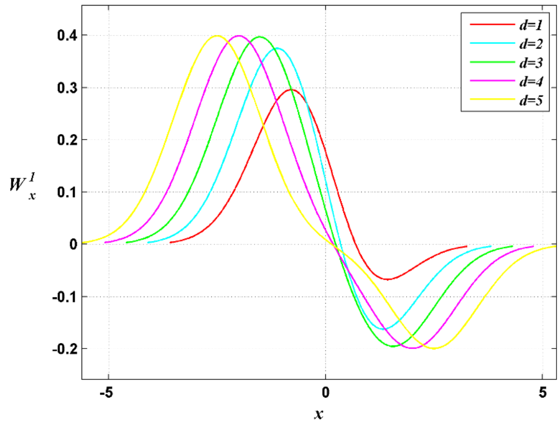
The wavelet coefficient of the nth basis function is calculated by the convolution with stair step:

$$W_{s,x}^{n} str(x,y) = str(x,y) * \psi_{s,x}^{n}(x,y) = h_1 \delta(x) * s^n \frac{\partial^{n-1} g_s\left(x + \frac{d}{2}, y\right)}{\partial x^{n-1}} + (h_2 - h_1)\delta(x) * s^n \frac{\partial^{n-1} g_s\left(x - \frac{d}{2}, y\right)}{\partial x^{n-1}} \quad (36)$$

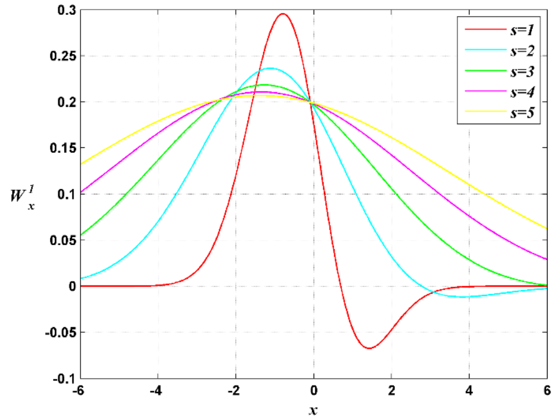
For example, if n=1, $W_{s,x}^{1} str(x,y)$ will be obtained by:

$$W_{s,x}^{1} str(x,y) = \frac{h_1 e^{-\frac{\left(x+\frac{d}{2}\right)^2}{2s^2}}}{\sqrt{2\pi}} + \frac{(h_2 - h_1) e^{-\frac{\left(x-\frac{d}{2}\right)^2}{2s^2}}}{\sqrt{2\pi}} \quad (37)$$

$W_{s,x}^{1} str(x,y)$ across x is shown with different distance d and scale s=1 in Figure 5a and different scale s and distance d=1 in Figure 5b.We find out that it is not possible to detect centric point x=0 in any diagram because x=0 is a normal point, which no extrema or zero-crossing happened at this point. According to Figure 5b, the scale variation does not change the situation. The centric dislocation in Figure 5a decreases with increasing d where the two steps lead to independence.



a)



b)

Figure 5: (a) Stair edge response with different distance (b) Stair edge response with different scale

Figure 6 shows a stair image with first order wavelet response at scale *s=1*.

C. Ramp edge response

Another edge type discussed here is the ramp edge. We used a ramp approximation function to simplify the results. Assume $g_\sigma(x,y)$ is a Gaussian smoothing function which is identified as:

$$g_\sigma(x,y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}} \quad (38)$$

If the step function is laid in the x direction, the blurred result will be:

$$u_{-1}(x,y) * g_\sigma(x,y) = \frac{\operatorname{erf}\left(\frac{x}{\sqrt{2}\sigma}\right) + 1}{2} \tag{39}$$

where $erf(x)$ is the error function [18]. The slope of output in origin will be driven as [19]:

$$slope = \frac{\partial}{\partial x} u_{-1}(x,y) * g_\sigma(x,y)\Big|_{x=0} = \frac{1}{\sqrt{2\pi}\sigma} = \frac{1}{\Delta L} \tag{40}$$

where the slope would be adjusted with *sigma*. We define the function $y(x,y)$ as a ramp edge:

$$y(x,y) = \begin{cases} 1 & \sigma\sqrt{\frac{\pi}{2}} \le x \\ \frac{x}{\sqrt{2\pi}\sigma} + \frac{1}{2} & -\sigma\sqrt{\frac{\pi}{2}} \le x \le \sigma\sqrt{\frac{\pi}{2}} \\ 0 & x \le -\sigma\sqrt{\frac{\pi}{2}} \end{cases} \tag{41}$$

Consider the blurred step function $r(x,y)$ as a ramp edge $y(x,y)$ approximation:

$$r(x,y) = u_{-1}(x,y) * g_\sigma(x,y) \cong y(x,y) \tag{42}$$

In this situation, wavelet coefficient will be obtained by:

$$W^1_{s,x} r(x,y) = r(x,y) * \psi^1_{s,x}(x,y) = \frac{e^{-\frac{x^2}{2s^2+2\sigma^2}}}{\sqrt{2\pi}} \tag{43}$$

As mentioned in Section III, $W^1_{s,y} r(x,y) = 0$ here. If the ramp edge be in y direction, the formulas will change conversely:

$$W^1_{s,y} r(x,y) = \frac{e^{-\frac{y^2}{2s^2+2\sigma^2}}}{\sqrt{2\pi}} \tag{44a}$$

$$W^1_{s,x} r(x,y) = 0 \tag{44b}$$

In general form of the Eq 24, wavelet coefficients in the *x* and *y* direction will be driven as:

$$W^1_{s,x} f(x,y) = \frac{-m}{\sqrt{2\pi(m^2+1)}} e^{-\frac{(y-mx)^2}{2(m^2+1)(s^2+\sigma^2)}} \tag{45a}$$

$$W^1_{s,y} f(x,y) = \frac{1}{\sqrt{2\pi(m^2+1)}} e^{-\frac{(y-mx)^2}{2(m^2+1)(s^2+\sigma^2)}} \tag{45b}$$

Where these results are similar to the blurred step response. Figure 6 ilustrates a ramp image with first order wavelet response at scale *s=1*.

IV. CONCLUSION

Since edges often occur at image locations representing object boundaries, edge detection is extensively used in image segmentation when we want to divide the image into areas corresponding to different objects. In this paper, a new group of wavelet basis functions were introduced in the edge detection issue. We evaluated the introduced bases in the step, ramp and stair functions as three main edge types and relevant formulas are derived. A smooth function as a pre-processing stage was used before the step detection and the results were obtained in the arbitrary direction.

a) Basic stair and ramp images

b) First order wavelet responses

Figure 6: Stair and ramp images with wavelet responses

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